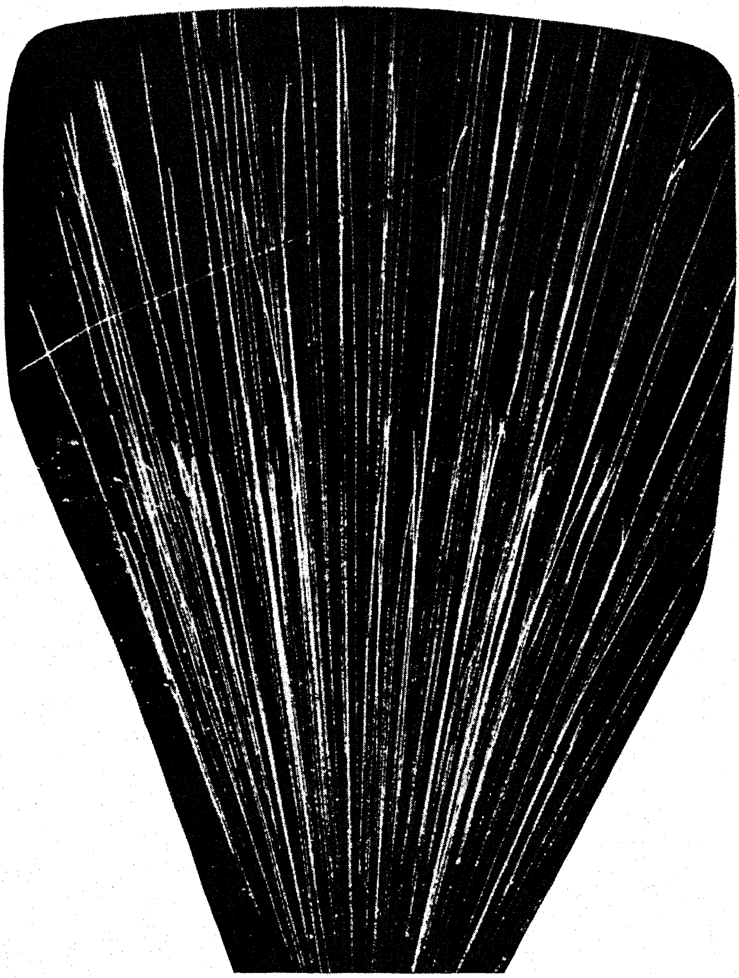
AN ELEMENTARY SURVEY OF MODERN PHYSICS

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO · DALLAS
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY
OF CANADA, LIMITED
TORONTO



EVIDENCE

In regard to the atoms from which these tracks diverge; in regard to the particles producing the tracks; concerning millions of minor accidents to atoms; about some rather serious accidents to a few; and regarding one catastrophe in which a helium nucleus drives into an inner recess of a nitrogen atom and causes a hydrogen nucleus to be ejected with great velocity in order to make way for the intruder.

The first photograph to show this particular kind of atomic warfare. We give it a more polite name. We call it atomic disintegration. It is the beginning of the Transmutation of the Elements. (Chapter 12, page 260.) (By permission of Blackett and Lees and courtesy of the Royal Society.)

AN ELEMENTARY SURVEY OF MODERN PHYSICS

BY

GORDON FERRIE HULL

PROFESSOR OF PHYSICS
DARTMOUTH COLLEGE

[redacted]

NEW YORK
THE MACMILLAN COMPANY
1936

Published June, 1936

SET UP AND ELECTROTYPED BY
THE LANCASTER PRESS, INC., LANCASTER, PA.

PRINTED IN THE UNITED STATES OF AMERICA
BY THE FERRIS PRINTING CO., NEW YORK

To
My Wife

PREFACE

The domain of physics has become so great that only a very small part of it can be covered even superficially in a one-year college course. It is to furnish material for a second-year course that this text has been written. However, emphasis has been placed not on the computational or technical side but on the development of the atom idea, the methods of estimating dimensions and properties of atoms, of electrons; tentative proposals regarding the structure of the atom; a study of photons; a survey of a few of the vast number of interactions between matter and radiation.

Every year for many years past the writer has given to seniors and graduate students a course originally called Modern Electrical Theory. But as the electron is everywhere in the physics of today, that course, changing from year to year, became one in Modern Physics. In that course mathematical operations were not avoided, but in a great number of cases they could be greatly simplified as compared with the original presentation. Finally it has appeared that, taking for granted some of the results of such operations, a very large part of modern physics can be presented without involving the student in mathematical difficulties. This necessitates, however, omitting reference to the work of Heisenberg and of Dirac and giving but the briefest introduction to that of Schroedinger.

The writer has no apology to offer for making the presentation very elementary. Experience has taught him that all students of physics, even advanced students, can profit by a statement of experimental fact and of physical principles in clear, direct terms. It is his hope that this feature will characterize this text.

Acknowledgments for permission to use photographs or figures as basis for drawings are due to the following:

Macmillan Company, a part of sodium absorption spectrum from Wood's *Physical Optics*.

Van Nostrand Company, in part Figures 9, 10, 11, 12, 16, of Chapter 7, from Compton's *X-Rays and Electrons*.

McGraw-Hill Company, data from Bacher and Goudsmit's *Atomic Energy States*.

One figure, 6–6, is based on a figure in Pauling and Goudsmit's *Structure of Line Spectra.*

Dr. Bainbridge for photographs of mass spectra.

Julius Springer, Berlin, for photographs and drawings from *Zeitschrift für Physik, die Ergebnisse der exakten Naturwissenschaften und die Naturwissenschaften.*

Professor H. E. White and McGraw-Hill Company for photographs illustrating the hydrogen atom.

The University of Toronto Press and H. Grayson Smith and F. G. A. Tarr for the illustration representing the superconductivity galvanometer.

The Cambridge University Press for the photographs, Figures 11–2 and 12–1.

The Royal Society and Professor Blackett for the frontispiece.

Professor Chadwick for photograph, Fig. 12–5.

S. Hirzel, Leipsic, for photographs from the *Physikalische Zeitschrift.*

Professors Eve and Keys for two diagrams from their text, *Applied Geophysics.*

The author is indebted to Dr. Dunning and Professor Pegram for data in regard to the experiments now being carried on by them and their associates; to Professor H. E. White for original photographs of "the hydrogen atom"; to Dr. M. S. Livingston for original photographs of the apparatus used by Professor Lawrence and himself, and also to Dr. Livingston for discussions in regard to Chapters 12 and 14; to Mr. Sanborn Brown for making many of the drawings.

G. F. H.

HANOVER, N. H.

TO TEACHERS

May the author be allowed to make some suggestions to teachers? Different groups of students may be interested in modern physics; those who are majoring in English, that they may get some appreciation of the modern world of science; those who are going into journalism, that they may report with reasonable accuracy the proceedings of a scientific convention; students of philosophy, that they may confound their opponents; certainly those looking toward chemistry or physics as their life work. It would be a rare teacher indeed who could interest equally all such students in all of the topics presented in this text. Yet every topic has many bonds connecting it with other topics, in some of which even the apparently listless student may have a lively interest. And it may be the highest art in teaching to seek out that lively interest of the student and to broaden, deepen, and encourage it. Modern physics must appeal to different kinds of minds.

For the reasons just stated, examinations should contain optional questions which will allow for difference in interests. When the class is small enough, it might be best to allow the students to summarize the arguments for or against a theory, or to bring out the points of excellence in an experiment.

And this brings the author to urge that demonstrations of experiments be given to as large an extent as possible. The glass spheres in mercury vapor, the Brownian movement, the oil drop, magnetic and electric deflection of electron streams, X-rays, even the inspection of the X-ray plant in a well-equipped hospital, electron tubes, thyratrons, Geiger counters and cosmic rays—students will find interest in all of these. The author was astonished to find that there was an unusual interest in his blowing and mounting extremely fine quartz fibers. There is no danger that these excursions into the experimental realm will detract from the vigorous pursuit of the arguments leading to the formation of a great theory. Rather they emphasize the qualities which have led to the building of the vast experimental structure upon which theories must be based.

Books may be read almost anywhere, but students may not become acquainted with scientific apparatus outside of a well-equipped laboratory.

In order to give the more advanced students some of the theoretical aspects of the subject, a few mathematical derivations are given in the Appendix. It will be observed that some of these operations are quite unorthodox and the author is well aware that they may call forth criticism from the exalted manipulators of mathematical symbols.

Similarly in the body of the text, Chapter 3 gives a simple unorthodox method of dealing with orbits. The question may arise why such a topic should receive attention. The answer is as follows: Generally the discussion of "orbits" is involved in great mathematical difficulty. But here, after assuming the relation for the work done by gravitational or electrical force, we use only simple algebra. And it ought to be a matter of interest to students to see how the motion of the smallest particle that we know in physics—inconceivably small even in comparison with a grain of sand—to see how the motion of that kind of particle is similar to or differs from the motion of a vast mass like that of Jupiter about the Sun. Bohr became an immortal in physics because, though having great mathematical power, he had also a clear physical concept of one of the properties of orbits—a property that ordinarily is hidden in mathematical symbols. Rutherford added greatly to his fame by the discovery of the *nuclear* atom and again the properties of an orbit—this time a hyperbolic orbit—were made use of. In this chapter the teacher ought to be completely at home in dealing, in a most elementary manner, with conic sections.

The teacher of modern physics—indeed the teacher of any aspect of physics—is unusually favored among mortals. For no other subject can have an equal appeal to the imagination, no other subject deals with such clear, concise ideas. Yet none can have a greater philosophic breadth. To assist in presenting these various points of view this text is written.

CONTENTS

CHAPTER 1

PAGE

MOLECULES—IN SWARMS............................ 1

Law of the conservation of energy of linear momentum; fundamental principles.——Illustrations: pendulum; impact of two spheres; rifle bullet and block of wood.——Loss of motional (kinetic) energy; this loss taking the form of heat.——Mercury vapor and glass spheres.——Pressure of the vapor, speed of the molecules, $p = \frac{1}{3} nmu^2$.——What is meant by this speed?——Impossible for all molecules to have same speed.——Law of distribution of speeds, Maxwell's law.——Most probable speed (or speed of most of the molecules); experimental confirmation; average speed, root-mean-square speed, temperature.——Root mean square speed of hydrogen—1.2 miles per second at ordinary temperatures; of oxygen, one-fourth $(1/\sqrt{16})$ of this. ——Brownian movement; particles in water, smoke in air; Einstein's theory and Perrin's experiment; precision out of chaos.——Avogadro's number from the Brownian movement; from the law of atmospheres.——Diameter of a molecule from departure from Boyle's law; mean free path; coefficient of viscosity.——Tables.

CHAPTER 2

WE DISCOVER AND WEIGH THE ELECTRON. WE WEIGH ATOMS AND DISCOVER ISOTOPES....... 21

The atom of matter and the atom of electricity.——Electrolysis, electric current, definition.——Ratio of electric charge to mass of carrier in electrolysis.——Early attempts to compute the amount of electricity in the electric atom; name "electron" given to this electric atom, 1874.——Passage of electricity through a gas; cathode rays.——Discovery of the "electron." ——Force on an electrified particle due to electric and magnetic field.——Measurement of v and e/m, 1897.——Measurement of electron charge; oil drop experiment, Millikan.——Stokes' law and correction.——Results of measurement of e; *accurate* value of N, Avogadro's number.——Rather accurate value of *average* mass of atoms of elements.——All neutral atoms contain electrons and *something else* which is positively charged and contains nearly all the mass.——The "pumpkin" atom.——More accurate measurement of mass of electron; it depends upon speed when this approaches that of light.——Momentum no longer proportional to velocity, nor kinetic

PAGE

energy to v^2.——Velocity selector applied to atoms; very accurate measurement of the mass of an atom; *isotopes*, J. J. Thomson, *Aston*, Bainbridge.——Extraordinary precision of measurement of mass of atoms shows that there is only one kind of O 16, of Ne 20 of Ne 21, and of Ne 22, etc.——Consequences of discovery of isotopes; there may be several hundred "pure" kinds of a very simple compound; elements of odd atomic number have about two isotopes each; of even, several.——Analysis of photographs of mass lines and of mass parabolas.

CHAPTER 3

SIMPLE PROPERTIES OF ORBITS; THE DISCOVERY OF THE NUCLEAR ATOM........................ 38

A mass falling towards the earth; motional (kinetic) and positional (potential) energy.——A gondola weighing one ton on the earth's surface weighs 1990 pounds when ten miles high.——Force = GMm/r^2.——How we may fire a bullet round the earth, necessary speed = 5 miles/sec. At 6 miles/sec. an ellipse is described.——At 7 miles/sec. a parabola (the velocity of escape).——At 8, a hyperbola.——Energy depends only on the major axis.——A very elementary way of considering the motion of Jupiter about the sun or of an electron about a positive charge.——One way in which the solar system could *not* have been formed.——In gravitation, force is always attraction, in electricity sometimes repulsion.——What happens when a positive charge is fired at or towards another positive charge?——Alpha particles as projectiles, atoms as targets.——The nuclear atom is discovered.——A "pumpkin" atom versus a nuclear atom (the botanical analogy would be a ripened dandelion flower, feathery material outside, heavy inside).——A positive core versus a negative core.——The diameter of the core of a gold atom is about 3×10^{-12} cm. How do we know?

CHAPTER 4

RADIATION.. 49

Radiation, a vast and wonderful domain.——Light—corpuscles or waves?——What do we mean by the "velocity" of light?——Why is it an important quantity?——Wave velocity and group velocity.——How do we account for the pressure of light?——Experimental determination of this pressure.——Light possesses momentum.——In some ways it behaves like a perfect gas.——Temperature radiation.——"How much" red, yellow, blue, light comes from a furnace at a certain temperature?——Is there distribution of energy in light as there is of energy in the molecules of a perfect gas?——

PAGE

Is light continuous or granular?——Planck forced to make an unwelcome assumption in order to "explain" an experimental fact.——Other attempts, more heroic, less successful.——The beginning of the quantum theory.——Einstein's contribution.

CHAPTER 5

THE PHOTOELECTRIC EFFECT........................ 65

Discovery of photoelectric effect by Hertz.——Electrons driven off a metal plate by light.——Ratio of charge to mass the same as for cathode rays.——Method of measuring the velocity of the escaping electrons.——A useful optical instrument—a constant deviation spectroscope.——Maximum kinetic energy of the electrons depends only on the *frequency* of the light.——Einstein's relation and its verification. Accurate measurement of h.——The energy of the electrons does not depend on the intensity of light.——Threshold frequencies.——Explanations based on the wave theory fail.——Photons; bullets of light, "explain" the photoelectric effect but fail to explain interference of light.——What is our picture of a photon?——Impossible to split a photon.——Impossible to measure the "dimensions" of a photon.——Some photoelectric phenomena require both points of view, that of an electromagnetic wave and that of a bullet of light.——Photons may have meridional planes of symmetry.——Sunfish photons.——Experimental details.——Enormous variations in photoelectric sensitivity.——Photo-electric, -resistant, -voltaic, photronic cells.

CHAPTER 6

ATOMIC SPECTRA AND THE BOHR THEORY......... 86

Spectra the thumb prints of atoms.——They may be exceedingly complex; therefore also the atom.——Accuracy of measuring wave lengths reaches nearly the limit of human precision.——Perplexities regarding the nuclear atom.——In one way we can prove that the electrons cannot be in motion, in another way that they cannot be at rest.——Then comes Niels Bohr who gets rid of one of the perplexities by simply denying that it exists.——Submicroscopic astronomy with orbits made to order.——Assumptions, correct from the point of view of dimensions, made with an eye to results.——Experiment (spectroscopy) directing the formation of theory and theory suggesting new experiments.——Astonishing success!——Measurements from various parts of physics gathered together to obtain a long-known optical constant.——But the "simple" Bohr theory is too simple.——More new astronomy.——More triumphs!——But does an electron spin? If so, why?——

PAGE

Simple atom models.——Then models which are not simple. ——Then abandonment of astronomical models.——Spectroscopists "don't speak our language."——Energy levels.——Algebraical atoms, the terms determined by spectroscopy. Success and failure of the Bohr theory.

CHAPTER 7

X-RAYS 105

Discovery of X-rays, 1895.——X-ray tubes old and new.——A dream as old as the human race partially realized; hidden things are brought to light.——Science leaps forward.——But what are X-rays?——Theories; longitudinal waves in the ether; pulse in the ether; Bragg's doublets; transverse waves in the ether; bullets of radiant energy, quanta, photons.——Crystals used to measure wave lengths, Laue and Bragg methods.——Moseley's historic work.——*Atomic numbers.*——Identification or discovery of elements by their X-ray spectra. ——A new physics, new chemistry; new crystallography; new metallurgy; new surgery; new industries.——Moseley's law compared with Bohr's formula.——Characteristic and general radiation.——Maximum frequency given by $hf = Ve$.——Scattered X-rays; pulse theory accounts for polarization; also gives right number of "loose" electrons (6 for carbon), but fails for heavy atoms.——Absorption, abrupt changes, accounted for on quantum theory; "breaks" in the absorption curve give "levels" in the atom.——Bombardment of electrons by photons; Compton effect.——Modified and unmodified lines for different elements.——Pressure of light again explained.——Impossible for a *free* electron to absorb the entire energy of a photon; impossible for a *bound* electron to emerge with all the energy of a photon, since the atom must share in the energy. What then is a photoelectron? And what about Einstein's relation for photoelectrons, $\frac{1}{2}mv^2 = hf - w$?——Reflection and refraction.——X-rays finally proved to possess all the properties of light.——Extraordinary range of law of dispersion based on old wave theory.——Electric waves to X-rays.——A range of a million million million.——History of X-rays of outstanding importance.——Properties of a photon.——Some experimental details; Bragg's spectrometer; the powdered crystal method.

CHAPTER 8

ELECTRON TUBES 135

Electron tubes; of extraordinary scientific and commercial importance.——Why do electrons come from hot metals?——Richardson's views; thermions, laws of emission; modern

PAGE

views; Fermi-Dirac law.——Extraordinary influence of surface layers of atoms; emission changes by factor of 100,000,000 due to temperature or to surface condition.——Parts of an electron tube.——Simple experiments; electric currents without batteries.——Currents limited by filament temperatures, plate voltages, space charges.——Amplification factor due to the grid.——Rectifiers, amplifiers, oscillators.——Barkhausen-Kurtz oscillators; Lecher wires.——By means of thyratrons we can control currents of large power and high voltage.——Oscillograph tubes, sweep circuits.——Millionfold extension of current measurement.

CHAPTER 9

ELECTRICAL PHENOMENA IN GASES AND SOLIDS... 170

Collisions of electrons with atoms; elastic, inelastic. An electron may lose none of its energy if this energy is smaller than a certain value, then it loses all; "resonance," ionization, recombination; the Franck and Hertz experiment.——Collisions of photons with atoms.——Absorption of quanta of the right kind; "pilfering" collisions between atoms.——Strange behavior of excited atoms; an electron striking an excited atom produces an explosive mixture; photons do not collide with photons.——Cloud tracks due to alphas, betas, gamma rays.——Electrical phenomena in gases; grid-glow tubes, voltage regulators; high temperatures of electrons in a cold tube.——Modes of testing electrical conditions in a gas.——Luminous intensity due to one gas greatly altered by admitting a trace of another gas.——Can we produce "light without heat"?——Electrical phenomena in solids.——The extraordinary property of electrical resistance.——Superconductivity.——Currents of hundreds of amperes flowing for days with no source acting, no heat produced.——Low temperatures, how obtained and measured.——Extraordinary interrelation between magnetic field, temperature, and superconductivity.——World's record to date 0.0044° absolute.

CHAPTER 10

THE ZEEMAN AND RAMAN EFFECTS............... 204

Influence of a magnetic field on a light source; the Faraday effect (1845); the Zeeman effect (1896).——Explanation; on the basis of the wave theory the picture is very satisfactory—at first—but not complete.——New point of view; the quantum idea.——Explanation—complete—but no picture.——The Gerlach and Stern experiment.——Another effect (Raman) which cannot be explained on the classical, but is explained on the quantum, theory.——Band spectra so explained.——The discovery of isotopes by spectra.——Raman spectra of liquids and

PAGE

gases.——Models of molecules.——How may the parts of a molecule move?——Energies due to rotation and to vibration. ——The specific heat of a gas explained on the quantum theory.

CHAPTER 11

RADIOACTIVITY: SPONTANEOUS TRANSMUTATION OF ELEMENTS.................................... 236

Radioactivity, discovery, 1896.——Radium, 1898; three radiations.——Methods of analysis.——Disintegration of atoms; probability of disintegration; half-life.——The element helium frequently given off.——Counting atoms; the Geiger counter. ——Energies of the particles.——Energy required to ionize an atom.——Rise and fall of activity.——Tables of data showing the complexity of the phenomena; radioactive families.—— Isotopes.——Alpha ray spectra.——Beta ray spectra, line spectra from the outer electrons, continuous spectrum from the nucleus.——The nucleus of only 10^{-13} cm. diameter has *energy levels.*

CHAPTER 12

THE BEGINNING OF ARTIFICIAL TRANSMUTATION 255

Alpha particles used as projectiles.——Elastic and inelastic collisions.——How we may identify particles by their cloud tracks.——A nitrogen atom capturing an alpha particle is forced to shoot out a hydrogen nucleus.——Rutherford's early experiments, 1919. Blackett's and Lees' photographs.—— Cockcroft and Walton's historic experiment, the first artificial transmutation.——Mass transformed into energy.——The discovery of the neutron (1932).——The discovery of heavy hydrogen (1932).——Methods of obtaining high velocities; new atom guns.——The cyclotron of Lawrence and Livingston. ——The new Van de Graaff electrostatic generator of enormous dimensions.——Ingenious application of this idea by Tuve, Hafstad, and Dahl.

CHAPTER 13

COSMIC RAYS.. 277

Early history.——High altitude measurements; great increase in intensity with height.——Three methods of testing and measuring: (1) the electrometer, (2) the cloud track, (3) the G-M counter.——A telescope for cosmic rays.——Photons or particles?——Enormous energies.——The magnetic effect; data compared with theory.——Great increase of magnetic effect with altitude, proving that practically all the rays are electrified particles. Positive particles predominate.——Discovery of the

PAGE

positron.——Apparatus sent up to the top of the atmosphere. ——Cosmic rays are not neutrons.——Do they come from a region 10^{10} light-years away?——Dilemmas.

CHAPTER 14

TRANSMUTATION OF THE ELEMENTS. INDUCED OR ARTIFICIAL RADIOACTIVITY 307

The discovery of induced radioactivity by the Joliots, 1934. ——New projectiles used for bombardment, protons, deuterons, neutrons.——The game of transmutation.——The chemical elements are now jumping up and down—but not far.——We cannot build heavy atoms out of hydrogen.——Strange properties of neutrons; slow neutrons effective projectiles.——Slow uranium used to produce fast uranium and element 93.——Potential barriers around, and energy levels in, the nucleus.—— Structure of atoms; protons and neutrons.——New table of the elements.——Mass has been changed to energy; we now change energy to mass, but with limitations.

CHAPTER 15

WAVES, PARTICLES, NEW ATOM PICTURES........ 337

A photon has momentum and frequency.——A particle has momentum; has it also frequency?——De Broglie's relation. ——The principles of least time and of least action.——The Davisson and Germer experiment.——Wave speeds and particle speeds.——Photographs taken by electrons and X-rays almost identical.——De Broglie's atom.——Schroedinger's equation. ——What do we put into and what do we get out of a mathematical equation?——The latest photograph of the hydrogen atom.

CHAPTER 16

LIMITATIONS IMPOSED BY NATURE—THE UNCERTAINTY PRINCIPLE............................ 358

Limitations imposed by nature due to the wave character of (a) light, (b) electrons; due to the unit nature of (a) electricity, (b) action.——The uncertainty principle, of philosophical importance.——Limitation in velocity of light.——The Michelson-Morley experiment and Miller's extension.——Explanations; the Fitzgerald-Lorentz shortening; the Lorentz equations. ——Concerning the theory of relativity: its success in two points.——Is the universe expanding?——Is space curved?—— The newest relativity theory.

CHAPTER 17

PAGE

MODERN APPLICATIONS OF PHYSICS 379

The all-metal radio tube. Amplification of 10^{1800} in a telephone circuit; 4,000 telegraph messages in the same wire.——Direction finder; audiometer; stethophone; photoelectric cells; the photronic cell.——The piezo-electric effect; control of radio station frequency; measurement of pressure in a gun.——Geophysical prospecting; gravity apparatus; the Eötvös torsion balance; seismic methods.——X-rays in industry, in surgery, in biology, in producing mutation.——Electron optics; electron lenses; images formed by electrons; methods of concentrating electrons.——The top of the world.

APPENDIX 405

1–1. Light. The intensity of light passing through an absorbing material decreases in a geometrical ratio, $I = I_0 e^{-\mu x}$. The number of photons follows the same law, $N = N_0 e^{-\mu x}$.——**1–2.** Atmospheres. The pressure of the atmosphere decreases with height in the same way, $p = p_0 e^{-\rho g h}$.——**1–3.** Energies. The number of the particles of a gas having *velocities* between v and $v + 1$ may be put in the form $N = \text{Const. } e^{\frac{-E}{kT}}$ where E is the kinetic energy of a particle.——**1–4.** Electron Densities. The electron densities in two adjacent regions is $N = N_0 e^{\frac{-(V-V_0)e}{kT}}$ where V's are the potentials.——**1–5.** Electron flow from a condenser.——**1–6.** To find the value of $\int_0^\infty e^{-x^2}dx$.——**1–7.** To find the law of the distribution of speeds from that of velocities.——**1–8.** To find Avogadro's number by observing Brownian movements.——**2–1.** Stokes' law correction.——**3–1.** The radius of curvature of an ellipse at the extremity of an axis.——**3–2.** The radius of curvature of a parabola at the vertex.——**3–3.** The law of equal areas.——**3–4.** To find the period in an elliptic orbit. Kepler's third law.——**3–5.** To find the energy in an elliptic orbit.——**3–6.** To find the energy in a hyperbolic orbit.——**3–7.** Orbits from the point of view of energy.——**3–8.** The scattering of alpha particles by a thin film.——**4–1.** The adiabatic expansion of radiation gives $pv^{4/3} = \text{constant}$.——**4–2.** Tentative laws of black body radiation.——**4–3.** To derive Wiens relation $\lambda_m T = 0.2885$ from Planck's formula.——**4–4.** To derive the Stefan-Boltzmann relation $E = aT^4$.——**6–1.** Comparison of Bohr's circular orbits.——**6–2.** Ratio of magnetic to mechanical moment.——**6–3.** Concerning fine structure.——**6–4.** The torque in a beam of circularly polarized light.——**7–1.** Concerning mass and energy. The derivation of $E = mc^2$.——**7–2.** The Compton effect.——**7–3.** The dis-

PAGE

tances between atom planes for rock salt.——8–1. Current controlled by space charge. $i = \frac{CV^{3/2}}{x^2}$.——8–2. The frequency of the Barkhausen-Kurtz oscillations.——9–1. The distribution of energies.——9–2. Kinetic energy of electrons; voltage and temperature.——9–3. Vapor pressure of helium at low temperatures.——10–1. The Zeeman effect derived from the quantum theory.——10–2. Change in major axis of an elliptical orbit due to velocity change.——10–3. The Stark effect.——10–4. The rotational energy of a molecule.——10–5. Vibrations of a molecule. Identification of isotopes from band spectra.——10–6. The g factor.——11–1. The law of radioactive decay.——12–1. Comparison of the Lyman series of the hydrogens H^1 and H^2.——12–2. Head-on collision of (a) a photon, (b) an electron, (c) a proton, with a particle.——13–1. Curvature of path of a high-energy electron in a magnetic field.——13–2. Circuits for cosmic ray counters.——14–1. Table of atom masses computed from transmutations.——15–1. The Schroedinger equation; simple derivation; transformation to spherical coordinates; methods of solution for the case of the hydrogen atom.——15–2. The Cavendish Society's songs.——15–3. "π to Thirty Places."——16–1. Uncertainty of position of an electron.

General—Nobel Prize Men in Physics.

General—Table of Physical constants.

INDEX.. 453

INTRODUCTION

"The essence of law is not logic, the essence of law is experience." Thus has spoken recently an eminent American justice. Similarly an eminent scientist has weighed the claims of scientific theory and experiment. In his presidential address before the British Association for the Advancement of Science (1934), Sir James Jeans, himself a theoretical astrophysicist, compares modern physics with the science of fifty years earlier. "In that interval the main edifice of science has grown almost beyond recognition, increasing in extent, dignity, and beauty, as whole armies of laborers have patiently added wing after wing, story upon story, and pinnacle to pinnacle. Yet the theoretical physicist must admit that his own department looks like nothing so much as a building which has been brought down in ruins by a succession of earthquake shocks. The earthquake shocks were, of course, new facts of observation, and the building fell because it was not built on the solid rock of ascertained fact, but on the ever-shifting sands of conjecture and speculation."

It is frequently stated even in educational circles that the building that has fallen is that of ascertained fact. Nothing could be further from the truth. The long-established experimental facts of physics are still established experimental facts. True, many new phenomena have been discovered and some theories have been altered or extended to be in agreement with new facts. But that is what a theory is—an elastic synthesis of experimental results. And since we can hardly hope, for many generations yet or even for eons, to possess all knowledge, we may expect theories to continue to be in a state of flux.

The term Modern Physics must always be a relative one. At present it generally implies a study of the phenomena which were brought to light near the close of the last century, phenomena including and resulting from the discovery of electric waves, of the X-rays, of the electron, of radioactivity, together with the theories which have been evolved in an attempt to bring all of these phenomena into a common philosophy. Atoms, electrons, photons, and the very new units of mass and electricity, neutrons

and positrons, the building bricks of matter, electricity, radiant energy, these, so far as we can see, form the universe and these will be our chief topics of study. A colossal program! Ours can be only a very elementary survey. In all cases the experimental viewpoint will be stressed. For example, there will be presented the experimental basis for our view that matter possesses a dual nature—of particles and waves; and that light possesses a dual nature—of waves and particles. And it will be shown that whatever theories we may adopt, they must be evolved under the strenuous compulsion of experimental fact.

The history of science, certainly since the time of Galileo, shows that if we would make progress we must make measurements. During his time and for two centuries or so after, the quantities to be measured were for the most part those of ordinary everyday affairs—the period and length of a pendulum, the speed of a ball rolling down a plane, forces comparable to those experienced by men. But after the telescope came into use, comcomputations based upon some ordinary measurements were made; for example, of the diameter and mass of the earth, of the distances to the planets, of the velocity of flight. These were not of the dimensions of human distances, masses, velocities; they differed in the direction of largeness. It is only in rather recent years that scientists have gone in the other direction. But in both cases we carry over, with certain alterations, the methods, the arguments, and the physical principles used and established in the more ordinary realm. Among the alterations found necessary in the microscopic world, one is outstanding, that energy changes are not continuous—they occur in steps, jumps, abrupt variations. This is one of the most important features of the new physics.

In order that the student may have a most concise summary of the discoveries and developments in physics since 1895, the following list, nearly chronologically arranged, is presented. Obviously if these topics were treated with any fullness the presentation would require very many volumes. But the student who gives some thought to the topics here presented will conclude that whatever may be said regarding the universe with which astronomers deal, that of physics is not only vast but rapidly expanding.

MODERN PHYSICS: A CHRONOLOGICAL PREVIEW

Discovery of X-rays by Roentgen, 1895; of uranium rays by Becquerel, 1896; of the electron by J. J. Thomson, 1897; of radium by Pierre and Madame Curie, 1898.

Spectral lines shown to be influenced by a magnetic field; Zeeman, 1896

First proposal of the "quantum" idea in light; Planck, 1900.

Experimental discovery of the pressure of light; Lebedew, Nichols and Hull, 1901–1903.

Richardson's work with "thermions" 1903; the beginning of the electron tube.

Special theory of relativity and the extension of the quantum idea; Einstein, 1905.

The invention of the screen grid electron tube; DeForest, 1907.

The liquefaction of helium; Kamerlingh Onnes, 1908.

Fairly accurate determination of the number of molecules in a given mass; Perrin, 1908.

Final identification of alpha particles from radium; Rutherford and Royds, 1909.

Accurate measurement of the charge on the electron; Regener, 1909; Rutherford, 1911; Millikan, 1910–1916.

Experimental proof that mass increases when speed approaches that of light; Bucherer, 1909.

Method of obtaining a "cloud track" of a single electron, alpha particle, or atom of any element; C. T. R. Wilson, 1912.

X-rays proved to be like light, Laue; measurement of wave length; Bragg, 1912. Crystal structure now open for study; new age in crystallography and metallurgy; new fields in physics.

THE NUCLEAR ATOM; Rutherford, 1912.

X-rays from the Elements; Moseley, 1913.

ATOMIC NUMBER.

A New Age in Chemistry.

Bohr's theory of spectra, 1913.

Cosmic rays—studied since 1903, altitude effect, 1912–1914.

Experimental proof of Einstein's quantum relation in photoelectricity; Hughes, 1912; Richardson and K. T. Compton, 1912; Millikan, 1916.

WAR. Few scientific discoveries from 1914 to 1920.

Accurate measurement of masses of atoms; *isotopes*; Aston, 1920.

Transmission of speech by radio, 1920. Vast development of electron tube devices, ushering in the "age of electronics"; creation of many new industries.

Experimental proof that photons, X-rays, have momentum, again establishing the fact that light exerts pressure. Photons are bullets of light,—the Compton effect 1923.

Applications of science to industry. New methods of detecting ore bodies and oil wells. High frequency electric waves. Oscillations of small quartz plate control a high power transmitting station. Gyroscopes and radio control and guide airplanes.

L. de Broglie's proposal that particles might have wave properties, 1924.

Heisenberg's "matrix" spectra, 1925.

Schroedinger's wave mechanics, 1926.

Dirac's quantum mechanics, 1926.

Cosmic rays, high altitude effect; Millikan, 1926.

Experimental proof of wave character of the electron; Davisson and Germer, 1927.

Interchange of components of molecular energies and light quanta; the Raman effect, 1928.

Discovery of latitude effect of cosmic rays, 1932.

Discovery of heavy hydrogen; Urey, Brickwedde, and Murphy, 1932.

Discovery of the neutron; Chadwick, 1932. Discovery of the positron; Anderson, 1932.	The neutron and positron are new constituents of atoms and will cause a revision of our ideas of atomic structure.

Radiation changed to matter; Curie-Joliot's, 1933.

Transmutation of the elements—begun by Rutherford, 1919; accelerated by Cockcroft and Walton, 1933; many others, 1933.

Generation of high voltages; 1,000,000 volts used, 1933; 10,000,000 in prospect.

Discovery of artificial or induced radioactivity, 1934.

Discovery of element of atomic number 93; Fermi, 1934.

AN ELEMENTARY SURVEY OF MODERN PHYSICS

AN ELEMENTARY SURVEY OF MODERN PHYSICS

CHAPTER 1

MOLECULES—IN SWARMS

What is the "speed" of a gas molecule, its size, mass; how far can it move without hitting another molecule?——We do not know the speed of any molecule yet we compute the "speed" of a great number.——Some fundamental principles.——Some points regarding averages and means.——A law of chance becomes one of precision.——Methods of "measuring" the above quantities.——The motions of smoke particles in quiet air, of gold particles in water.——Brownian movements.——The number of molecules in a given mass.

Fundamental Principles.

There are two physical principles or laws which have constant application throughout the whole realm of physics: the law of the conservation of energy, and the law of the conservation of momentum. Let us review some elementary phenomena in physics illustrating these laws.

The Simplest Illustration of the Law of the Conservation of Energy.

When a simple pendulum is drawn aside, its center of mass is raised slightly; work is done in raising it; the pendulum then is said to have potential energy. If released, the pendulum swings with increasing velocity until it reaches the vertical position, after which the velocity decreases. At the bottom of the swing the energy of motion is just equal to the original energy of position. In fact at any point of its path its motional energy is just equal to the loss in positional energy, measured from its highest point. Or, as it is usually stated, the sum of the kinetic and potential energies is constant. To put this in algebraical form, $\frac{1}{2} mv^2 = mgh$ where m = mass, v = velocity at any point, h = vertical distance below end of swing, g = acceleration of gravity. This relation is identical in form with that obtained

when a mass is dropped from rest. At a distance h below the starting point the velocity v is given by $v^2 = 2\,gh$.

Thus in the case of a pendulum or in that of a body thrown up from the earth or falling down, we have the simplest illustration of the law of the conservation of energy.

Now let us consider two elastic balls of equal masses (Fig. 1–1) hung by equal strings, the balls touching when at rest. Let one ball be drawn aside and released. Work done on it $= mgh$ where h = the height its center of mass had been raised. Its kinetic energy at the bottom would be equal to mgh, also to $\frac{1}{2}\,mv^2$, where v is the velocity just before it strikes the other ball. It can be seen that it comes nearly to rest at once and that the other ball takes up the motion and swings out until its center of mass has risen very nearly a distance h. Here then the energy of No. 1 has been given up to No. 2, and there will be a corresponding transfer of energy at every impact. But there is obviously a slight loss of energy at every impact, and the less elastic the bodies the greater is the loss. Let us then take the case where the elastic reaction is zero—let us put a little wax on the balls so that they stick together. It will then be seen that if No. 1 is drawn aside a distance x_1, the two balls after the impact (now sticking together) move out a distance x_2 where $x_2 = x_1/2$. (It is assumed that the length of the strings is large compared with x_1.) Now it can be easily shown that $v_1 : v_2 :: x_1 : x_2$, hence that $v_2 = v_1/2$. The kinetic energy before impact is $\frac{1}{2}\,mv_1^2$; after impact it is $\frac{1}{2}\,(2\,m)v_2^2$ or $\frac{1}{4}\,mv_1^2$. Thus it can be seen that the total kinetic energy immediately after striking is only one-half that of the single mass before striking. Here then is a case where energy is apparently lost. If the law of the conservation of energy holds, what has become of half of the energy of the first ball? We say—and we know it is true by experimental tests—that the mechanical energy lost took the form of heat in the balls. Very frequently in later

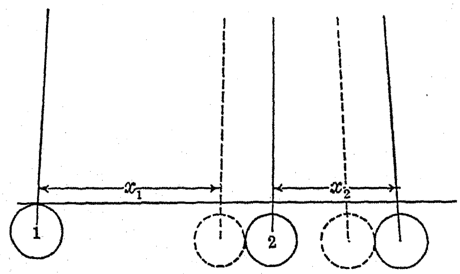


FIG. 1–1. Momentum is unchanged by impact.

work in this text we shall come to a similar situation, *mechanical energy or energy of a well-known form disappears; the question arises what has become of it and the answer will be, it has taken a new, or unusual, or non-mechanical form.*

Newton's "Quantity of Motion."

And now let us consider another quantity connected with the motion of these masses. This quantity, mass × velocity, we call *momentum*. In the case of the equal elastic spheres, at every impact the momentum of one mass was given up to the other—no momentum was lost. Similarly in the case of the completely inelastic spheres, since the velocity of the two spheres after impact was half that of the first sphere just before impact, the mv was unchanged. We could continue this experiment to include spheres of very unequal masses and we would find that in every case the momentum is unchanged by impact. Hence we state the *law of the conservation of linear momentum*, that in every case of interaction of matter upon matter the total momentum of the masses concerned is unchanged in every direction.[1] (This is really an illustration of Newton's third law of motion.) Later we shall see that the above two principles of energy and momentum will be called upon to help explain what happens when light or X-rays strike an electron, or when an electron or a hydrogen atom hits a nitrogen atom, or when a cosmic ray pierces our atmosphere.

There is another illustration of the law of the conservation of linear momentum which should be given. A block of wood is suspended by four long parallel strings and a rifle bullet (22 caliber) is fired horizontally into it. The velocity of the rifle bullet just before it enters the wood can be measured (by photographic methods, for example). The velocity of the block (with the bullet in it) can be computed from the height to which it rises. It will be found that the momentum of the block and

[1] This law is sometimes illustrated thus. "Momentum equals mass × velocity. A tennis ball thrown against a wall returns with (nearly) the same speed. Hence momentum is constant." This of course is absolutely wrong. Momentum is a directed quantity. Moreover the momentum of one body taking part in a reaction is not constant. Here the momentum of the tennis ball has been changed by (nearly) twice its original amount. The total momentum of the wall (earth) and tennis ball is constant. It is an absurd illustration, since the motion of one of the bodies cannot be measured.

bullet after impact will be equal to [1] that of the bullet before impact. But the kinetic energy after impact is only about 1/1000 of the kinetic energy before impact. Almost all of the energy of the bullet takes the form of heat. The motional or mechanical energy is almost all lost during the impact but the momentum is conserved. We have corresponding results when a rifle is suspended by long strings—then fired. The total momentum before the explosion is zero, it is zero after the explosion, for the rifle bullet of mass m_1 has a velocity v_1 in one direction, giving a momentum m_1v_1 in that direction, while the rifle of mass m_2 moves with velocity v_2 in the other direction and $m_1v_1 = m_2v_2$ (we neglect the motion of the issuing gas). But almost the entire energy of the explosion is given to the bullet—only a very small fraction to the rifle. (Work out the ratio when $m_2 = 1000\ m_1$.)

We have phenomena almost identical with the above when an electron going with great speed is "captured" by an atom or when a radioactive atom explodes, firing out an electron or an alpha particle.

Swarms of Molecules.

We have been considering collisions between bodies of ordinary size. Let us proceed to the case of very small particles. A glass tube, containing some mercury and very small glass spheres, has been evacuated and sealed, then gently heated. Presently the spheres are seen moving in all directions, striking against one another and against the glass wall. It is evident too that mercury molecules must be striking against the glass wall and presumably against the spheres and one another. We have clear evidence that the wall of the tube is being bombarded by very small particles. We can idealize the case and think of a 1 cm. cube vessel containing n gas particles, each of mass m and having velocity u. Making the plausible assumption that one-third of these are moving normal to one pair of faces of the cube, it follows that every one of the $n/3$ particles will make $u/2$ collisions per second on one face; the momentum change per collision (the particles being perfectly elastic) is $2\ mu$; hence the total momentum change per second on one face is $nmu^2/3$. Now, force is defined as the rate of change of momentum, or momentum change

[1] In many laboratory courses there is an experiment on the "ballistic pendulum" in which the momentum law is assumed and the student is asked to find the velocity of a rifle bullet.

per second, and pressure is the force per unit area. Hence the pressure due to gas particles is $nmu^2/3$. But nm is the mass in 1 cm.^{3} or the density, ρ. Hence $u^2 = 3\,p/\rho$ is a relation from which we can compute u. For example, if the gas were oxygen at 76 cm. pressure and at 0° C., ρ would be about 0.00143 gm./cm.^{3}. Then

$$u^2 = \frac{3 \times 76 \times 980 \times 13.59}{0.00143}$$

and u = 46,000 cm./sec. = 460 m./sec. The velocity for hydrogen particles under similar conditions would be 184,000 cm./sec. or nearly 1.2 miles per second.

We have suddenly come to a derivation of an important relation in regard to molecules—that they exert a pressure, $p = nmu^2/3$, upon the walls of the containing vessel. But what kind of a speed is u, here assumed to be the same for all the particles? It is very easy to see that if by any miracle the particles all had a common speed at a certain instant, a moment later the speeds would not all be equal. For consider two equal particles A and B moving as shown in Fig. 1–2. If A should strike B head on, it can be seen that A will be brought to rest and B will have the velocity given by the resultant of v_1 and $v_2 = \sqrt{v_1^2 + v_2^2}$. (Both momentum and energy are thus conserved.) Consequently A, having a large speed just before collision, has been brought to rest while B's speed has been increased. It is clear that u must be some sort of mean speed, and this brings us to a very large topic—how are the speeds of molecules distributed? Or to put it another way, what fraction of all the particles has speeds lying between certain limits, say between 1000 and 1001 ft./sec.? To answer this question in any completeness would require a discussion of the laws of probability—a topic that may appear to have no place in an exact science. But many natural processes are dependent upon the operation of those laws: the inheritance of characteristics, the expectation of life, the distribution of grades in a large college class, the percentage of inners (dependent however upon

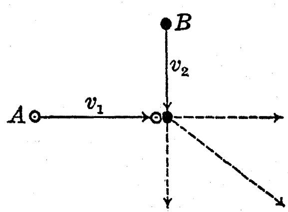


FIG. 1–2. One particle may lose, another gain momentum.

the diameters) made by a large regiment of expert marksmen, the distribution of speeds in many molecules. Here is a very simple case.

A Law of Chance.

Suppose that the length of a certain steel rod has been measured many times, that the arithmetic mean (a. m.) of all the measurements has been found, and that the errors or differences between the a. m. and the various measurements have been plotted. A curve similar to A in Fig. 1–3 will be obtained, where distances along x represent the errors e_1, e_2, and the area above e_1e_2 is equal to the number of the errors of magnitudes including and lying between e_1 and e_2. The equation to the curve is

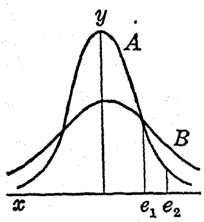


FIG. 1-3. The distribution of errors.

$$y = \frac{N}{a\sqrt{\pi}} e^{-u^2/a^2}$$

(where N is the total number of observations) and it has many times been experimentally confirmed. The quantity a is a measure of the accuracy of the measurements, for the smaller a is, the greater is the percentage of small errors and the sharper is the fall towards zero. In curve A the errors are small; a is small. In curve B both are large.

The Distribution of Velocities and Speeds.

Now consider an enclosure containing gas and let the motions of the particles at any instant be resolved along the x axis. The curve showing the number of particles having velocities between definite limits is identical with that just described. In other words, the relation

$$y = \frac{N}{a\sqrt{\pi}} e^{-u^2/a^2}$$

gives the number of particles having velocities along any line lying between u and $u + 1$, where N is the total number of particles. When we consider speeds, motions without regard to direction, the corresponding relation is (Appendix 1–6)

$$y = \frac{4\,N}{a^3\sqrt{\pi}} c^2 e^{-c^2/a^2}$$

where c is the speed. The two relations above are known as the law of distribution of velocities and of speeds respectively. To Maxwell we owe both of them.

An analysis of the relation above shows that the maximum ordinate of this curve occurs for $c = a$. In other words, a is the most probable speed. The arithmetic mean of all the speeds is shown by $\bar{c}$. On the other hand, if we compute the total kinetic energy of all the molecules with their various speeds, the speed C which all the particles would be required to have in order to possess the same energy is shown by $C = \sqrt{\overline{c^2}}$. In other words,

$$\frac{NmC^2}{2} = \frac{1}{2}m(c_1^2 + c_2^2 + c_3^2 + \cdots \text{ to } N \text{ terms}),$$

or

$$C = \sqrt{(c_1^2 + c_2^2 + \cdots)/N} = \sqrt{\overline{c^2}}.$$

C, then, is the square root of the sum of the squares of all the speeds divided by the number of particles, or, more briefly, it is the root-mean-square speed. Now this is the speed that appears in the relation $p = nmu^2/3$, where u must equal the C above.

A further analysis of the curve gives the relations which hold for a, $\bar{c}$, and C. They may be summarized thus, $a : \bar{c} : C = 1 : 2/\sqrt{\pi} : \sqrt{3/2} = 1 : 1.128 : 1.224$. We may now re-interpret the curve. For oxygen at 0° C. the root-mean-square speed C is 460 m./sec. Hence a in the above formulae is $C/\sqrt{3/2} = 376$ m./sec. and $\bar{c}$, the average speed, is nearly 420 m./sec. The number of particles having speeds between 376 and 377 m./sec. is (substituting a for c) $4\ N/a\sqrt{\pi}e$. Putting in numerical values $a = 376$, $e = 2.718$, this equals $0.00220\ N$. Hence if the total number of particles is 100,000, we find that there are 220 which have speeds (Fig. 1–4 a) between

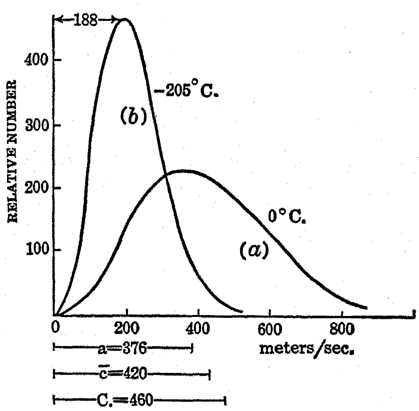


FIG. 1–4. Maxwell's law of distribution of speeds. For oxygen at 0° C. the most probable speed, a, is 376 meters/sec.; the average speed, $\bar{c}$, is 420, and the root-mean-square speed, C, is 460.

376 and 377 meters per second. Now since C^2 varies as T if we take the temperature at 273/4 or 68 $K = -205°$ Centigrade, the value of C would be $460/2 = 230$ m./sec. and $a = 376/2$. And since a is the denominator, the number of particles having speeds between 188 and 189 m./sec. is 440, twice what it was at the maximum of the first curve; that is, there are now 440 particles having speeds between 188 and 189 meters per second (Fig. 1–4 b).

Experimental Confirmation.

The student may note that we appear to have passed from a condition of confusion—that of a gas in which a great number of molecules are going in all possible directions with a great variety of speeds—to one of order and precision. And he is justified in asking whether we have merely been computing something by means of a formula—anyone can write down a formula—or whether this formula has been put to an experimental test. Within certain limits it has been verified experimentally in various ways. The most accurate ways are indirect. The principle of the various direct methods that have been used during very recent years is shown in Fig. 1–5. Silver or some other metal

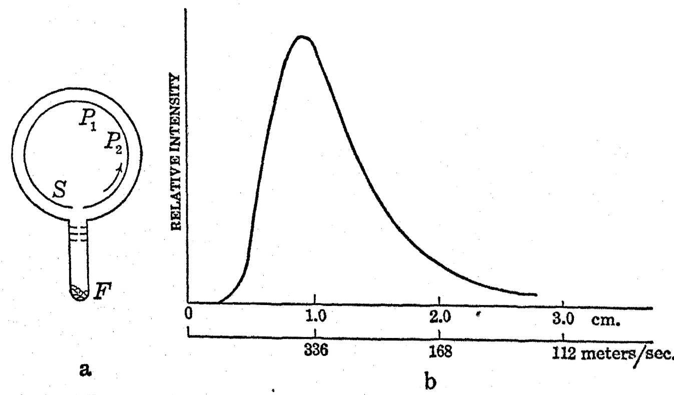


FIG. 1–5. The distribution of molecular speeds is experimentally determined.

that can be readily volatilized is heated in F to a definite high temperature. The vapor can be released at any chosen time so that it passes through a series of small openings, then, once with every revolution of the cylinder SP_1P_2, through a very fine slit at S. The molecules, having passed through S, continue on and condense on a thin sheet of mica. This can be withdrawn from

the cylinder later, spread out, and examined for density of deposit. Since time is required for the molecules to pass across the diameter of the cylinder, they arrive behind P_1, the slow ones spread out to P_2. The density of the deposit can be measured and plotted. The agreement of the plot with the Maxwell curve is shown in Fig. 1–5 b.

It is necessary that the cylinder be rotated at high speed, therefore that it be perfectly balanced. The highest vacuum must be maintained. Altogether the experiment calls for the best of technique. The curve here given is from the article by Zartman in *Physical Review*, 37, 383, 1931.

But an indirect method has perhaps more interest for us. Electrons come from a hot wire—as everyone knows who has seen a radio tube. Richardson proved to a fair degree of satisfaction that the distribution of speeds of the electrons *outside* [1] the wire was in accord with Maxwell's law.

The Significance of Temperature.

It is still necessary to show the dependence of the above relation upon the temperature of the gas. The combined Boyle's and Charles' laws give us the relation $pV = RT$, where p is the pressure, V the volume, T the absolute temperature of the gas, and R a constant. If the mass of the gas is equal to the molecular weight, we are then dealing with a *gram molecule*, or mole of the gas. The constant R then has the value 8.317×10^7 ergs. But in the relation $p = nmC^2/3$ the n molecules are the number per cm.^{3}. In a gram molecule the number is N, Avogadro's number. Hence

$$p = \frac{NmC^2}{3\,V}$$

where V = volume of 1 gram molecule. Hence

$$\frac{NmC^2}{3} = RT \qquad \text{or} \qquad \frac{1}{2}mC^2 = \frac{1}{2}\cdot\frac{3\,RT}{N},$$

or $T \propto \frac{1}{2}mC^2$. Now this is the mean kinetic energy of the molecules. Hence we have a very clear concept of absolute temperature—it measures the *random* translational energy of the molecules of a substance. If we were to throw from the top of a high

[1] However, it will be shown in Chapter 8 that for electrons *inside* the wire this is not at all a complete statement of the case.

cliff a sealed vessel containing a gas, the speed of the vessel would constantly increase but the random translational energy of the gas molecules would not change until the earth was struck, then part of the recoil action would cause this random energy, therefore the temperature, to increase.

In further illustration of the principle just stated, let us consider two similar vessels, one containing oxygen and the other hydrogen, and let them be at the same temperature and pressure. The principle above would require that $\frac{1}{2} m_1 v_1^2$ for hydrogen would equal $\frac{1}{2} m_2 v_2^2$ for oxygen. But the mass of the oxygen molecule is nearly sixteen times that of the hydrogen; hence the speed of the latter would be nearly four times that of the oxygen. If a valve were opened between the two vessels, the gases would diffuse into each other. However, the temperature would not change and therefore the mean kinetic energy of the molecules would still be equal. We might extend this illustration to include extremely heavy molecules, finally particles very large compared with molecules, and it would still be true that the mean kinetic energy of the particles, perhaps visible in a microscope, would be equal to that of the very small invisible gas molecules.

The Brownian Movement.

The topic just considered is also illustrated by the so-called *Brownian movement.* A microscope (one of ordinary power will do) is used to look down into a very small vessel containing water. A very strong sidelight is thrown into the water in such a way that no direct light enters the microscope. Particles suspended in the water, very large compared with molecules, scatter sufficient light into the microscope so that, if the focus is right, they appear as miniature stars. They are in constant zigzag motion. Their mean kinetic energy is equal to that of a molecule of the water. Since the mass of the particle is very large compared with that of a molecule, its speed is correspondingly slow. The relation holds, $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$. It is often stated in texts that the motion of these particles is due to the bombardment of the water molecules. But the bombardment is mutual; many molecules strike the particle and it strikes many molecules. However, the water molecules have very much the larger velocity and may be said to do the bombarding. But the statement that the motion of the particles is due to collisions—to unbalanced collisions—tells

us nothing more than is told us by Newton's first law of motion. All random motion is due to collisions. The important point is that the mean kinetic energy of the particles is equal to that of molecules at that temperature.

Instead of using water, we may use smoke particles in air. Any smoke will do. All that is necessary is to have a strong sidelight on the smoke—with no direct light entering the microscope.

The phenomenon we have been discussing was named (1827) after its discoverer, an English botanist, Robert Brown. It remained as a thing apart, an unexplained phenomenon, until about 1880 when various scientists proposed that the motion of these particles was dependent upon the thermal agitation of the surrounding liquid or gas. But the most definite quantitative work has been done in very recent years. Einstein in 1905 developed a formula which could be tested by experiment. The tests were carried out by Perrin (1908) and by deBroglie (1910) in France, and by Millikan (1910–1913) and his associates, Fletcher and Eyring, in America. Einstein's theoretical work was based on two assumptions: (1) that the mean kinetic energy of a particle suspended in a fluid is the same as that of a gas molecule at that temperature, (2) that the resisting force after the particle has suddenly acquired energy is due to a property of the fluid later to be discussed—the viscosity of the medium.

As a result of this analysis he obtained this relation (Appendix 1–7), $\overline{x^2} = 2\,(RT/NK)t$, the experimental test of which we now consider.

In the eyepiece of the horizontal microscope with which we are observing the motion of a particle, there is a scale of equidistant parallel lines. We take the x axis at right angles to gravity and perpendicular to these lines. We note the x displacement of the particle, positive or negative, in a short time t, perhaps 20 seconds. This is repeated thousands of times. (The temperature T of the liquid should be kept constant.) We then square the x's (this removes the difference between positive and negative) and average the squares. Thus we find $\overline{x^2}$. Einstein's theory indicates that $\overline{x^2}$ should be proportional to the interval of observation, t; also to the absolute temperature T and to the gas constant R. N is Avogadro's number and K a constant of the medium. We shall show in the next chapter how we may use this relation to find N. The relation has been verified.

HENCE WE SEE THAT WE MAY, FROM THE BROWNIAN MOVEMENT, DETERMINE N, AVOGADRO'S NUMBER, ONE OF THE MOST IMPORTANT CONSTANTS [1] OF NATURE.

The amazing thing is that any quantitative relation should be found to hold for this phenomenon. It appears completely chaotic. Figure 1–6 shows the positions of three different par-

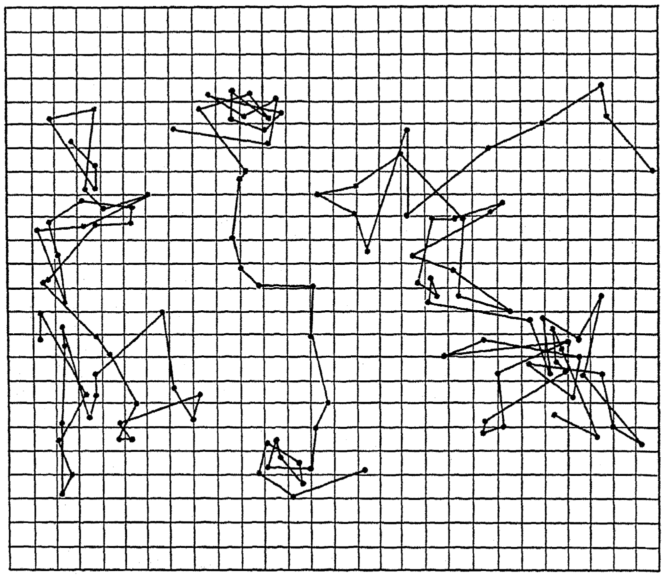


FIG. 1–6. Order in chaos. The dots represent the positions of a particle of gum mastic in water at the beginning and end of a certain period of time, 20 seconds. By measuring the displacements along a horizontal line an important constant of nature, Avogadro's number, was computed.

ticles successively observed by Perrin. The round dots represent the positions of a particle at the beginning and end of 20-second intervals. Straight lines join these dots, though the particles pretty certainly did not move *along* these lines. That the motion is completely irregular is evident.

Again it should be emphasized that in this experiment the entire liquid or gas should be in thermal equilibrium. There should be no convection currents. There should be no boiling.

[1] Avogadro's number is the number of molecules in a gram molecule, and a gram molecule has a mass in grams equal to the molecular weight: 2.016 gm. of H, 32 of O. etc.; in the case of a gas under standard conditions, 0° C. and 76 cm. of Hg, the volume is 22.414 liters.

In this connection let us consider again the motion of the glass spheres in the tube containing mercury. When the tube is heated, the mercury molecules in contact with the heated glass acquire large kinetic energy. They drive back slower molecules. Several million of the high energy molecules may form a bubble of vapor which rises and bursts into the relatively free space above the liquid. This bubble may give energy to a glass sphere, causing it to jump a few centimeters above the surface. Mercury vapor driving up from the surface may cause these spheres to bounce back and forth several centimeters above the liquid surface. The energy of the glass spheres is due to thermal agitation, but they are in a region in which there is great energy below as compared with above. Had spheres of this size been immersed in a liquid of their equivalent density they would have had random irregular motions, Brownian movements. The average energy of a sphere would have been equal to that of a molecule at that temperature, but the movements would have been too small to be seen in any microscope. Colloidal particles, however, can be seen in motion.

Avogadro's Number and the Law of Atmospheres.

Let us suppose that two glass tubes open at the ends dip into the ocean and stand several hundred miles vertically. Let the air be pumped out and one tube be *filled* with hydrogen, the other with oxygen so that the pressure at the ocean surface is ordinary atmospheric pressure. The pressure would decrease, as we ascended in the tubes, sixteen times as rapidly in oxygen as in hydrogen. The law would be $p = p_0e^{-kh}$ where p is the pressure at height h but the k for oxygen would be sixteen times that for hydrogen (Appendix 1–2). A practical statement of this law is illustrated thus: for any two points in our atmosphere vertically different by 800 feet, the pressure at the upper point is 0.97 of the pressure at the lower point. (At the top of a mountain 6400 feet high, the pressure would be $(0.97)^8 = 0.784$ of that at sea level.) For an equal fractional decrease of pressure in the oxygen and hydrogen tubes the vertical distances would be 725 and 11,500 feet. In oxygen, for the pressure or density to be halved, it would be necessary to rise a distance of 5 kilometers or 5×10^6 mm.; in hydrogen 80 kilometers.

Let us suppose that we are able to see and count the oxygen and hydrogen molecules. Then we could substitute for pressure, in the above statement, the *concentration* or *number* of molecules per cm.^{3}. The lighter the molecule, the greater the height for a certain fractional decrease in concentration. Let us now suppose that we are able, using a microscope pointed in the direction of the arrows (Fig. 1–7), to count particles of some definite material suspended in a liquid. The same law holds. And if we find that it is necessary to rise only 1/20 of a millimeter in order for the concentration to be halved, the mass of these particles would be $5 \times 10^6 \times 20 = 10^8$ times as great as the oxygen molecule.

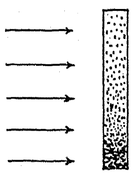


FIG. 1–7. The rate of change, with height, of the concentration of particles suspended in a liquid leads to the determination of the mass of an oxygen molecule.

Perrin (1910) with excellent technique performed the experiment suggested above. With alcoholic solutions of gum mastic and gamboge in water, and after obtaining particles of nearly the same size by centrifuging, he measured the concentrations at various heights. This gave him the ratio of the mass of the particle to that of the molecule of oxygen. If he could measure the former, he would be able to compute the latter and therefore the mass of a molecule of any element. He measured the density of the particles by placing some of them in a solution (potassium bromide) and altering the density of the latter until the particles would neither rise nor fall. The density of the particles was then equal to that of the liquid. He measured the diameter of the particles by evaporating some of the solution on a glass plate. The particles, apparently clinging to one another, arranged themselves in lines so that by measuring the length of a line and counting the number of particles the diameter of one could be determined. The mass of the particle, equal to the density times the volume, is then known. Hence the mass of the oxygen molecule is known and also Avogadro's number.

The value of N as obtained by Perrin, using different methods of measuring the mass of the particles, varied from 6.5 to 7.0 $\times 10^{23}$. It will be seen in the next chapter that Perrin's determination of N, though outstanding in accuracy up to that time (1910), fell far short of the accuracy obtained by later electrical methods. The value of N now accepted is 6.064×10^{23} per mole.

One Method of Measuring—or Computing—the Diameter of a Molecule.

We have been dealing with molecules as if they were like idealized glass spheres, perfectly elastic, uniform, of definite boundary, all molecules of any one substance, identical. This has been, since the middle of the nineteenth century, the picture of a molecule in the development of the kinetic theory of gases. In the derivation of Boyle's law, $pv = RT$, it was assumed that these spheres were very far apart in comparison with the molecular dimensions—that they occupied mere points in the volume v. But if we think of them as possessing some volume then as the gas would be compressed we would expect a departure from that law; that it would now take the form $p(v - b) = RT$, where b would be a limiting volume below which the gas could not be compressed; $v - b$ could not be negative. Now we can obtain in a rather crude way a relation between b and the volume of the spheres as follows. In Fig. 1–8 we picture two molecules A and B in contact. Their centers cannot approach closer than a distance d equal to the diameter of a sphere. Hence the molecule A would be excluded from the volume indicated by the dotted sphere or $\frac{4}{3}\pi d^3$. For every *pair* of molecules this volume would be one of exclusion. Hence in a gram molecule the total volume of exclusion b would be $\frac{2}{3}\pi d^3 N$. Now experimentally by measuring the volume and the pressure (carried to large values) *we may measure* b. Hence we can get an estimate of d, the diameter of a molecule.

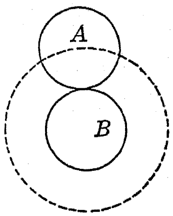


FIG. 1–8. Molecules cannot occupy the entire volume.

The Mean Free Path of a Gas Molecule.

Another quantity ought to be here considered. It is the *mean free path* of a molecule or the average distance a molecule goes without striking another molecule.

Let us think of a long cylinder, Fig. 1–9, of diameter $2d$, of total volume 1 cm.^{3}. Since the cross-section is πd^2, the length of the cylinder is $1/\pi d^2$. The total number of molecules in this cylinder is n, the number per cm.^{3}. Some of these molecules will be completely inside the cylindrical surface, some will be far

enough out so that their centers are on this surface. But a molecule A going through the total length of the cylinder will strike

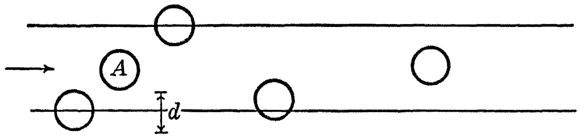


FIG. 1-9. How many collisions would a molecule make in one centimeter?

them all. Thus the total number of collisions will be n in a distance $1/\pi d^2$ and the average distance between collisions will be $1/\pi n d^2$. This then is the mean free path L.

But this picture is a rather crude one. It regards all the spheres as at rest except A. When allowance is made for the motions of the spheres with speeds given by Maxwell's law, it results that

$$L = \frac{1}{\sqrt{2}\,\pi d^2 n}.$$

It is seen that for a gas the mean free path is inversely proportional to n, the number of particles per cm.^{3}, and therefore to the pressure when the temperature is constant.

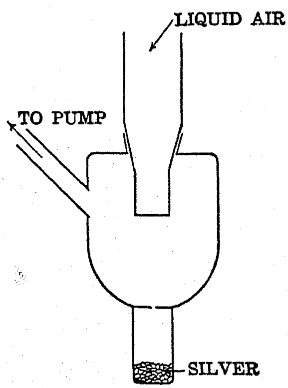


FIG. 1-10. The mean free path of silver atoms in air is determined by experiment.

We have direct experimental evidence that this is an approximate value of the mean free path. A small mass of silver in the bottom of a quartz tube (Fig. 1-10) in a vessel which can be highly evacuated is vaporized. The molecules are driven out and when the vacuum is high they are deposited on a glass or quartz surface cooled by liquid air or solid CO_2 above the apertures shown. If gas is let into the vessel or if the upper surface is drawn back, the deposit is no longer a sharp image of the openings. Collisions are responsible for the diffusion. Theory gives a relation for the density of deposit in terms of the mean free path, the pressure of the gas, and the distance between the opening in the furnace

and the cool surface. Thus we have a rather direct experimental method of testing the relation

$$L \doteq \frac{1}{\sqrt{2}\,\pi d^2 n}.$$

But the outgoing molecules are of silver and the other molecules of air. What is d in the above relation? and L?

Using tables, the student can compute the mean free path and will find that in ordinary air it is of the order of 10^{-5} cm.; for a very high vacuum it is a kilometer.

Viscosity.

We come to the discussion of another topic having great theoretical and industrial importance.

In the apparatus of Fig. 1–11, an outer cylinder A containing oil or gas can be rotated by a motor with considerable speed. An inner cylinder B is suspended by a wire or fiber so as to be concentric with A. When A is set in motion, B will be dragged in the same direction but will take up a new position as shown by the mirror M attached to the rod above. The angle through which B has been turned depends on the speed of A, the radii and lengths of the cylinders, the stiffness of the supporting fiber, and the *coefficient of viscosity* of the liquid or gas between the cylinders. The picture is this: as A is speeded up, the molecules striking its inner surface are given a forward motion; this is passed on to the outer surface of B, hence the drag. Again the kinetic theory gives a relation for this quantity $\mu = (1/3)\, nm\bar{c}L$. This may be put in another form if a gas is in the cylinder. Then

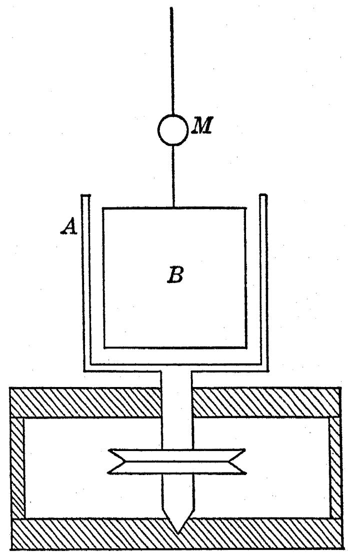


FIG. 1–11. The viscosity of a gas or of a liquid may be measured.

$$L = \frac{3\,\mu}{\rho\bar{c}}.$$

Since

$$\bar{c}^2 = \frac{8\,C^2}{3\,\pi} = \frac{8\,p}{\pi\rho}$$

and since μ can be measured, it follows that L can be measured.

Or if we substitute for L the value previously found, we have the relation

$$\mu = \frac{2\,m}{3\,\pi^{3/2}d^2}\sqrt{\frac{p}{\rho}}.$$

It is seen that if the temperature (and therefore p/ρ) is constant and if the diameter of the molecule does not change with pressure, the *coefficient of viscosity should be independent of pressure.* Within limits this has been experimentally confirmed.

Summary.

The total momentum of colliding bodies is unchanged by collision.

The energy of an explosion goes into the small mass.

There is a law of distribution of velocities of gas particles which is similar to the law of errors.

The above law has to do with motion along one direction, but from it we derive a law of distribution of speeds; this includes all directions.

From the law of distribution of speeds we obtain relations between the most probable speed a, the average speed $\bar{c}$, and the root-mean-square speed C.

From the kinetic theory we obtain $C = \sqrt{3\,p/\rho}$. This enables us to compute a, $\bar{c}$, and C for any gas for which Boyle's and Charles' laws hold.

We are also able to compute the percentage of the molecules having speeds within certain limits.

Experiments designed to test the validity of these results have been described.

It follows that the *random* energy of *translation* of the molecule of a gas is independent of its mass.

The Brownian movement illustrates this point. And from this chaotic motion a precise relation has been deduced from which we can compute N, Avogadro's number. From N we can compute n, the number of molecules in 1 cm.^{3} of a gas.

We may also compute N, Avogadro's number, by applying the law of atmospheres to colloidal particles in suspension.

The elementary theory and the experimental method of determining three quantities—the diameter of a molecule, the mean free path, and the coefficient of viscosity—are presented. The tables below gather together some of the data.

With N known, then n, the number of molecules in 1 cm.^{3} of a gas for standard conditions, is also known. Hence we can use the relations

$$(1)\ b = \frac{2\,\pi d^3 N}{3};^{1} \qquad (3)\ \mu = \frac{nm\bar{c}L}{3};$$

$$(2)\ L = \frac{1}{\sqrt{2}\,\pi d^2 n}; \qquad (4)\ \bar{c} = \sqrt{\frac{8}{3\,\pi}}\,C = \sqrt{\frac{8\,p}{\pi\rho}}$$

to compute the diameter of a molecule, the mean free path, and the mass of a molecule. The last quantity is of course most directly computed from the relation $Nm = M$ where M = molecular weight. Thus we have cross-checks on the validity of the above relations.

In Table I will be found some values of the above quantities for various atoms and molecules as found by different methods. The unit of distance is 10^{-8} cm.

TABLE I

Gas	Mean Free Path	Molecular Diameter	
	L from (3) and (4)	from (2) and (3)	from (1)
Hydrogen	1125	{2.72 2.47	{2.32 2.76
Helium	2850	2.20	{2.64 2.30
Oxygen	{ 640 1000	{3.62 3.40	2.91
Carbon dioxide	{390 630	{4.64 4.20	{2.62 3.40
Mercury vapor		3.64	{2.37 1.93

[1] This implies that the volume "of exclusion" or volume reserved for a molecule is four times that of the sphere. But the logic here is far from faultless. See footnote, p. 109.

In Table II will be found a few constants taken from Birge, *Physical Review*, Supplement, 1929.

TABLE II

Volume of a gram molecule of a perfect gas at standard temperature and pressure....	22413.5 cm.^{3}
Avogadro's number, N....................	6.064×10^{23}
Gas constant R, in $pv = RT$, for one gram molecule..........................	8.3136×10^{7} erg
Boltzmann constant $k = R/N$..............	1.3708×10^{-16} erg
Mass of hydrogen atom....................	1.6622×10^{-24} gram
Mass of standard atom or "statom".........	1.6489×10^{-24} gram
Kinetic energy of translation of a molecule at 0° C. $E = \frac{3}{2}\frac{RT}{N}$	5.621×10^{-14} erg
Change of translational energy per degree C. $\frac{3}{2}k$..............................	2.058×10^{-16} erg

CHAPTER 2

WE DISCOVER AND WEIGH THE ELECTRON. WE WEIGH ATOMS AND DISCOVER ISOTOPES

The phenomena of electrolysis prove that electricity is atomic.——Confirmed by discharge of electricity through a gas.——Cathode rays—"the fourth state of matter."——Discovery of the electron, measurement of its velocity, charge, mass.——A new universe.——The architecture of the atom becomes interesting. A region in which Newton's laws must be modified.

The Atom of Matter.

The English chemist Dalton and the Italian physicist Avogadro in the very early years of the nineteenth century placed the atomic theory of matter on a firm foundation. The underlying argument was this—if two elements *A* and *B* unite in various ratios of masses to form various compounds, the ratios are given by simple whole numbers; one unit of *A* unites with one, two, three of *B*. Faraday (1833) showed that this idea was in complete accord with the experimental results obtained when an electric current passed through an electrolyte. But he also showed—though his ideas were not clearly grasped for some sixty years—that if matter was atomic, so also was electricity.

If a solution of sodium chloride (NaCl) is decomposed by an electric current, we picture an atom of sodium to carry one unit of positive electricity, one atom of chlorine to carry one negative electric unit. In barium chloride solution ($BaCl_2$) the barium atom carries two units and each of the chlorine one. In general we picture all monovalent ions as carrying one electric unit, all bivalent two, etc. It is now necessary to define a unit of electricity.

The Atom of Electricity.

The ordinary ammeter measures a current in amperes, and an ampere is defined in two ways—the legal and commercial way is that it will deposit on the proper electrode 0.001118 gram of silver per second when it flows through a definite silver salt

solution; the other, the scientific, absolute way, that it is one-tenth of the absolute electromagnetic unit. It is obvious that the legal definition is based on or derived from the absolute. When an ampere has passed for one second, we say that one coulomb of electricity has passed through the instrument. Hence one coulomb of electricity and 0.001118 gm. of silver are associated together. Since N atoms (Avogadro's number) of silver weigh 107.88 gm., we can find the number of atoms in 0.001118 gm. and hence the charge in coulombs on one atom. Thus Faraday in 1833 could have found the charge constituting the atom of electricity had N been known. Johnston Stoney in 1874 made this computation but the value of N based upon kinetic gas theory was very much in doubt. He obtained the value 10^{-20} of a coulomb for this electric atom. He called it the *electron*. The correct value is about 16 times his computed value.

But without knowing N we can easily compute an important quantity concerned in the phenomenon of electrolysis It is evident that the ratio of the charge on an ion to its mass is greater the smaller the mass. When this ion is the silver atom and when we measure the charge e in coulombs and the mass m in grams,

$$\frac{e}{m} = \frac{1}{0.001118} = 894.4;$$

for the hydrogen atom it is

$$\frac{1}{0.001118} \times \frac{107.88}{1.008} = 95{,}730.$$

This latter value is the greatest that e/m can have in electrolysis. But in the passage of electricity in a gas it was found that this ratio for the cathode ray carrier was about 1800 times the value for the hydrogen atom. This point will be considered in more detail later.

Discovery of the Electron.

The atomic nature of electricity was very slow of acceptance. It was not until a study was made of the discharge of electricity through gases that it was established. Sir William Crookes had discovered in 1870 the phenomenon of cathode rays. With extraordinary intuition he called these rays the "fourth state of

matter." In 1897 Sir J. J. Thomson found that these rays consisted of negatively charged particles (Fig. 2–1), and he measured their velocity and the ratio of the charge on a particle to its mass. The velocity, depending on the voltage in the discharge tube, was very large, several thousand miles per second. The ratio of charge to mass, e/m, was independent of the nature of the electrodes or of the gas in the tube. It seemed to be a new constant of nature. It appeared fairly certain that the particle in motion had a mass about 1/1800 that of a hydrogen atom and that the charge was that on a monovalent ion. Thus was the *electron* isolated. It became part, and an exceedingly important part, of the world of the physicist.

Measurement of Its Velocity, Charge, and Mass.

Let us return to electrical units. The absolute electromagnetic unit of current may be easily defined as follows. Let us consider a long straight wire of length l carrying a current i. Suppose a magnetic field H is at right angles to it. There will be a force F on the wire at right angles to it and to the field equal to $H \cdot i \cdot l$ dynes if H is measured in ordinary magnetic units and i in electromagnetic units. This gives the definition of the electromagnetic unit of current, the e.m.u. Now let us picture a procession of electrified particles, each of charge e (e.m.u.), having a velocity v cm./sec., and let n = number of particles in 1 cm. length of the procession. The amount of electricity that would pass a point per second $= n \cdot e \cdot v$. This would equal i, the current. If there is a magnetic field H at right angles to v, there will be a force on every centimeter length equal to $H \cdot n \cdot e \cdot v$. Hence the force on one particle carrying charge e with velocity v would be $H \cdot e \cdot v$ dynes at right angles to H and v.

Similarly if there are two parallel metal plates with distance d cm. and electromagnetic potential E between them, the electric intensity X between the plates is E/d. If there is a particle with charge e between the plates, the force acting on it due to the electric field is Xe dynes. Ordinarily this potential is measured in volts. If V be this quantity, then $V \times 10^8 = E$ and the force on e follows as above. Thus if we would apply a voltage to two metal plates as in Fig. 2–1 a, an electric stream entering in the direction of the arrow would be deflected up. But if a magnetic field were imposed at right angles to the plane of the paper and

were directed away from the reader, the stream would be deflected down. With both fields operating (and both terminated sharply

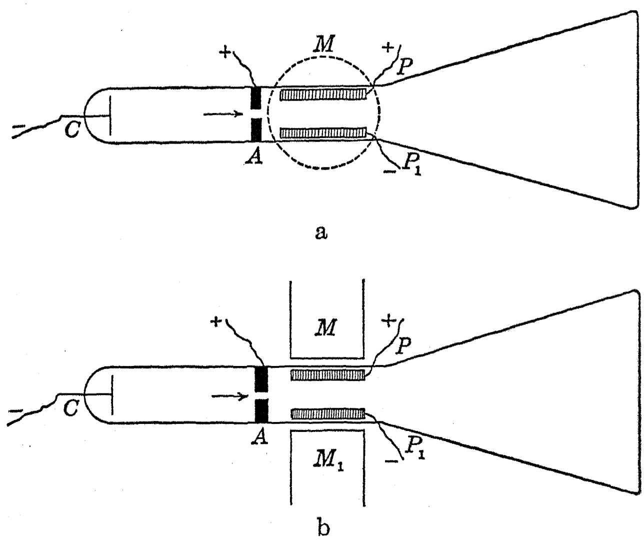


FIG. 2–1. Electrons moving along the arrow are, beyond A, deflected by electric and magnetic fields; in (a) the fields are at right angles to one another; in (b) they are parallel.

by lines joining the ends of the plates), it would be possible so to adjust them that the electric stream would not be deflected. It is understood that these plates would have to be in a discharge tube as in Fig. 2–1. The fact that the stream was not deflected would be shown by a blue spot on the fluorescent screen at the end of the tube. When there is no deflection, the two forces are equal and opposite; then $Hev = Xe$. Hence the velocity is determined at once and equals X/H.

It was in this way in 1897 that J. J. Thomson measured the speed of the particles constituting cathode rays.

If only the electric field operates, the force on the electrons is constant in amount and direction as long as the electron is between the plates. Hence the curve described is a parabola, the curve that is described by a ball thrown into the air or by a rifle bullet—if air resistance be neglected. But there are certain important differences—the vertical acceleration imparted to the ball or rifle bullet is independent of the mass of the body. In the

electrical case the acceleration perpendicular to the direction of the plates is dependent upon the mass, as well as upon electrical quantities. It can be very easily shown that, given a certain electron velocity, the deflection of the blue spot on the fluorescent screen when only the electric field operates is proportional to e/m. All other quantities involved can be measured and the deflection gives us the value of e/m for a cathode ray particle.

Suppose that the magnetic field can be limited on the left, as before, by the line joining the left edges of the plates, but that it extends to the right at least as far as the fluorescent screen and that both fields operate and are balanced between the plates. The electrons would move along a line parallel to the plates as far as the line joining the right edges but thereafter they would be under the influence of the magnetic field alone. The force now, Hev, would always be at right angles to v, the direction of motion. The resulting curve would therefore be a circle. Now in elementary physics it is shown that when a body of mass m describes a circle of radius r, there must be a force pulling or holding the mass towards the center and $= mv^2/r$. Hence we have the relation $Hev = mv^2/r$ or $e/m = V/Hr$. The velocity is given above and H and r can be measured. Again we measure the ratio e/m.

In the early years of the electron's history, in the years 1896–1910, while the ratio e/m was rather accurately measured, difficulty was experienced in measuring e accurately. In the Cavendish laboratory at Cambridge, J. J. Thomson, Townsend, C. T. R. Wilson, and H. A. Wilson performed various ingenious experiments with this objective. The last observer, building on the work of the others, noted the rate of fall of a cloud—a fog of electrified vapor particles—when formed between two horizontal metal plates which could be electrified. The cloud particles could be made to fall fast or slow or some of them to rise, depending on the manner of charging the plates.

The Oil Drop Experiment.

Millikan, taking up this study in 1906, very greatly improved the technique of the older methods and devised the method now extensively followed in physics laboratories. It is known as the oil drop experiment. Two brass plates about 5 cm. square (Fig. 2–2) are spaced so as to be accurately parallel about 5 mm. apart

by hard rubber and glass. The top plate has a few fine (1/2 mm. diameter) holes in it; a light source and low power microscope are

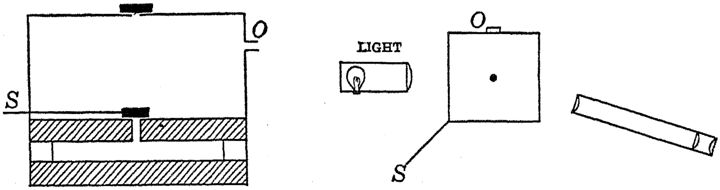


FIG. 2–2. The electron charge may be measured by means of simple oil drop apparatus.

arranged as shown. Above the upper plate is a brass cap with a small opening O into which oil (very good cylinder oil will do) is sprayed by an atomizer. The atomizer is first operated away from the apparatus until it is seen that a fine spray issues from it. Then a very slight puff is sufficient when the opening of the atomizer is placed in O to give many fine oil drops. Some of these drops will finally fall through the holes and get between the plates. A radio battery of 90 or 180 volts is connected to the two plates through a commutator so that the potential can be quickly reversed. Also it ought to be possible quickly to change the voltage.

Oil drops between the plates will scatter light from a flash lamp into the microscope (no direct light should enter) and the position of the drop on the microscope scale can be noted. With no electric field between the plates, the velocity of fall can be measured. Now the Cambridge workers named above had brought out of obscurity a formula, worked out by Stokes fifty years earlier, giving the force experienced by a sphere in moving through a viscous medium,[1] viz., $F = 6\ \pi\mu av$, where μ = coefficient of viscosity, a = radius of the sphere, and v its velocity. When the sphere falls with uniform speed, as it can be seen to do in our case, the above force must just equal its weight. If ρ = density of the oil used, then $\frac{4}{3}\ \pi\rho a^3 g = 6\ \pi\mu a v_g$ where v_g is the velocity due to its weight and g = acceleration of gravity. There is one obvious small correction; the density of air σ must be considered, in which case ρ must be replaced by $\rho - \sigma$. The above relation

[1] The law holds for hail or raindrops falling in air or for small metal spheres falling in water or oil.

then gives the radius of the drop and therefore its mass. Now when the electric field is applied, we can adjust the voltage so that the drop neither rises nor falls. In that case if e is the charge on the drop, $Xe = mg$. Thus e can be determined. But in general it is difficult exactly to balance the drop. It is easier to apply a voltage so that the drop may be pulled up with a velocity v_e. Then

$$\frac{Xe - mg}{mg} = \frac{v_e}{v_g} \quad \text{or} \quad Xe = \left(\frac{v_g + v_e}{v_g}\right) mg.$$

Since mg may be measured, as above, e can be determined.

Stokes' Law Corrected for Small Drops.

Some very beautiful results came out of the experiment as performed by Millikan. In the first place, he found that the charge measured was not the electron charge but 10, 18—a whole number of times that charge. But dividing the charge (on different drops or on the same drop at different times) by whole numbers, he obtained a constant quantity—unless the drop under gravity had a *small* velocity. Now Stokes himself pointed out certain limitations of his law—the sphere must be large compared with the mean free path, its velocity must be small compared with molecular velocities. When Millikan plotted the value of e, the electron charge, against the velocity of fall under gravity, he obtained the curve of Fig. 2–3. It showed that the computed value of e was constant for velocities greater than about 0.1 cm. per second, but for velocities less than this it rapidly rose. The value of e becomes constant if Stokes' law is corrected[1] as indicated by theory and as found by Millikan.

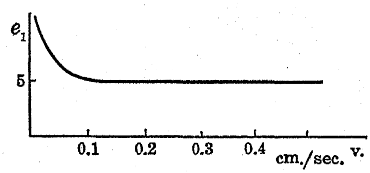


FIG. 2–3. Stokes' law holds for drops falling faster than 0.1 cm./sec.

It is shown in Appendix 2–1 that if there are n electric charges on the drop, then the total charge $e_n = C(v_g + v_e)$ where C is a constant. Now the charge on the drop can be changed by passing X-rays through the oil drop chamber or by holding polonium above the openings in the

[1] Any attempt to measure e without using the correct law would give a value perhaps 20 or 30 per cent of the smaller value in error.

upper plate. Then we would obtain a new value of v_e but the same value of v_g. Then if we divide the $(v_g + v_e)$'s by whole numbers[1] obtained by trial, we ought to be able to get a constant quotient.

The Charge on the Electron.

The accurate measurement of e by Millikan made possible the computation of other important quantities. When we were dealing with electrolysis it was stated that if N were known, e could be computed. Now knowing e we compute N. (In the tables will be found its accepted value expressed to an accuracy of a fraction of one per cent. Compare this accuracy with that obtained for N by the kinetic gas theory of 50 per cent and by Perrin in 1910 of 5 to 10 per cent.) Knowing N, we can compute the *average* weight of a molecule of any element. Thus the discovery of the electron, and its accurate measurement within twenty years after its discovery, shed new light upon the nature of atoms and molecules.

It is easy to write down the quantity representing the charge on an electron, $1.59/10^{19}$ coulomb, and representing its mass, $9/10^{28}$ gram. But to comprehend these quantities is rather difficult. In a 110-volt 100-watt lamp 5.5×10^{18} electrons must pass a point in the filament per second, and if we could get a gram weight made up of electrons there would have to be more than 10^{27} of them. (*But what would happen if we could get that number of electrons together*, with nothing but electrons?) As far as mass is concerned, a hydrogen atom is equivalent to 1840 electrons, but obviously it cannot *consist* of that number of electrons. Yet all atoms must contain some electrons—and what else?

The Atom Becomes Complex—The "Pumpkin" Atom.

An atom, electrically neutral, must contain as many positive units of electricity as negative. If we were to say that hydrogen, the lightest atom, presumably the simplest, contains one electron,

[1] For example, here are three laboratory values of different $(v_g + v_e)$'s (constants left out): 11.91, 18.36, 17.31. Dividing these by 11, 17, 16, we get 1.083, 1.081, 1.083. We then assume that the drop in the first case had on it 11 electron charges. Similarly for the other cases. Then we find the value of e_1. Of course we would have obtained a constant had we divided by 5.5, 8.5, 8, or by 22, 34, 32. But we must divide by whole numbers and we are guided by the known approximate value of e in choosing our dividers. For the best illustration of this matter the student should consult Table IX, Millikan's *The Electron.*

then the remainder of the atom must have a unit of positive charge and possess almost the entire mass. Now in the days of the kinetic theory of gases we thought of one atom as a uniform elastic sphere with definite boundary. This idea was partly carried over to the picture of the atom after the discovery of the electron and then the atom became a sphere of positive electricity, in some way containing large mass, together with one or more electrons embedded in it, the number of negative units just balancing the positive charge. We might call it the "pumpkin" atom—the seeds representing the electrons. If the atom lost one electron it became positively electrified by one unit. If it lost two electrons the positive charge was two units. If the neutral atom acquired an extra electron it was negatively electrified by one unit. As far as these simple ideas were concerned this picture was satisfactory, but it failed to give any idea of the way in which an atom of one element differed from that of another. The picture was abandoned when Rutherford and Moseley, 1912–1914, established the concept of the nuclear atom.

The Mass of the Electron Increases as Its Speed Approaches That of Light.

Let us return to Fig. 2–1 and consider again a stream of electrons passing between the plates of a condenser. We showed there that we could arrange an electric and a magnetic field so that electrons of only one definite velocity could get through. That velocity was given by $v = X/H$. And now instead of obtaining electrons by a high electric potential in a discharge tube, let us use a speck of radium on the end of a needle. As all students of elementary physics know, radium gives out three different kinds of rays, a positively charged stream of helium atoms (nuclei), a stream of electrons, and gamma rays which are composed of extremely short light waves. On the end of the fine wire R (Fig. 2–4) is a minute trace of radium. The electrons coming from R have various velocities, some of them very large, approaching the velocity of light. While the electrons are between the plates they move

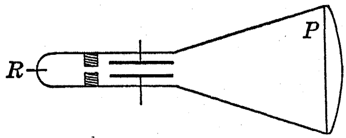


FIG. 2–4. The mass of an electron from a radioactive source, R, is found to increase with its velocity. All mass increases with speed.

parallel to the plates if the condition above is satisfied, but after they leave the condenser they are subject only to the magnetic field and thereby take a circular path to the screen or photographic plate at P. The relation which holds is $Hev = mv^2/r$. Since v is known by the above relation and e and H are known, and r can be measured by the deflection of the spot on the screen or photographic plate, then m can be measured. It is now found that for very high velocities the value of m is not that previously given, 9×10^{-28} gm., but a larger value depending on v. In fact this relation is found to hold,

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where m_0 is the ordinary or "rest mass," m is the new mass, and c is the velocity of light. If v is of the order of 20,000 miles per second, m is greater than m_0 by only about one part in two hundred. But for a speed of 90,000 miles/sec. $m = 1.15\,m_0$; that is, it is fifteen per cent greater than the mass at low velocity (called the rest mass).

The above experiment was performed by Neumann and was a modification of a very beautiful experiment by Bucherer—which unfortunately we cannot here reproduce.

Here is a new idea. Physicists had been accustomed to look upon the law of the conservation of matter as one of the everlasting laws—that the total quantity of matter (mass) in the universe is a constant. And now if the quantity of matter in a body depends upon its speed, what are we going to do about it? The answer is, the increase in mass must have been taken out of something equivalent. Does some other body lose an equivalent mass or is there a loss of energy somewhere that can compensate for the change? In any event we have the experimental fact that as matter approaches the velocity of light, its mass, as we use that quantity in all computations in which force is involved, increases very rapidly and would become infinite if the velocity of light could be reached. The inference is that *matter* cannot have a speed greater than, nor as great as, that of light. But it does not mean that we cannot have some kind of wave velocity greater than that of light.

Other new points of view grew out of this dependence of mass upon velocity. It results that the momentum of a mass as its speed approaches that of light is no longer proportional to its speed nor is its kinetic energy proportional to the square of the speed.

The Velocity Selector.

Again referring to the condenser and electrified particles passing through it under balanced electric and magnetic forces, we see that the balancing depends only upon speed, not upon the mass of the particle or its charge, whether positive or negative. We call the device a velocity selector. Particles which get through must have a certain velocity equal to X/H. Hence we might measure the mass of electrified atoms of matter or of alpha particles instead of electrons.

We Measure the Mass of an Atom.

Bainbridge has done this. Figure 2–5 represents the principles of his apparatus. Atoms or molecules are apt to be positively electrified in a discharge tube. Hence we reverse the terminals as compared with the apparatus for the study of cathode rays. Here the anode is at A, the cathode perforated by a fine hole is at C. The gas to be examined is admitted just below the cathode and diffuses into the discharge tube. Positively charged particles are drawn through the cathode and continue through a fine slit into the velocity selector. The two condenser plates (the two heavy black lines) are connected to radio batteries. The magnetic field is perpendicular to the plane of the paper and is between two soft iron pole pieces facing one another about 1 cm. apart and of cross-section $BDEFGH$. Above EF the discharge tube is magnetically shielded by a heavy soft iron cylinder not shown. After the

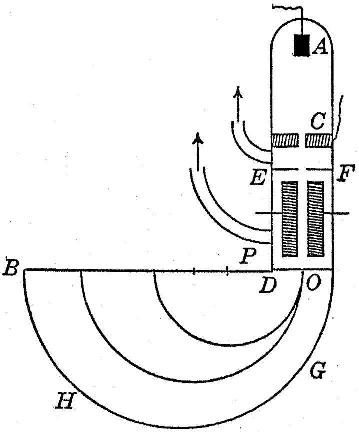


FIG. 2–5. Atoms are weighed by magnetic fields.

rays get through the selector, they are bent by the magnetic field through 180° to the photographic plate, *BP*. From the relation

$$r = \frac{mv}{He} = \frac{mX}{H^2e}$$

it follows that the mass of the atom or molecule is proportional to the diameter of the semicircle—or the distance from *O* to the line (produced by the rays) on the photographic plate. Now since this distance is proportional to m/e, the same line would be made by a helium atom doubly charged (written He^{++}) as by a hydrogen molecule with one unit (H_2^+), if the helium atom had exactly four times the mass of the hydrogen atom. But the photograph (Fig. 2–6, Plate E) shows the two lines separate. Taking the oxygen atom as 16, the masses of all other atoms can be measured—and with an astonishing accuracy.

Isotope of Neon Discovered by J. J. Thomson by the Parabola Method.

It would be very wrong to give the impression that Bainbridge was a pioneer in this work. The first attempt to measure the masses of atoms using electric and magnetic fields was made by J. J. Thomson in 1910. In his apparatus (Fig. 2–1 b) the pole pieces are at MM^1. Thin mica sheets separate these from soft iron plates PP^1 which not only continue the magnetic field but also form the condenser plates. In this case, since the electric and magnetic fields are in the same direction, the *forces* due to these fields are at right angles to each other. The gas particles in *A*, electrified or ionized by the high potential between the cathode *C* and the anode *A*, are drawn through the fine hole in the cathode and continue through a very fine hole in a brass tube and pass on between the condenser plates to the photograph plate. The deflection x due to the electric field and y due to the magnetic are related thus:

$$\frac{y^2}{x} = C\left(\frac{e}{m}\right),$$

where c is a constant. Hence for any definite e/m, but for various velocities, the curve on the plate is a parabola. Figure 2–7 shows some photographs. Various known atoms gave parabolas which

could be easily identified and which demonstrated the accuracy of the analysis. But there was on the early plates one parabola which was perplexing. It was near the known parabola for neon 20 (oxygen 16 being the standard). By a long process of eliminating possible combinations of atoms which would give a mass of 22, viz., NeH_2, CO_2 doubly charged, the revolutionary view was put forth that neon had two kinds of atoms, of masses (very nearly) 20 and 22. Now ever since the time of Dalton it had been assumed that the atoms of any element were identical. The kinetic theory of matter rested on that assumption, as did all chemical formulae. But that an element might have atoms of different masses and yet of the same chemical properties was proposed by Soddy in 1910 as a result of his study of radioactive elements. (A similar proposal was set forth by Sir William Crookes in 1886 but his argument was based on experimental results which later were found to be in error.) Soddy gave the name *isotopes* to elements chemically identical but of different atomic weights. The discovery of a neon 22 was the first verification of this idea among elements other than radioactive.

Aston's Important Work. Dempster Also a Pioneer.

The outstanding name connected with the weighing of atoms and the discovery of isotopes is Aston. Working in the Cavendish laboratory, he designed apparatus for "focussing" particles having a definite ratio of charge to mass and of separating out or dispersing different masses. His text *Mass Spectra and Isotopes* stands alone as authoritative and illuminating in this field and should be consulted. He has examined practically all of the known elements and has found isotopes for about 70. The most abundant isotope of oxygen was taken as 16; all others were referred to it. It is to be noted that though oxygen has 3 isotopes—16, 17, 18 of relative abundance 99.81, 0.03, 0.16—only one was used as a standard. In this method the different isotopes are separated; in chemical methods of obtaining densities only averages of all the isotopes present can be measured. The extraordinary precision obtained in the measurement of the mass of atoms shows that there is only one kind of O 16 as well as one kind of atom of any isotope. For example, Aston's value for H^1 is 1.007775 and Bainbridge agrees with this even in the last

figure.[1] If there is doubt of 1 in the last figure, it would mean that we know the (relative) mass of H^1 to an accuracy of 1 part in 1 million. However, this may be going quite a bit too far. But the student is asked to compare this precision with that of the early kinetic gas method of measuring the mass of an atom.

Another pioneer in this work was Dempster of Chicago. His apparatus was similar to that of Bainbridge but without the velocity selector.

Spectroscopic Methods of Detecting Isotopes.

It ought to be noted that the oxygen isotopes 17, 18 were not originally found by the mass spectrograph but by means of optical spectra as later described. The masses of the atoms computed from the optical spectra are 17.0029 and 18.0065. We speak of the two kinds as having "mass numbers" of 17 and 18. Similarly for all other isotopes.

Chemical Compounds Become Complex.

Attention should be called to the enormous complexity introduced into chemistry by the discovery of isotopes. For example, tin has 11 isotopes, mercury 9, $\cdots$. But let us take some compounds of simple elements, say zinc chlorate, $Zn(ClO_3)_2 4H_2O$. Zn has 5; Cl, 2; O, 3; H, 2 isotopes.[2] It can be shown that ClO_3 has twenty different forms or combinations. It occurs twice in the compound, giving forty different cases. Then putting in the other elements with their various isotopes it is seen that there are 1200 different kinds of "pure" zinc chlorate—this as far as arithmetic is concerned. However, the question regarding the abundance of the various isotopes places in doubt the arithmetical computation.

The Packing Fraction.

In the early days of isotopes and mass numbers, i.e., in the 1920's, it was believed that there were only two building bricks

[1] When this chapter was written (1933) it was believed that Aston's value for H (1.007775) was correct to an accuracy of 1 part in 100,000. It was known that the close agreement of Bainbridge's value with his was accidental. But now (July, 1935) it is known that Aston made an error in his first measurement. The value of H is now given by Aston as 1.0081. This error does not detract from the great value of Aston's work.

[2] Very recently it has been found that hydrogen has 3 isotopes. The student is asked to recompute the above number. With H and O each having 3 isotopes, the number of kinds of *pure water* is 18!

for atoms, the proton or positive part of the hydrogen atom, and the electron. Consequently attempts were made to account for the masses of the isotope atoms using those two bricks. The discovery of the neutron and positron in 1932–1933 puts this matter in a new light but, even so, there is undoubtedly great significance in Aston's "packing fraction" curve. The mass of H^1 is 1.0081 (nearly); its packing fraction is 81. The mass of Ne^{20} is 19.9967; its packing fraction is $[(20 - 19.9967)/20] \times 10^4 = -1.6$. In other words, the difference between the nearest whole number and the atom mass is multiplied by 10,000 and divided by the mass number to give the "packing fraction." It is not a very satisfactory name, since it implies that some one unit, perhaps the hydrogen atom, is the building brick of atoms. However, the curve obtained in plotting these packing fractions and mass numbers is a very smooth curve. (See Fig. 14–12.)

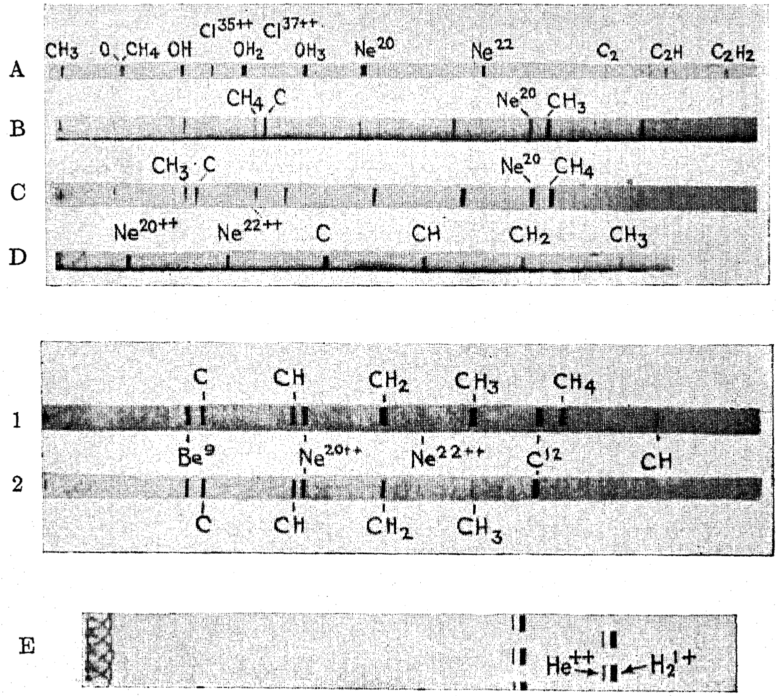


FIG. 2–6.

The great importance of atomic masses in problems connected with cosmic rays and with the transmutation of the elements will be taken up later.

Analysis of Photographs.

In Fig. 2–6, Plate A shows the positions of the lines due to CH_3, O, CH_4, OH, Cl^{35}, etc. Distances are to be measured to a fiducial line on the left. Cl^{35++} should have a position of 17.5 (very nearly) compared with O 16.

In Plate B, CH_4 and Ne^{20} are placed on the plate for a certain magnetic field; then C and CH_3 for another field. Then the ratio of CH_4/Ne^{20} is compared with C/CH_3. Similarly for Plate C.

In Plate D, Ne^{20++}, Ne^{22++} should have positions 10, 11 (nearly). Bainbridge finds for Ne^{20} the mass number 19.9967 for O 16 and for Ne^{22} 21.9947.

In Plates 1 and 2 (Fig. 2–6) the mass of beryllium Be^9 is compared (by the method of Plate B) with C, CH, CH_2, etc. He finds Be^9 to be 9.0130 ± 0.0007. Another check is obtained for Ne^{20} and Ne^{22}.

In Plate E the helium atom doubly charged and the (old or ordinary) hydrogen singly charged are compared. If He is taken as 4.00216, that of hydrogen is 1.007775.

Figure 2–7 a is a "parabola" photograph by Bainbridge. The masses from the bottom up are C, CH, CH_2, CH_3, etc., CO, CO_2. The y distances are given in the table below and the masses M_x, computed from the formula

$$\frac{M_x}{C} = \left(\frac{Y_c}{Y_x}\right)^2$$

where $C = 12$, are given in the fourth column.

Ions	Mass Numbers	Y	Computed Masses
C	12	74	12
CH_1	13	71	13.02
CH_2	14	68.7	14.2
CH_3	15	66.2	15.0
CH_4	16	64	16.1
CO	28	50	28.2
CO_2	44	38.6	44.1

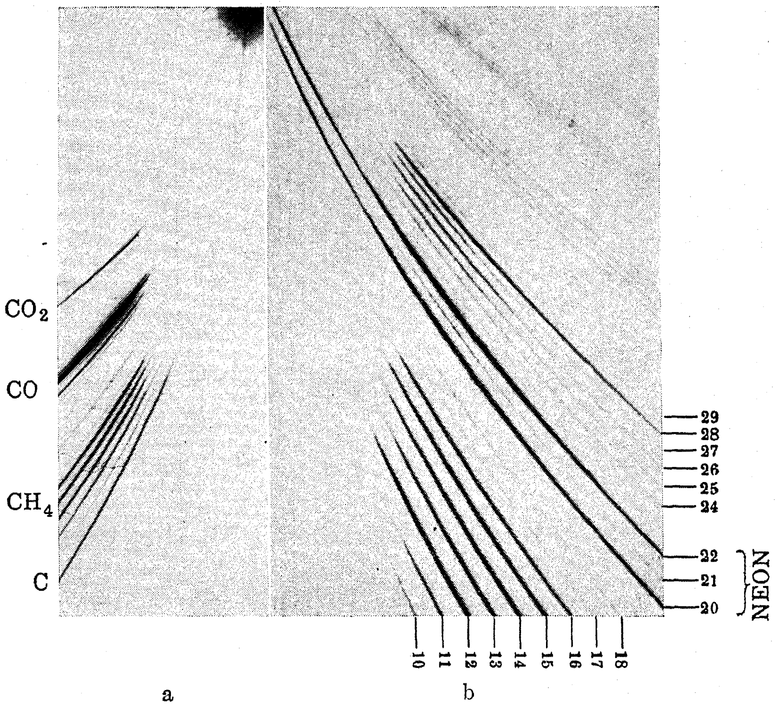


Fig. 2–7. Atoms masses shown by parabolas (a) C, CH_2 – CO_2 (Bainbridge); (b) C(12) CH(13)—especially neon 20, 21, 22 (Harmsen).

It is seen that this method is merely able to identify masses but not to measure them accurately.

Figure 2–7 b is a beautiful photograph by Harmsen (*Zeit. f. Phys.* 82, 1933). *Note most especially neon 20, 21, 22.*

CHAPTER 3

SIMPLE PROPERTIES OF ORBITS. THE DISCOVERY OF THE NUCLEAR ATOM

A very elementary way of considering the motion of Jupiter about the sun or of an electron about a positive charge; in what ways these motions are alike; and what happens when a positive charge is shot at another positive charge.——A new picture of an atom.

We Want to Fire a Projectile so that It Describes a Circle Around the Earth.

Let us think of a mass dropping from a high tower towards the earth. Its speed increases, its motional (kinetic) energy increases, its positional (potential) energy decreases in such a way that the loss in the latter is equal to the gain in the former. The relation expressing this is $\frac{1}{2}mv^2 = mgh$ if it drops from rest at height h and if we neglect air resistance. In this the assumption is that the weight of the body, the mutual attraction between it and the earth, remains constant. This is very nearly true. But as a body is taken away from the earth this attractive force decreases. The gondola of a stratosphere balloon which weighs one ton on the surface of the earth will pull down on the balloon with a force of only 1990 pounds when ten miles high. The law of force is $F = GMm/r^2$ when M and m are masses of earth and gondola, r distance to center of earth, and G the Newtonian constant of gravitation.

But if the gondola were only one mile high and if a bullet were fired horizontally from it with a very considerable speed, the bullet might describe a circle round the earth always one mile above its surface. For when a mass held by a string is describing a circle about an axis, the string must pull in towards the center [1] with a force mv^2/r. Consequently the pull of the earth upon the bullet which is equal to its weight (mg) and also equal to GMm/r_0^2

[1] In this chapter it is understood that either the velocity of the body is small compared with that of light or that our methods of observation are not sufficiently accurate to note the departure from the Newtonian law. We ignore too the curvature of space in the neighborhood of large masses.

must be just equal to mv^2/r_0. Therefore, in order that the motion may be as proposed, v^2 must equal $r_0 g$. Now at the surface of the earth r_0 = radius of the earth $= 10^9/(\pi/2)$ cm. $= 6.37 \times 10^8$ cm. and $g = 980$. Therefore[1] $v = 7.9 \times 10^5$ cm./sec. = 4.9 miles/sec. From the above relations $GM = r_0^2 g$. (If the earth were to turn seventeen times as fast as it does, or one revolution in 1.4 hours, a body at the equator would have no weight.)

If the body were farther out from the earth, r would be greater but g would decrease in the ratio of $1/r^2$. Hence the necessary velocity would vary as $1/\sqrt{r}$. The kinetic energy necessary for the motion might be most easily obtained from the relation

$$\frac{mv^2}{r} = \frac{GMm}{r^2}.$$

Then

$$\frac{1}{2}mv^2 = \frac{GMm}{2r};$$

that is, the kinetic energy necessary to keep the body in the circular orbit would decrease as the distance to the earth increases.

We Fire the Projectile with a Speed of 6 Miles per Second; Then 7.

What would happen if at the earth's surface a bullet were fired horizontally with a velocity greater than that computed above, say 6 miles/sec.? Obviously it would get some distance from the earth but it would always be under the law of attraction and the force would be GMm/r^2 where r is now the instantaneous distance to the earth's center. It can be rather easily shown that it would describe an ellipse with the center of the earth as one of the foci. The semi-major axis a of the ellipse would be given by

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{GM},$$

where r is any radius vector from this focus to the ellipse, v is the velocity at that point, and G and M are as before. At the starting point of our motion, r is the radius of the earth and $v = 6$ miles/sec. $= 9.656 \times 10^5$ cm./sec., and a comes out $= 1.25 \times 10^9$

[1] Obviously if one were at the equator and fired the mass east, the required speed would be less than 4.9 by the speed of rotation 0.29 miles/sec. If west, greater by the same amount.

cm. = 7767 miles; or $2a$ the major diameter of the ellipse = 15,534 miles. The semi-minor axis b is 1.08×10^9 cm. or 6900 miles.

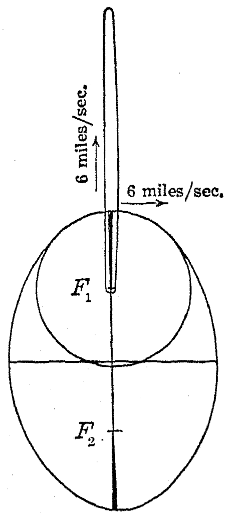


FIG. 3–1. A particle fired horizontally on the earth's surface with a speed of 6 miles/sec. describes an ellipse; fired vertically it describes a thin ellipse of the same major axis.

From the data just given it is seen that this projected mass would describe an ellipse which, starting out tangential to the earth, would be 7500 miles from the earth's surface when at the other end of the major axis (Fig. 3–1). As the velocity of projection increases, the ellipse becomes more elongated until a becomes infinity, in which case $v^2 = 2\,r_0g = 2\,GM/r_0$ or $v = 7$ miles per second. In other words, a body projected from the earth at a speed of 7 miles/sec. would leave the earth. This is called the velocity of escape. We have taken the direction of projection as horizontal, but it makes no difference what the direction is so long as it is not towards the earth—the body will escape from the earth. Attention is called to this matter of direction since in a number of recent texts on modern physics it is erroneously stipulated that the direction must be upward, that if the direction is not vertical the speed must be such that the vertical component is 7 miles per sec. This is incorrect.

The Diameter of the Elliptic Orbit Is Independent of the Direction of Firing.

Let us return to the case of the 6 miles per sec. speed—and suppose this is vertically upwards. How far *up* (h) would the particle go? We equate the kinetic energy $\frac{1}{2}\,mv^2$ to the gain of potential energy; that is, to

$$GMm\left(\frac{1}{r_0} - \frac{1}{r_0 + h}\right).$$

When we substitute the values of v, GM, and r_0, h comes out to be 11,530 miles. Adding the radius of the earth, the distance from the earth's center is 15,530—which is the major axis of the ellipse in the case of horizontal projection. If immediately after firing

the particle from the surface of the earth we caused the earth to shrink to a very small body (retaining all its mass, however) the body fired up would, after reaching a distance of 15,530 miles from the earth's center, stop, return, pass the original boundary of the earth with a speed of 6 miles per second, and continue to gain speed. If it was fortunate in not hitting the central great mass, it would turn very rapidly around that body and start out again practically parallel to its original direction and go out to the distance of 15,530 miles from the earth's center. This is a special case of the previous elliptic motion.

It can be seen that the velocity as the particle rounded the central body would be very great. In consequence of this great velocity its mass would be increased (page 30). But an increase in mass, obtained at the expense of its energy, would cause a variation in the orbit. This is an illustration of a point that must be considered when we deal with the Bohr atom.

Hydrogen Escapes from Our Atmosphere; All Gases from the Moon.

In Chapter 1 it has been shown that the mean speed of hydrogen molecules in our atmosphere is more than 1 mile per second and that there would be a fraction of those molecules with speeds greater than the above "velocity of escape." Consequently we should expect that hydrogen would gradually escape from our atmosphere, as it apparently does. On the moon the velocity of escape owing to the small mass of the moon is small. Given that the moon's diameter is about 1/4 and that its mass is about 1/80 the corresponding earth quantities, it can be seen that the velocity of escape on the moon is about $7/\sqrt{20}$ miles/sec. The moon has lost all its gases.

The Orbit of Escape Is a Parabola.

For the critical speed of 7 miles per second (see Appendix 3–2) the orbit of the projected body is a parabola. At infinity the speed of such a body would be zero. The kinetic energy of the body at the earth's surface is just sufficient to overcome the continuous pull of the earth all the way out to infinity. Had the speed been greater than this critical speed of 7 miles per second, the body would have some energy left over at infinity. The orbit would have been an hyperbola.

The Smaller the Orbit, the Less the Major Axis of the Ellipse, the Less the Energy.

This discussion has been introduced here to make it clear that when a body is describing an ellipse or a circle about an attracting mass it does not have sufficient energy at any point to escape to infinity. Let us now reverse the problem and start with a small mass at rest at an infinite distance from a large attracting mass. There is mutual attraction. The center of mass remains fixed. The total momentum at any time is zero. But if the large mass is very large compared with the small one, we may take the center of mass of the two as identical with that of the large mass. As the small mass falls in towards the large, it gains kinetic and loses potential energy. The work done by the attracting force equals GMm/r, and this must equal $\frac{1}{2}mv^2$. Now it is obvious that a body with this energy could not describe an ellipse or circle about the large mass. The energy would be too great. How much too great? Suppose we shunt the motion sideways and the body starts describing a circle of radius a. Then we know that the force of attraction, GMm/a^2, must equal mv^2/a. Hence $\frac{1}{2}mv^2$ must equal $GMm/2a$ or the kinetic energy of the body in the circular motion must be just half that which it would have been had the body come from infinity under this law of attraction. *So one-half of the work done in pulling the body in from infinity must in some way be lost or thrown away in order that a circular orbit may be described.* If we take the total energy when the body is at infinity as equal to zero, then the total energy when the body is describing a circular orbit is less than zero, that is, a negative quantity and equal to $-GM/2a$.

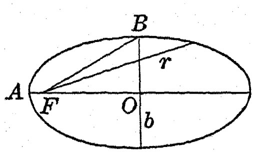


FIG. 3-2. In an elliptic orbit the attracting mass is at F.

In the case of an elliptical orbit the distance r is changing within certain limits but analysis shows that the total energy is still equal to $-GMm/2a$ where a is the semi-major axis. (See Appendix 3-5.) In the ellipse (Fig. 3-2) F is the focus;

$$OA = FB = a; \quad OF = ae; \quad AF = a(1-e); \quad ae = \sqrt{a^2 - b^2}.$$

The student can prove that the kinetic energy at B is equal to that in a circular orbit of radius a.

The total energy may be written

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a} \qquad \text{or} \qquad v^2 = k\left(\frac{2}{r} - \frac{1}{a}\right)$$

where $k = GM$ and v is the velocity at any point on the ellipse distant r from F.

Energy Depends Only on the Major Axis.

Thus the *total energy* of a mass (in reality of both masses) describing an ellipse is the same as that for the same bodies when a circle is being described if the major axis of the ellipse is equal to the diameter of the circle (Fig. 3–3). The minor axis does not count in the energy—it helps to fix the focus. The student probably knows that in our solar system the planets revolve about the sun in ellipses which are nearly circles. Now it is obvious that our solar system could not have been formed by the drawing in by the sun of the various planets from an infinite distance—not unless they came through a viscous medium which robbed each planet of half its energy, shunted the motion off sideways, and then disappeared from this part of the universe.

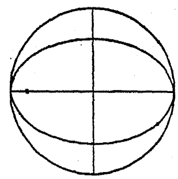


FIG. 3–3. The energy in the circle about its center is the same as that in the ellipse about its focus.

In the special kind of ellipse for which a is very great, $1/a$ becomes equal to zero and we have parabolic motion, then the velocity at any distance r is given by $\frac{1}{2}mv^2 = GMm/r$. This case has been discussed above.

Energies in the Various Orbits.

In the case of hyperbolic motion the energy must be greater than in the parabolic motion. The relation then is

$$\frac{1}{2}mv^2 = \frac{GMm}{r} + \frac{GMm}{2a}, \qquad \text{or} \qquad v^2 = k\left(\frac{2}{r} + \frac{1}{a}\right).$$

Thus under the law of *attraction* varying as $1/r^2$ we may have as an orbit an ellipse, a parabola, or a hyperbola according as the kinetic energy is less than, equal to, or greater than GMm/r. Again it might be well to point out what this frequently recurring quantity, GMm/r, is. It is the work necessary to take to infinity the body, m, which is at a distance r from the center of attraction.

Hence if we regard the energy when at infinity as equal to zero, the potential energy at distance r is $-GMm/r$. Since the kinetic energy is $\frac{1}{2}mv^2$, then the total energy is $\frac{1}{2}mv^2 - GMm/r$. In a circular or elliptical orbit this equals $-GMm/2a$. Hence the relation

$$v^2 = k\left(\frac{2}{r} - \frac{1}{a}\right),$$

where $k = GM$.

If now we replace gravitational by electrical forces, we may have identical relations. The bodies still must possess mass. Hence we picture a small mass m and charge $-E'$ revolving round a large mass M and charge $+E$. The attractive force is now EE'/r^2. Again we shall have as orbit an ellipse, a parabola, or a hyperbola depending upon whether

$$v^2 = \frac{EE'}{m}\left(\frac{2}{r} - \frac{1}{a}\right), \quad \text{or} \quad v^2 = \frac{2EE'}{mr}, \quad \text{or} \quad v^2 = \frac{EE'}{m}\left(\frac{2}{r} + \frac{1}{a}\right).$$

Again it is true that if the body is describing one ellipse and desires to go to another of smaller major axis, it must lose energy. The kinetic energy increases but the gain in kinetic energy is not as much as the loss in potential energy.

So far the results for the gravitational and electrical forces seem identical. But there is possibly one great difference. The former are always attractive, the latter may be repulsive. If the two charges are of the same sign this is the case.

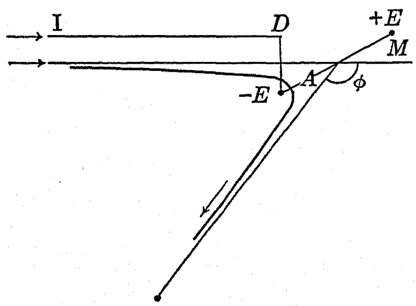


FIG. 3–4. A positive charge driving straight at $+E$ may get as far as D then go straight back. But starting towards A it describes a hyperbola.

Hyperbolic Orbits Are "Two Faced"; They May Be Due to Attraction or Repulsion.

Now consider a small mass m and charge $+E'$ which has been projected from a great distance directly towards a large mass M and charge $+E$ (Fig. 3–4). The kinetic energy will continually decrease, the potential increase,

until m arrives at D (where $ED = d$) when it comes to rest, then retraces its path towards infinity. Now it is seen that the kinetic energy at infinity, $\frac{1}{2} mv^2$, must equal EE'/d since this is the potential energy at D.

If the line of projection is not directly towards D, but is to one side, the path described is a hyperbola with M or E as outside focus. The other focus can be shown to be on a line drawn perpendicular to ID at D. Now this hyperbola due to repulsion from E is the hyperbola that would have been described had m, with a charge of $+ E'$, started inwards with the same velocity at infinity but under a law of *attraction* about $- E$. This is an important point. It shows that an α particle with a positive charge of two units will in general describe a hyperbolic path when fired towards a heavy mass charged positively. It will lose speed until it goes through A nearest the heavy mass. But it would have described the same path if there had been an equal mass with an equal negative charge at a different place, at the other focus of the hyperbola. In this case, however, its speed would have been very great as it rounded the negatively charged pylon.

Alpha Particles as Projectiles; Atoms as Targets.

Now let us see what would happen if an α particle were projected so as to pass near or through an atom. To begin with, we know the speed, approximate mass, and charge of the α particle. But what about an atom? We have evidence to show that the chief mass of an atom is associated with the positively charged part. Let us then picture the atom as a "pumpkin" atom, a sphere of positive electricity uniformly distributed, with negative electrons in shells inside. While the α particle is outside the atom there is no electric force—the positive electricity acts as if all its charge were concentrated at its center and presumably the electrons symmetrically arranged on a shell would act in the same way. The α particle therefore would not be disturbed from its straight line motion nor would its speed be altered as long as it is outside. If it enters the atom, presently some of the positive electricity is in the shell outside of the α particle and that much of the positive electricity would exert no force on the particle. If the electron shell is still inside, there would then be an attraction towards the center of the atom. Then if the particle gets inside the electron shell, there would be small repulsion since now

but a fraction of the electricity would be "inside" the particle. Altogether this would be a difficult kind of motion to follow. Moreover since the α particle has a mass more than 7000 times that of an electron, there would be a large probability of the latter being given a great speed which might throw it out of the atom.

Now several years before the discovery of the electron, a conclusion had been reached regarding the *mass* of an isolated electric charge. It appeared that the mass was inversely as the radius of the charge. The formula that was found was $m = \frac{2}{3} e^2/a$ where e is the electric charge in e.m.u. and a is the radius of the spherical shell upon which the charge is uniformly distributed. [For an electron of charge 1.59×10^{-20} and $m = 9.03 \times 10^{-28}$ gram, $a = 1.9 \times 10^{-13}$ cm.] Hence if we regard the mass of an atom as due to its charge, its radius must be small compared with that of an electron. Let us then picture a neutral atom as one containing a number of electrons in a relatively large shell and a positive charge neutralizing the combined charge in the electrons but containing almost all the mass of the atom and therefore having a small diameter. If an α particle were projected towards this atom, there would be no force as long as it was outside the electron shell. But immediately after it got inside this shell it would be repelled by the concentrated positive charge. This would have a large mass and the particle would describe hyperbolas due to repulsion, as outlined above.

The Nuclear Atom Is Discovered.

Rutherford devised an experiment (1912) to test this idea. He would allow a stream of alpha particles from radium to fall upon a sheet of gold foil and he would measure the relative number of the particles which came from the foil at various angles to the original direction of the stream. In his laboratory Geiger and Marsden counted by the scintillation method the number of such particles scattered between certain angles. In Appendix 3–8 will be found the deduction of the following relation:

$$P = \frac{\pi}{4} nt(2\,a)^2 \left(\cot^2 \frac{\phi_2}{2} - \cot^2 \frac{\phi_1}{2} \right)$$

for the probability that a particle will be scattered between the

angles ϕ_1 and ϕ_2 (Fig. 3–5). Here 2 a or d is the distance of nearest approach, the major axis of the hyperbolas due to repulsion, n is the number of repelling centers per cm.^{3} and therefore equals Avogadro's number N divided by the atomic volume or $N\rho/A$; t is the thickness of the foil. All of these quantities are measurable, and since P was found experimentally, then 2 a or d could be computed. This gave the *distance of nearest approach*—therefore it gave a superior limit to the sum of the radii of the alpha particle and the radius of the gold nucleus.

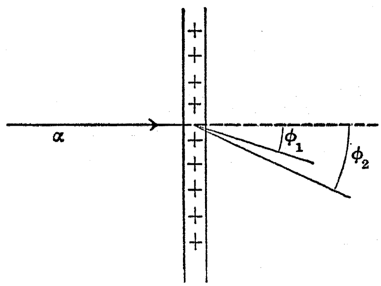


FIG. 3–5. The nuclear atom was discovered by observing how alpha particles were scattered in passing through gold foil.

In the other relation we have used,

$$\frac{1}{2}mv^2 = \frac{EE'}{d},$$

or

$$E = \frac{mv^2d}{2\,E'},$$

E'/m is the ratio of charge to mass for the alpha particle, v its speed. These are known from the electric and magnetic deflections of that particle. Since d for gold has been found, then E is known. It was found that E was approximately proportional to the atomic weight, and finally was put in the form of Ze where Z is the atomic number. Mosely's work, later to be described, assisted in establishing this extremely important relation—the charge on the nucleus is equal to the atomic number (H = 1, He = 2, $\cdots$ *etc.*) times the electron charge.

Geiger and Marsden not only showed that P varied as t, the thickness of the film, but also that the relation above held for scattering angles from 150° to 5°. Now for $\phi = 150°$ the collision is nearly "head on" and the alpha particle must approach the gold nucleus nearly within the distance d or 3.4×10^{-12} cm. But for $\phi = 5°$ the distance is $a(1 + \epsilon)$ where ϵ is the eccentricity and this equals $a(1 + \text{cosec}\,\phi/2)$ or 23 a or 40×10^{-12} cm. Hence

they concluded *that the law of the inverse square held for the force between the distances 3 and 40 $\times$ 10^{-12} cm.; therefore that there could be no negative electricity within shells of those radii.*

Thus arose in 1912–13 the idea of the *nuclear atom* which surely is in strange contrast to the elastic sphere atom of the kinetic theory and the "pumpkin atom" of J. J. Thomson. The idea that an atom has such a positively charged nucleus is now an established feature throughout the realms of physics and chemistry.

There is however the necessity of presenting the view that the atom might have the electrical charges reversed; that there is an outer ring or shell of positive charges and a central core of negative charge. The force is still zero while the α particle is outside the atom, then becomes attractive when it is inside. There would be the same hyperbolic path (neglecting the reversal in magnitude of the masses), the same deflection—but what about the mass of the core? As has been stated, the mass of an atom is associated with the positive charge. Consequently the α particle would produce a very great displacement of the negative core—so great as to produce large variations in the so-called hyperbolic orbit and large disruptions in the atom. The results of the experiment do not satisfy this picture.

The experiment described above was of importance not only because it confirmed the idea of the nuclear atom and gave us the approximate values of the atomic number; it was of importance also since it was the first attempt to explore the inner recesses of the atom. It opened up a universe of a new dimension; it increased "experimental space" from that of the atom, or 10^{-24} cm.^{3}, to that of the nucleus, or 10^{-36} cm.^{3}. Now the volume of the earth is about 10^{27} cm.^{3}; the universe, that which can be seen by the most powerful telescope, is only about 10^{72} cm.^{3}. It is seen that the ratio of the volume of the earth to that of the nucleus of an atom is a million million million times greater than the corresponding ratio of universe to earth. Throughout the vast realm of the universe and down towards atomic dimensions the Newtonian law of gravitation holds. In the minute regions of the nucleus, forces of repulsion of like charges and presumably forces of attraction of unlike charges follow the law of the inverse square of distance.

The universe of the physicist has been expanding.

CHAPTER 4

RADIATION

Some extraordinary facts about the most ordinary phenomenon with which humans are acquainted—light.——What do we mean by the "velocity" of light?——Why is it an important quantity?——How do we account for the pressure of light?——In some ways light behaves like a perfect gas.——Is there distribution of energy in light as there is in the molecules of a perfect gas?——Perplexing properties—continuous or granular?——Planck forced to make an unwelcome assumption in order to "explain" an experimental fact.——The beginning of such assumptions.

Light—Corpuscles or Waves? Its Velocity.

Every schoolboy knows that Newton, when he didn't think of light as a wave motion, thought of it as a flight of particles. As such a flight the straight line propagation and reflection of light were easily explained. To account for the bending of light towards the normal as it passed from air to glass, it was necessary for Newton to assume that it was attracted towards the so-called denser medium and that its speed was greater in glass or water than in air. But Huyghen's explanation of the same phenomenon about the same time required that it should be less. That particular point was not put to test until it was done by Foucault in 1854, and more definitely two hundred years after Newton's proposal by Michelson in 1880. The latter found that yellow light travels slower in carbon bisulphide, CS_2, than it does in free space (air) in the ratio 1/1.758. Now on the basis of the wave theory, the ratio of the speed of the wave in CS_2 as compared with air should have been very accurately 1/1.640. What was it that Michelson measured? The answer is, he measured the "signal" or "group" velocity, not the wave velocity.

Wave and Group Velocities.

Let us think of two trains of waves travelling through a medium, the longer waves moving faster than the shorter. Suppose two

crests (Fig. 4–1) are superposed at A and B. There will be heaped up maxima at A and B with smaller maxima between. If the two sets had the same speed, if the medium were non-dispersive, these maxima crests would have that speed, but if Set I moves faster than Set II, then it will be necessary for crest 4 in Set I to overtake crest 5 in Set II since crest 5 in Set I has got ahead of crest 6 in Set II. It is not difficult to work out the relation

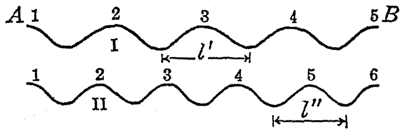


FIG. 4-1. The long waves may travel faster than the short. If so, what is the speed of the combined wave motion?

$$U = v - l\left(\frac{v' - v''}{l' - l''}\right)$$

for the velocity of this maximum crest or the "group" velocity where v is the velocity of yellow waves and the v's and l's are velocities and wave lengths of the red and blue waves. We have used the terms "red" and "blue" though in reality the wave lengths may differ by only infinitesimal amounts. Now "wave" velocity is measured by finding the amount light is bent in passing through a prism, and when Michelson substituted the values so measured in the relation above he found that he had measured accurately the "signal" or "group" velocity. It is to be noted that this signal velocity is less than even the slower of the two velocities if the long waves travel faster than the short. In the case of deep water waves, for example, the wave velocity is proportional to the square root of the wave length or $v \propto \sqrt{\lambda}$ and the group velocity is one-half the wave velocity. But in capillary waves where $v \propto 1/\sqrt{\lambda}$ it is 3/2 the wave velocity. Now the student may not like this splitting of hairs in dealing with velocities. But Michelson's measurements of the velocity of light have been preëminent. What would have been thought of the accuracy of his work or of the wave theory if the discrepancy between 1.758 and 1.640 had not been accounted for? Moreover in one of the very latest developments in physics great importance attaches to the relation between the two velocities. In Chapter 15 we discuss the ideas of de Broglie, that a *particle* is a crest having a group velocity very different from that of *its waves*.

Michelson's Experiments.

Michelson's first measurement of the velocity of light, though perhaps one hundred times as accurate as any previous measurement, was made when he was a young man. About forty-five years later he greatly increased the accuracy. Figure 4–2 gives a rough idea of his method. Light from an intense arc lamp passing through a slit S is reflected from one face of an octagonal mirror M, thence to a mirror on a mountain top twenty-two miles away, returning to another face as shown, then into an observing telescope. The adjustments are first made with the mirror at rest. Then when the mirror is rotating the light in the observing telescope cannot again fall where it first fell in the telescope unless one face of the mirror has exactly replaced the face (or the second, third, ···, face) ahead of it. In other words, while the light was going 22 miles and back, the mirror must have turned through one-eighth (two-eighths, three-eighths, ···) of a revolution. With the mirror rotating 530 turns per second, this coincidence occurred. Then the computed velocity of light in air was 299,728 km. per sec.; reduced to a vacuum this was 299,796 km. per sec., an accuracy of better than 1 part in 10,000.

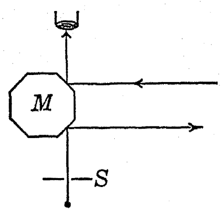


FIG. 4–2. Michelson re-determines the velocity of light.

Why attach importance to this matter? Because this velocity is generally regarded as a constant of nature. That is one of the assumptions in the theory of relativity. But is it constant? The most recent measurements indicate the possibility that it may have a periodic variation. If it should definitely be determined that the velocity of light increased or decreased during the years, or that it was periodic or changed in any way, the announcement of that fact would produce a most perplexing situation in physics and astronomy. However, nothing but the most refined methods will detect this variation if it exists. Let us note what these refinements are—an extremely intense source of light, a method of producing very rapid rotation of an optically perfect mirror in which the mirror is supported on an air jet, a method of measuring to a high accuracy the speed of rotation of the mirror by comparing it with the period of a tuning fork kept continuously in vibration by an electron tube so that it can be

calibrated with the utmost accuracy, and finally the extremely accurate measurement of a base line from which the distance to the mirror 22 miles away can be obtained by triangulation. The most modern devices in science were utilized in this measurement.

Phenomena Establishing the Wave Theory.

The phenomenon of interference, discovered about 1800 by Dr. Thomas Young and greatly extended by Fresnel during the following years, established the wave theory of light. This phenomenon can be seen by anyone for himself by looking at the filament of an incandescent lamp or any fine strong light source at a distance through a fine piece of silk. A central white filament with colored filaments on the sides will be seen. In the laboratory, light from a fine slit is passed through or reflected from a grating, a surface with finely ruled lines on it. Generally there are 5000 or 10,000 lines per cm. ruled with great precision. Assume that the wave trains (Fig. 4–3) in 1 and 2 agree in condition of motion—that they are in the same phase, as we say—then along the lines 3 and 4 they will also be in the same phase if 3 falls behind 4 by one wave length. Similarly for all lines parallel to 3 and 4, they will be in the same phase. A lens will bring all this light to a focus. Hence the position of the first maximum on one side of the central maximum is given by the relation $\lambda = a \sin \theta$ where a is the grating space. The measurement gives λ to an astonishing accuracy.

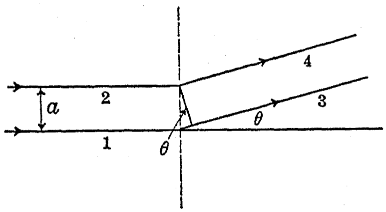


FIG. 4–3. The wave length of light may be measured by means of a grating.

Spectra, Wave Lengths, Frequencies.

By either a prism or a grating it can be shown that light from a furnace or from the filament of an incandescent lamp gives a continuous spectrum; light from a glowing vapor or from a gas through which an electric current is passing, as a neon light, gives strong, rather sharply defined, bright lines; light from the sun may at first appear continuous but really contains a great number of dark lines in the continuous background. It is the wave lengths

of the lines bright or dark that are measured. And to what accuracy! The wave length of the red line of hydrogen for example is 0.0000656285 cm. This is carried out to the ten-thousand-millionth of a centimeter and it is not a sharp line! It is convenient to take as a unit of length $1/10^8$ cm., called an angstrom (Å), in which case we roughly give this wave length as 6563 angstroms. Instead of wave length we may identify the line by the number of waves in 1 cm. Thus the wave number of H_α is 15,233. The visible spectrum extends from about 7500 to 3900 angstroms. But only a very small part of the energy emitted by a hot furnace is in the visible. The wave lengths run all the way from 1000 times to 1/10 of the visible. Instead of either wave length or wave number, we may identify the lines by frequencies. In sound, for example, we know that when a tuning fork vibrates 100 times per second, and the velocity of sound is 1100 ft. per sec., the wave length is 11 ft. In general the relation holds, $fl = v$ where f is the frequency (here 100 per sec.). For the red hydrogen line the frequency is $3 \times 10^{10} \div (6563/10^8)$ which is $15{,}233 \times 3 \times 10^{10}$ = nearly 457 million million, or 4.57×10^{14}. Rather fast. Perhaps this is the most definite way in which to identify lines, for the velocity of light changes from one medium to another but the frequency is an everlasting constant for any one radiation.[1]

Wave lengths? Frequencies? In what kind of a medium? Why, the ether of course. But what is it? It must fill all space and permeate all matter. Since the velocity of a disturbance through an elastic medium must be of the order of the square root of the elasticity divided by the density, and for light this equals 3×10^{10} cm. per sec., it follows that if we make the density of the ether one one-thousandth of that of air its rigidity must be one thousand times that of steel. It must be able to transmit the feeblest ray through vast interstellar spaces with absolute fidelity, it must respond to the tiniest impulse, yet it must not be disturbed in any way whatever by the earth whirling through it at a speed of nearly 20 miles per second. We laugh at the tale of the old monks who would debate at length the important question as to how many angels could stand on the point of a needle. Yet during the latter half of the nineteenth century many of the foremost physicists of the world were engaged in devising new

[1] Except for Compton encounters or other disturbances later noted.

models that would harmonize the conflicting properties of the ether.

Light Is an Electric Phenomenon. Maxwell's Theory.

An epoch-making theory was put forth by Maxwell in 1866–1873, the electromagnetic theory of light. In that he prophesied the existence of electric waves which would possess all the properties of light—in fact, light was of electric origin. This gave us a new mode of picturing the cause of light—it was now thought to be due to the vibration of an electrified particle; the electric field due to the particle extending out through space was put into vibration as air is put into motion when a tuning fork vibrates. A vast amount of physics has been written on that basis.

Electric Waves Are Discovered by Hertz.

The electric waves foretold by Maxwell were discovered by Hertz some twenty years later. The enormous industry of radio resulted. We in America may now listen to sounds due to electric waves originating in Europe, Asia, the South Pole—waves which may be a few miles in length or only a few meters. To those who have known the difficulties which have been overcome in bringing Maxwell's ideas to fullfilment, radio will never cease to be a miracle.

It is not possible here to list all the accomplishments of Maxwell's theory, but there was another prophecy which has had a large place in recent physics—that light should exert a pressure on any surface on which it fell. Why should it exert a pressure, and how much? Maxwell gave an electrical proof which we shall not here consider and he computed that full sunlight falling on an absorbing surface would press with a force of two and a half pounds per square mile, rather less than five one-hundred-millionths (5×10^{-8}) of a gram per square centimeter.

The Pressure of Light Is Measured.

The experimental discovery of this pressure was made by Lebedew in Russia and Nichols and Hull in America in 1901–1903. The latter verified to an accuracy of a few per cent the relation given by Maxwell—that it should be equal to the energy density of the light.

The "torsion balance" used in the original apparatus of Nichols and Hull is shown in Fig. 4–4 a. *C* and *D* are two thin circles of cover glass each brightly silvered on one face. They are suspended on small glass hooks, held up by a fine vertical glass rod *ab*. This again is suspended by a very fine quartz fiber. The pressure of the air in the bell jar in which the torsion balance was mounted was adjusted so that the disturbing action of the air molecules on the vanes was a minimum. For with a certain pressure of the air in the bell jar the exposed vane, starting out as if pushed by the light, would turn back towards the light; obviously the gas action was suction. For a lower air pressure the vane would continue to turn from the light, the increasing deflection depending on the time of exposure. Clearly the gas action was pressure. Thus between these two conditions the most favorable air pressure was found. The moment of force necessary to turn the balance through a definite angle was determined from its period of vibration. Then the deflection when the light was thrown on one vane gave the force due to the light beam, therefore the pressure of the light.

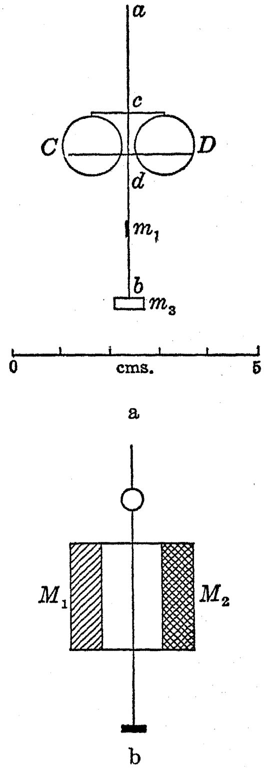


FIG. 4–4. Thin glass vanes used in the measurement of the pressure of light.

Theory required that the pressure should equal the energy density of the light beam in front of the surface. When light falls on a totally absorbing (a black) surface, the energy density in front of the surface is (very nearly) equal to that of the incident beam. In that case the surface would experience a maximum of heating and the air molecules in its neighborhood would be given a lively motion. "Gas action" then would be much in evidence and would militate against an accurate measurement of the light pressure. But when the surface is brightly silvered, the light is almost totally reflected, hence the energy density and the pressure

should be (nearly) doubled. Moreover in this case, as there would be small absorption, there would be small heating and therefore small disturbing gas action.

In order to compare the experimental results with Maxwell's view it was necessary to measure the reflection coefficient of the surfaces and the intensity in the incident light beam. This was done by finding the rate at which a fine strip of blackened platinum or a small silver disc was heated. This part of the experiment proved most difficult of accurate measurement. An error of computation crept into the original papers, so that the accuracy attained was not as great as first reported. However, it was shown that Maxwell's theory was verified to an accuracy of about 6 per cent.

The author extended the experiments and modified the apparatus. Figure 4–4 b shows two rectangles of thin glass mounted as before. But now the greatest care was taken to make the surfaces vertical. It resulted that gas action was of very small importance, in the bell jar used, for changes in air pressure from 2 to 20 cm. After all vapors had been driven from the surfaces by light, it was found that the deflection of the torsion balance when the light fell upon the silver surface was to that when it fell on the black surface in the ratio of 1.90 to 1. When light falls on a clean silver surface about 92 per cent is reflected, hence the energy density in front of that surface is 1.92 of that of the incident light. On the black surface it is all absorbed except about one per cent. Hence the ratio 1.90.

Dr. J. D. Tear, using very different methods, has successfully eliminated gas action and has demonstrated the action of light pressure itself through large ranges of gas pressure in the vessel in which the torsion balance was suspended.

Why Should Light Exert Pressure?

But how is it possible for waves in a medium, the ether (which is "imponderable" according to the long-held views of physicists), to exert a pressure? (Incidentally these waves are transverse—that makes matters worse.) There is one proof or explanation which gives some satisfaction. Let us imagine a train of waves 1 square centimeter in cross-section, $c + v$ cm. long (Fig. 4–5), falling upon a reflecting mirror M which is moving forward to meet the waves; c and v are velocities of light and of the mirror.

In one second all these waves will have struck the mirror and the first wave will be back a distance c from the first position of the

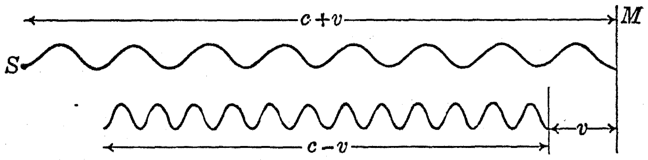


FIG. 4-5. The pressure of light may be explained on the basis of the wave theory.

mirror, a distance $c - v$ from the mirror at the end of the second. Hence all the energy in the original train of length $c + v$ is compressed to a length of $c - v$. The wave lengths are changed in the same way. Now it is well known that in ordinary motions of elastic bodies the energy density varies as $1/l^2$. Hence the energy density is now

$$\left(\frac{c+v}{c-v}\right)^2 E,$$

where E was the original energy density. Hence there is an increase of energy

$$(c-v)\left(\frac{c+v}{c-v}\right)^2 E - (c+v)E = \left(\frac{c+v}{c-v}\right) 2\,vE = 2\,vE$$

(since v is very small compared with c). Hence work must have been done by the mirror in advancing a distance v. And if p is the pressure, this must equal pv. Therefore the pressure equals $2\,E$ and this is the energy density in the total, direct and reflected, light.[1]

The Molecules of a Gas Also Exert Pressure. Comparison with Light.

It ought to be noted here that unless the wave motion is of such a nature that the energy density varies as $1/l^2$, this proof does not hold. In certain very recent texts the following "proof" is given. Let E be the energy density of the light falling upon an absorbing surface. Let the mirror be pushed forward a distance

[1] See also proof of pressure of light in the discussion of the Compton Effect, Chapter 7. In Appendix 6–4 will be found the author's proof that in a beam of circularly polarized light there is a torque about the direction of propagation. The experimental detection of the torque in a beam of circularly (or elliptically) polarized light has been made by Richard A. Beth (*Phys. Rev.*, 48, 471, 1935).

d. The extra energy dE has been received on account of the mirror advancing a distance d. Hence the work done equals pd; therefore $p = E$. That that argument is fallacious may be seen by merely allowing the increased energy to go into the mirror as before. Then there is no gain in energy and no work done in pushing the mirror forward. Hence p would equal zero. The same kind of argument might be used for a flight of bullets against a target. The argument would give the pressure equal to the energy density, whereas it is twice that value. Let there be a procession of bullets, of n per cm.^{3}, falling on an *absorbing* surface of 1 cm.^{2} area. The momentum lost per particle $= mv$: the number falling on 1 cm.^{2} per second $= nv$. Hence the pressure, which equals the momentum lost per sec. per cm.^{2}, equals nmv^2. But the energy density $= \frac{1}{2}nmv^2$. Hence $p = 2E$. For light on an *absorbing* surface $p = E$.

If the particles are going in all directions, $p = nmv^2/3$ as found in Chapter I or $p = 2E/3$. If light is going in all directions, $p = E/3$. Here then is a way in which a beam of light resembles a flight of particles—the light also possesses momentum but there is a curious difference, the light momentum is only half of that of the particles of equal energy density. On the other hand, we must remember that if ordinary matter could attain the speed of light the mass, therefore the momentum, would be infinite.

"Temperature" Radiation Resembles a Perfect Gas.

Let us now picture an enclosure, the walls of which are at a high temperature. The enclosure will be filled with light, radiant energy. This will exert a pressure on the walls just as would gas particles if they were present, only with the restriction of the one-half factor, as just stated. We can show that the radiation itself is quite a little like a perfect gas. There are the similarities just stated. Further, if a perfect gas is quickly compressed, the pressure and volume are related thus: $pv^\gamma =$ constant where γ generally $= 1.41$. If a volume of radiation is suddenly compressed the relation $pv^\gamma =$ constant where $\gamma = 4/3$. (See Appendix 4–1.) If a perfect gas expands, with temperature constant, the pressure decreases as the volume increases. For radiation the pressure remains constant since the energy density which depends on the temperature must remain constant. Is light then atomic or molecular like a gas?

If there is a hole in the wall of a furnace, light, radiation, will issue. It can be analyzed by a prism spectroscope with fluorite prism and lenses, and the distribution of its energy according to wave length can be determined. Here one would have to use an energy-measuring device, a thermo-element, to measure the energy of the light. In a perfect gas we found that there was a definite distribution of molecules according to speeds. Is there a similar law for radiation? Partly yes. Outstanding is this—the distribution of speeds of molecules depends only on temperature, also the distribution of energy of this kind of radiation. The walls of the enclosure have nothing to do with it. It is called *temperature*, or *black body*, *radiation*.

Laws of "Black Body" or "Temperature" Radiation.

This temperature radiation has certain well-known laws. The energy density $E = cT^4$ where c is a constant and T is the absolute temperature. This is the Stefan-Boltzmann law, originally an empirical relation merely suggested by Stefan in 1879, later (1885) derived by Boltzmann using the principle of the pressure of light. A similar law with a different constant holds for the energy radiated from one square centimeter of the surface of a body per second at temperature T. Thus it is important in the case of a fireplace, a stove, or an incandescent filament, if we want a large amount of energy radiated, to have the temperature high.

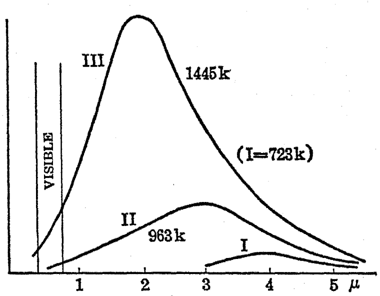


FIG. 4–6. Curves showing how the energy from a "black body" is distributed among the wave lengths. The visible energy lies between 0.4 and 0.75 μ, where μ = 0.001 mm.

Another law is that the wave length of the maximum energy decreases as the temperature increases, or $l_mT = 2885$ if T is in degrees absolute and l_m is in 0.001 mm., or 1 μ.

Both laws are illustrated in Fig. 4–6. The ordinates represent the intensity of the radiation of a certain wave length. In other words, curve III represents the "distribution of the energy according to wave length" for a body at absolute temperature 1445 (1172° C.). The wave length of the maximum ordi-

nate is at 0.002 mm. or 2 μ. At half this temperature the maximum of the curve is at 4 μ. The area under the curve represents the total energy per cm.^{3} or radiated per cm.^{2} at that temperature. The further importance of having the temperature of an incandescent filament high is seen in the fact that, if we want the chief part of the radiation in the visible, we would have to heat it to a temperature of 2885 $\div$ 0.6 (since yellow light has a wave length nearly equal to 0.6 μ) or 4800° K or 4530° C. No filament that the author is acquainted with can stand this temperature. So all that we can do is to heat it as high as possible without melting it—or to compromise between luminous efficiency and life of the filament. The student ought to compute the wave length of the maximum energy for the radiation given out by a block of ice, the walls of a room, a vessel of boiling water, and place these wave lengths in the wave length chart. Similarly find the temperature of the sun and of the filament of an incandescent lamp given that the maxima of their radiations are at 0.5 μ and 1.4 μ respectively.

We have called attention to the similarity between a perfect gas and temperature radiation. Is there any relation between the distribution of energies in a perfect gas and energies in radiation? In the former we know the number of molecules having speeds lying between two limits, in the latter the energy lying between two wave lengths. But obviously speeds of molecules cannot be likened to wave lengths; perhaps they might be likened to frequencies.

In Appendix 4–1 are compared certain points of resemblance. But why should there be any resemblance? Why not emphasize differences?

Is Light Continuous or Granular?

A gas is composed of discrete molecules. We have shown how their masses can be determined with high accuracy. But in 1900 and for one hundred years previously physicists had regarded the ether as a "continuous" medium. At least its graininess was regarded as very fine compared with matter. Moreover light was continuous, its intensity could be altered by infinitesimal amounts —this for any color, any wave length. But light originated, so it was thought, in minute electric oscillations. Hertz had used two rods placed end to end, separated slightly, which, charged up, sent out electric waves when they discharged. (Hertzian

dipoles are used today for short—5 meter—electric waves.) So, though light was continuous, it was supposed to originate in molecular dipoles. The intensity could vary continuously.

Planck Is Forced by Experimental Results to Make a Strange Assumption.

This was the picture when Planck came to this problem in 1900. But in trying to account for the distribution curves of Fig. 4–6 he found that it was necessary to assume that the electric oscillators radiated or absorbed energy in definite bundles, the energy in every bundle being proportional to the frequency of radiation in that bundle.

The Beginning of the Quantum Theory.

The *quantum theory* which now enters as a most important feature of practically all branches of physics had its beginning in the nebulous, uncertain ideas put forth by Planck. His formula for temperature radiation may be said to contain the birthplace of the quantum theory.

Let us now see how Planck's assumption affects the continuity of light. If an oscillator can emit energy only in chunks, bundles, quanta, how can these coalesce so as to give us complete continuity of wave motion out in space? Or if light can be absorbed only in those units, does it pile up near an absorbing medium until the proper amount has arrived and then disappear, be absorbed, in bundles? Obviously his assumption was disturbing to him and to physicists generally. Was there not some other way of accounting for those distribution curves?

Attempts to Derive the Radiation Laws Based on the Idea of Continuity Failed.

In the Appendix there are noted various formulae. Besides Wien's, the formula which has received the most attention is that of Rayleigh-Jeans. The assumptions made by those authors show what "illogical" methods theoretical physicists will use in trying to make a theory fit the facts. For example, here is a string one meter long fastened at the ends. It can oscillate in a variety of ways, possibly as a whole so that the wave length of the disturbance in the string is twice its length; possibly in two parts with a node halfway between the ends, in which case the fre-

quency now would be twice that of the first or fundamental mode; or it may oscillate so as to have frequencies three or four times the fundamental. Now the energy in any oscillation is proportional to the square of the amplitude and the square of the frequency. The amplitude might be exceedingly small, in which case the energy for that frequency would be almost infinitesimal. The Rayleigh-Jeans method attempted to carry over the following idea from vibrations in space to light vibrations. If the string were to break up into 1000, 1001, · · · 1010 parts, these would be different modes of vibration. The Rayleigh-Jeans method ignored the influence of amplitude and assumed that the energy of any mode would be the same as that of any other mode and would be equal to twice the energy of a "degree of freedom" of a molecule of a gas or twice $\frac{1}{2} k_0 T$ where k_0 is Boltzmann's constant. Now in ordinary elastic matter it is not true that the vibrating body is apt to break up into 1000, · · · 10,000, 10,001 vibrating parts, nor would the energy in one of these modes be likely to equal that in another. Yet these illustrious mathematical physicists were willing to make an assumption which was not true for ordinary matter in the hope that it might lead to a relation which would be correct for radiation in the ether! However, their formula fits the experimental curve for very long waves, for very low frequencies, far lower than those of visible light.

There is one other criticism of the Rayleigh-Jeans method. They apparently pictured the distribution of the radiant energy as dependent upon the geometrical configuration of the volume occupied by the radiation, whereas it is entirely independent of the extent and nature of the volume. Moreover, there is no possibility that the distribution would depend on collisions or interactions of elements of the radiation upon themselves or upon other elements. Such interactions do not exist. There is only one source of the law of distribution of thermal radiation; it lies in the interaction of radiation and matter.

Einstein's Important Extension of the Quantum Idea.

Planck after his first success in 1900 gave other methods of deriving his formula and Jeans and others continued too in their attempts to obtain explanations of the distribution of black body radiation, but in the meantime (1905) Albert Einstein put his

finger in this pie and pulled out a huge plum. To change the figure, he went the whole distance, he assumed that light was not only radiated and absorbed in bundles of energy, in quanta, but that these quanta kept their identity as they moved through space. Light could no longer be a phenomenon of continuity, it became one of discontinuity. It returned, in part at least, to the kind of light that Newton pictured, a flight of corpuscles. Einstein used this view of the quantum nature of light to account for one of the outstanding facts connected with the photoelectric phenomenon, a fact which could not, so far as we could see, be explained on the basis of the wave theory. His explanation and the mathematical relation which he gave in 1905 were completely verified by experiment about five years later.

Computations and Discussion of *h*.

Returning to Planck's equation, it can be easily proved by anyone having a slight knowledge of the calculus that Wien's formula, l_mT = constant, can be derived from the equation. In trying to find the wave length that gives the top of the curve (Appendix 4–3), i.e., that for which E is a maximum, we obtain the relation

$$l_mT = \frac{ch}{k_0 \times 4.965}$$

where c is the velocity of light and h is the constant in Planck's relation, $e = hf$. Obviously since all quantities on the right of the equation are constant, so also is l_mT. Now in 1900 the value of k_0 was not very accurately known since N, Avogadro's number, was not accurately known. But had its approximate value and the experimental value of $l_mT = 0.2885$ (cm. degrees) been used, the value of h could have been computed. However, further use was made of the equation. The area under the curve representing the equation must be equal to the total energy in 1 cm.^{3} of the radiation. Performing the integration we verify the Stefan-Boltzmann law, $E = aT^4$, and we get a relation between the constants of the equation and the experimentally determined constant a. The two operations carried out as indicated give $h = 6.42 \times 10^{-27}$ erg-seconds and $k = 1.34 \times 10^{-16}$ ergs per degree. The latter value was known to be correct within a few

per cent. The former was the first computation of the value of Planck's h.

But what is h? Planck, be it remembered, merely saw that, to get the right mathematical formula, he had to let his energy bundle e that he was distributing to the electric oscillators be proportional to the frequency of those oscillators. So he wrote $e = hf$. At first that was its only significance. As far as dimensions are concerned it is seen that h equals *energy divided by frequency* which is equivalent to *energy multiplied by time*. If we write energy $= mv^2$ and $v =$ distance/time, it results that h may also be represented by *momentum* multiplied by *distance*. These two aspects of h are utilized frequently in later work in this text. Expressed in symbols, they are $h = E \times t = mv \times l$.

The rather extraordinary fact may be noted that, whereas Planck's formula fits all the facts connected with temperature radiation, no other formula, even that derived by Wien, viz., $E = c\lambda^{-4}e^{-c/T}$, fits the clearly proved experimental fact expressing Wien's law, $l_mT =$ constant, and the Stefan-Boltzmann law, $E = aT^4$ (Appendix 4–3 and 4–4). This shows the complete superiority of Planck's law over all others.

CHAPTER 5

THE PHOTOELECTRIC EFFECT

Hertz, while trying to find out if an electric discharge produces waves like light, as Maxwell predicted and as he verified, finds that light produces an electric discharge—of a sort.——Confirming theories requiring an ether, he discovers a phenomenon which cannot be explained by those theories. Yet light and electricity are more than ever interwoven.——But light has a new quality.——For the maximum energy of the escaping electrons does not depend on the intensity of light.——Einstein makes a guess.——His photons—or bullets of light—"explain" the photoelectric effect but fail to explain interference. Bullets, all alike and without structure, cannot produce no-bullets.——Are they large or small?——Impossible to measure their dimensions. Some phenomena require photons to have both wave properties and bullet properties.——Photons may have meridional planes of symmetry.——Sunfish photons.

Light Produces an Electric Effect. Hertz Discovers the Photoelectric Phenomenon.

In 1887 while Hertz was immersed in his attempts to generate and detect electric waves, he noticed that a spark could more easily take place between two oppositely charged rods end-on-end facing one another if one of the opposing surfaces was illuminated by ultraviolet light. Later it was seen that a negatively charged metal plate could be discharged by the proper kind of light beam. This experiment can be easily demonstrated. A very well-insulated zinc or aluminum plate with clean surface is connected to the leaves of a gold-leaf electroscope. The plate is negatively charged. Light from a bare arc lamp is allowed to fall upon the plate. The leaves come slowly together. If a plate of glass is interposed between the arc and the plate, the leaves remain separated. If a plate of quartz is substituted for the glass, the leaves continue to come together. Ordinary glass cuts out light a little below the visible, in other words it cuts out ultraviolet light. Quartz lets this light go through. Hence the experiment shows that the plate loses a negative charge under the action of ultraviolet light. Under similar conditions a positively charged plate is not discharged.

Further study shows that, due to the light, negative electrons are driven from the plate. Gas surrounding the plate does not prevent the discharge but it complicates the phenomenon; so it is best to work in a vacuum. Figure 5–1 shows a bulb which can be exhausted and which contains a metal plate P, a hollow electrode A, various insulated plates 1, 2, 3, 4, any one of which can be connected to an electrometer, and a quartz window W. Light of definite wave length is admitted through W and falls on P. A small potential difference is applied to $A - P$. If the earth's magnetic field does not enter to disturb matters, or if electric charges on the glass walls or neighboring bodies are negligible, the electrode 2 connected to the electrometer will show, within limits, an increasing negative charge. If a magnetic field is imposed at right angles to the plane of the paper in direction away from the reader and localized between A and the electrodes on the right, plate 3 or 4 may be seen to become charged, 3 for a weak field, 4 for a stronger. If the direction of the field is reversed, plate 1 will be charged. All these results are in harmony with the view that some of the negative electrons starting out from P pass through the opening in A and are deflected by the magnetic field.

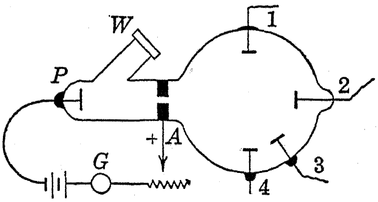


FIG. 5–1. Light passing through W and falling upon P causes electrons to be given off.

Photoelectrons Are Like Cathode Rays and Other Electron Streams.

If the tube be lengthened and a velocity selector be placed behind A, the deflection by the magnetic field will give both the velocity of the electrons[1] and the ratio e/m. Lenard, and Merritt and Stewart,[2] as early as 1900 found this ratio to be approximately the same as the rough determination by J. J. Thomson of the same ratio for cathode rays. Since that early date the name "photoelectrons" has been given to these charged particles.

[1] In Chapter 2 we used a velocity selector to obtain a stream of electrons or atoms of a certain velocity. (See pages 24, 29, 31.)

[2] Later measurements of e/m for both kinds of rays give a value of 1.76×10^{-7}, e being in e.m.u. and m in grams.

Electric Charges May Have Various Energies Due to "Potential" Differences.

It is now necessary to consider certain electrical ideas. Referring back to the oil drop experiment, let us suppose that there is a positively charged drop between the plates and that the lower plate is positive, the upper negative. If the potential be large enough, the drop may be pulled up against gravitation. Work would be done by the electric force against the gravitation force and also in this case against air friction. We say that the drop has been moved from a place of high to one of low *electric potential.* We therefore get the view that the potential difference between two points may be measured by the work done in taking unit charge from one point to the other by the electric force. Now it is necessary to exercise care in the matter of units and if we specify that potential difference is *equal* to the work done, then all units must be absolute. To illustrate—if Q e.m.u. (absolute electromagnetic units) of charge be moved from one point to another of potential difference V e.m.u., the work done $= QV$ ergs. (If 1 gm. be raised vertically 1 cm., the work done is 1 gm. cm. or 980 ergs.) This is the fundamental and everlasting definition of potential difference. Now it turns out that 1 volt, the everyday unit of potential difference, is equal to 10^8 e.m.u. of potential; and 1 coulomb, or 1 ampere flowing for 1 second, the corresponding unit of charge or quantity, is 1/10 of 1 e.m.u. of charge. And as we have instruments which measure voltage and current with precision, we can always determine the corresponding values in absolute units.

We have illustrated the idea of potential difference using electromagnetic units. An identical relation would have been obtained had absolute electrostatic units (e.s.u.) been used. Again, if Q e.s.u. of charge be taken between two points of potential difference V e.s.u. of potential, the work done $= QV$ ergs. It is obvious then that if the e.m.u. of charge be large compared with the e.s.u. (it is 3×10^{10} times the e.s.u.), the e.m.u. of potential must be correspondingly small compared with the e.s.u. of potential (it is $1/3 \times 10^{10}$ e.s.u.). One e.s.u. of potential equals 300 volts. However, it is not necessary to carry the two systems in our work; we shall use the electromagnetic units, or we shall show how to make the appropriate transformation.

We Measure the Number and Speed of the Photoelectrons.

Instead of a charged oil drop in air, let us think of an electron, in a vacuum, leaving a plate upon which light has fallen. Does it leave with much speed? If so, how shall we determine the speed? Let us arrange matters so that we can put an increasing positive potential on the photoelectric plate P and negative on A. The photoelectric current as measured by a very sensitive galvanometer Q will decrease—finally become zero. Or if we start with A quite negative compared with P, the current, at first zero, will increase in a fashion indicated by the curve in Fig. 5–2. The vertical lines represent intensity of current, horizontal distances voltage on A compared with P. It is seen that for a certain small positive potential on A the current gets up to a value which it does not much surpass—it becomes a *saturation* current. All of this implies that a very constant light source be used, that it should not vary either in the intensity or in the character of the light sent out.

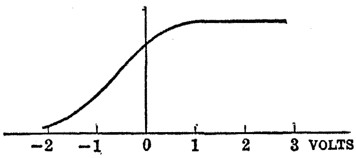


FIG. 5–2. The emission current may be made zero by a few volts negative on the receiving plate; or it may become constant for a few volts positive.

The picture here is that, given a constant light source, the number of electrons sent out per second is definite. When A is sufficiently positive, all of these electrons arrive at A. But as A becomes negative, fewer electrons arrive there—finally none. To account for this last fact let us assume that the maximum speed of the electrons leaving P be v. Then the kinetic energy of one electron is $\frac{1}{2}mv^2$. If an electron were to start with zero speed and if A were positive, it would be accelerated towards A. The work done by the force would give it a kinetic energy $\frac{1}{2}mv^2$ and it would also be equal to Ve where V is the potential difference between A and P and e is the electron charge. Similarly if it starts with a speed v and is brought to rest by a negative potential we must have the same relation $Ve = \frac{1}{2}mv^2$. Now V, the potential which reduces the current to zero, can be measured in volts, therefore computed in e.m.u., by multiplying by 10^8; and we can find the maximum velocity of the electrons from the relation $v^2 = 2\,Ve/m$.

For example, for $V = 3$ volts, since $e/m = 1.76 \times 10^7$, $v^2 = 2 \times 3 \times 10^8 \times 1.7 \times 10^7$ or $v = 10^8$ cm./sec. = 600 miles/sec. However, this experiment requires the finest technique and there are corrections to be applied which might change considerably the computed value.

But the maximum velocity of emission of the electrons is not in itself the most important quantity. The question is, what determines this velocity? The answer is—the frequency of the incident light. Let us now see how we can most conveniently alter this frequency.

How We May Vary the Frequency of the Incident Light.

Consider a 60° glass prism. Figure 5–3 shows a ray of light passing through it under the condition that the deviation of that ray is a minimum. In that case the emerging ray makes the same angle with its face as the entering ray does with its face. The ray passes symmetrically through the prism and is therefore perpendicular to the dotted line which bisects the angle A. Ray No. 1 might be a red ray. Ray No. 2, a blue ray, would enter and leave the prism at smaller angles when its deviation is a minimum. Now suppose we have a block of this glass cut so that the angle of $A = 30° + 45°$, etc.; then ray No. 1 entering at the same angle as in Fig. 5–4 would be totally reflected at the face AD; it would then have a direction 90° from the first direction in the glass and would emerge from BC, making the same angle with that face that it made with AB when entering. Hence its emerging direction would be at 90° to its incident. Similarly for ray No. 2. If then a telescope and collimator of a spectro-

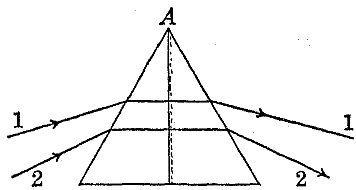


FIG. 5–3. The deviation is a minimum for light passing symmetrically through a prism.

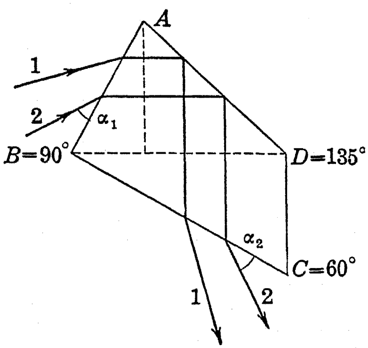


FIG. 5–4. A constant deviation prism. The 1's are at right angles to each other; also the 2's.

scope be placed at 90° to each other, both directed towards this block of glass placed on a rotating table, light of any desired wave length can be brought to the cross-hairs of the telescope by rotating the glass block. If the eyepiece of the telescope be removed and an adjustable slit substituted, we have the means of throwing light of any desired wave length into any opening—say that of a photoelectric cell. The consideration that the deviation of the light in Fig. 5–3 should be a minimum follows from the fact that the sharpest focus results from that condition. In Fig. 5–4, α_1 must equal α_2. It is only for that case that the incident and emerging rays are at right angles to one another. The apparatus just described is called a constant deviation spectroscope or a monochromator. The drum driving the turntable is generally calibrated in wave lengths. The frequency of the light can be computed. With quartz lenses and a quartz prism one can go out to ultraviolet light.

Einstein's Famous Equation.

When various frequencies are used as sources and the retarding voltages necessary to reduce the current to zero are measured and the data plotted, we have a straight line as shown in I, Fig. 5–5. Now the plum, to which reference has been made on page 62, which Einstein pulled out of the pie in 1905, and which was largely responsible for his receiving the Nobel prize, was this: Picturing light as a flight of bundles of energy of amount hf, and an electron in its path absorbing that energy, he said that it should leave the illuminated surface with a maximum kinetic energy $\frac{1}{2}mv^2$ equal to hf less an amount w_0 which would represent the work done in escaping from the surface. So we have $\frac{1}{2}mv^2 = hf - w_0$, the famous Einstein equation. Now we have shown that, if V is the stopping potential,

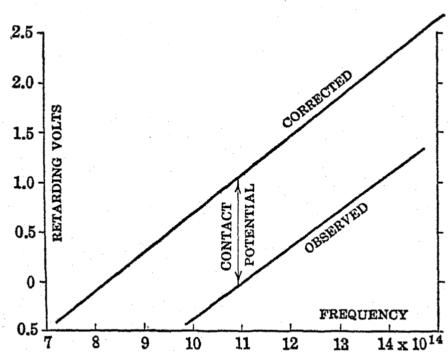


FIG. 5–5. For every frequency of the light there is a definite retarding potential necessary to stop all electrons. The slope of the line gives Planck's constant h.

$Ve = \frac{1}{2} mv^2$. Hence $Ve = hf - w_0$. Whatever w_0 is, the plotted curve should be a straight line from which h can be determined since e is known.

Confirmation. Millikan's Measurement of *h*.

Lenard in 1902 showed that the maximum speed of the electrons depended (astonishing fact!) upon the frequency of the light. A. L. Hughes in 1912 brought out the linear relation, as did at the same time O. W. Richardson and K. T. Compton, whose data [1] are plotted in Fig. 5–5.

But Millikan in 1916 made the work especially precise. It is not our purpose to burden the reader with experimental details. Indeed, we have written about these experiments in photoelectricity as if they were of a rather simple nature. But in order that the student may have some appreciation of the precautions which had to be taken in the testing of Einstein's equation, some of the details connected with Millikan's experiment ought to be given. Figure 5–6 represents merely the vacuum part of his

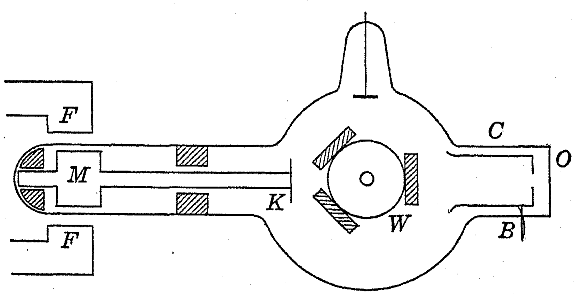


FIG. 5–6. Millikan's apparatus for measuring h. A lathe in a vacuum.

apparatus. Cylinders of sodium, potassium, and lithium are placed on the wheel w which can be rotated by an electromagnet necessarily outside the vacuum tube. Another electromagnet F can move forward or rotate a cylinder M and with it a knife edge K. In this way any one of the alkali cylinders can have a shaving cut off, a *clean* surface formed, then the wheel w is rotated so that that surface can be exposed to the light which enters at O. The electrons leaving the surface can be caught in the gauze cup which is connected at B to an electrometer. A retarding potential on C

[1] Their value of h from the slope, however, is too low.

is increased until no charge is received by the electrometer. Then the V of Einstein's equation is known. But is it? Well, not quite, for there is a quantity called the contact difference of potential to be measured and taken into account. But the consideration of that point will be taken up later. Numerous other rather perplexing experimental details are here omitted.

But why was it necessary for Millikan to use freshly shaved surfaces of the metals? The answer lies in these questions. What is a *surface*? From what depth in the metals do the electrons come? If the electrons actually come from the atoms on the surface, or if these atoms influence other atoms on the next deeper layer, it will be necessary to use extreme care in "cleaning" a surface. We cannot "see" atoms or molecules but we can detect their presence by various physical effects—one of which is the photoelectric. Illustrations of the very great change in the photoelectric current which may be brought about by exposing the surface to the action of a minute amount of some other material will be given later.

One of the very important results of Millikan's experiment was his accurate measurement of h, Planck's constant. He found $h = 6.56 \times 10^{-27}$ erg seconds. Now Planck in 1901 had computed the value of 6.55×10^{-27} from a consideration of black body radiation. True he had used some experimental data, at that time the best available, which were later found to be in error by several per cent, but curiously the errors compensated one another so as to give him a value which is now known to be very nearly (within 0.2 per cent) the present accepted value. But consider the different methods of approach. Planck who found it necessary to make an assumption which he distrusted, in order that his mathematical equation should fit the facts of black body radiation, and who computed from that equation the value of his distrusted h, and Millikan who was concerned with the maximum speed of electrons escaping from a surface illuminated by light! But these are only two of various different ways in which the value of h may be computed. It would appear then that h is a constant of nature.

Discussion of Einstein's Equation; Threshold Frequencies.

Let us return to Einstein's equation, $\frac{1}{2} mv^2 = Ve = hf - w_0$. It is seen that if v, the maximum velocity of emission of the elec-

trons, is zero, then $hf - w_0 = 0$; or there is a limiting frequency for which electrons are emitted. This is called the threshold frequency. Call this f_0; then $w_0 = hf_0$ and Einstein's equation takes the form $\frac{1}{2} mv^2 = Ve = h(f - f_0)$. The speed of the escaping electrons then depends on the difference between the frequency of the light used and the smaller threshold frequency. Volumes have been written on this topic—threshold frequency. The latest text, an excellent one,[1] lists 115 articles on this subject, and the observed data there tabulated show large discrepancies; but even these discrepancies show that the greatest care must be exercised in defining a "surface." For example, the threshold wave length for aluminum as obtained by different observers runs all the way from 3460 to 5000 A.u., that for silver with no outgassing from 3200 to 3400 A.u., with partial outgassing 2900 to 3150 A.u., with extending outgassing 2600 to 2700 A.u. Truly when we think we are dealing with a silver surface we may be dealing with one with a layer of gas molecules on it which may "conceal" the silver.

There is another way in which Einstein's equation should cause us to be on guard as to placing our faith in mathematical symbols. Since the threshold frequency is supposed to vary from one substance to another, one would suppose that the stopping potential (apparent) would change if, using the same illuminating wave length, we replace substance A by substance B. But experiment shows that when two different substances are in turn exposed to the light as in Fig. 5–6, the stopping potentials are the same! Why so? It lies in the word "apparent" or measured in the ordinary way, and this again suggests that the apparent stopping potential does not take into account the contact difference of potential—which we promised to consider later.

But in spite of all the numerous difficulties connected with the complete verification of Einstein's equation, the data, retarding potential vs. frequency, when plotted, give a straight line, the slope of which gives h/e and therefore h.

There is another way in which this relation can be illustrated. Let us use as light source a mercury arc lamp and one line after another of its spectrum as our illuminating light.[2] Then plot the photoelectric current (for sodium) versus retarding or accelerating

[1] Hughes and DuBridge, *Photoelectric Phenomena*.

[2] We may use filters which transmit only one mercury line or we may use the spectroscope of Fig. 5–4.

volts. We have the curves of Fig. 5–7. It is seen that the curves for the various lines cut rather sharply the line of volts and that

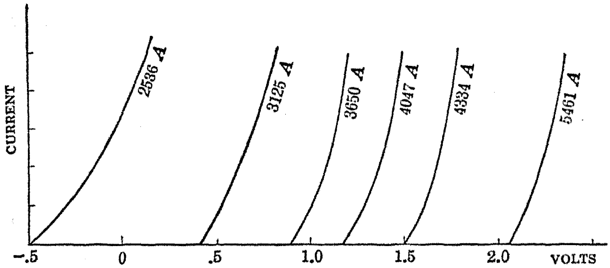


FIG. 5–7. The longer the wave length, the less negative the potential necessary to stop electrons.

the longer the wave length the less negative or more positive the necessary stopping potential, or the greater the apparent accelerating potential.

The Number of Photoelectrons Is (Nearly) Proportional to the Number of Absorbed Photons.

We have written much about threshold values, stopping potentials, maximum velocity of emission, the measurement of h. The other point of view is, how great is the photoelectric current, how many electrons emerge? Obviously that depends on the wave length used. It also depends on the intensity of the light. It is assumed here that the voltage accelerating the electrons is sufficient to give the saturation current, that is, that no further increase in current takes place for increased voltage. This implies also that the photoelectric cell is of the high vacuum type. If we have such a cell and if we use a definite light source, say a 100-watt nitrogen-filled tungsten lamp, and vary the intensity of illumination by changing the distance between lamp and cell, the photoelectric current is then proportional to the illumination. The performance of cells containing gas and of other photoelectric devices will be taken up later.

What Is Our Picture of a Photon?

And now let us return to the general photoelectric phenomenon; how do we explain it? Up to the time of Einstein's proposal there had been only one mode of attack. Light was a wave

motion; intensity for light of a certain frequency (given a certain medium) could be altered only by the amplitude of the wave motion; energy then was dependent upon this amplitude; since the law of the conservation of energy held, the kinetic energy of the emitted electrons could come only from the intensity of the light. But experiment showed that the maximum energy of the photoelectrons did not depend upon the intensity of the light. It is true that there was another possible explanation. Perhaps there was a mechanism in the atom that was triggered off by a special frequency in the light and that the atom then fired out the electron. That would necessitate a condition of resonance between a frequency in the atom and that of the light and would require the atom to provide the energy of expulsion. But that explanation would fail to explain the fact that if a higher frequency were used the electron would still be triggered off with an energy proportional to the increased frequency of the light. On the whole there was no explanation. Einstein's explanation, leaving out details [1] regarding the energy required for the atom to escape from the surface, fitted the facts. Hence, when his equation was experimentally verified in 1912, the quantum theory of light received large support. In 1913 Niels Bohr set forth his celebrated theory regarding spectra. Since then the *quantum idea*—the idea that energy is tied together in bundles of amounts hf where f is a characteristic frequency of the bundle—has spread so as to demand consideration in all parts of physics; perhaps it may be said, in all parts of science.

By the term "photon" we picture a bullet or bundle of light, whether visible or invisible. If its wave length is known, its frequency is easily computed; it is the velocity of light divided by its wave length. The reader will notice that no sooner do we start out boldly to accept the view that light is composed of bullets than we at once use the wave theory in order to find their frequency! For orange-yellow light of wave length 6×10^{-5} cm.

[1] It might be pointed out that, had Einstein known anything about the interaction of photons and electrons, he probably would not have made his famous proposal. For later (Chapter 7) it will be shown that a *free* electron cannot absorb all the energy of a photon. And had he known anything about *bound* electrons he would have seen that there might be a variety of energies of emission for an electron. So his proposal may be credited to courage and keeping one phenomenon in sight, also to the fact that he was undaunted by his rather large ignorance regarding atoms, electrons, photons, an ignorance which is only partial in comparison with that of nearly all of the rest of the human race.

the frequency is 5×10^{14} and the energy of the photon is 3.27×10^{-12} erg. The work done in raising a very small insect one centimeter is, say, one erg. One million millionth, 10^{-12}, of this is, as judged by humans, inconceivably small. Yet on the view put forth by Einstein and amply confirmed, it is the absorption of this energy in one gulp that causes an atom to eject an electron. And it must be an exceedingly quick gulp. For we have methods of measuring time, in certain cases, down to one ten-thousand millionth (10^{-10}) of a second and we can prove that electrons start away from a surface within about that time after light of the proper frequency falls on it. And this brings out another reason why the wave theory fails completely to account for the photoelectric effect. For we can measure the number of ergs falling upon one cm.^{2} due to a monochromatic radiation; then, knowing that the diameter of an atom is of the order of 10^{-8} cm., we compute the corresponding quantity of energy falling upon an atom, then the time required for the atom to accumulate energy equal to the known energy of ejection of an electron. It comes out that that time is several hundred days instead of 10^{-10} second, too great by a factor of perhaps a million million million, 10^{17} or 10^{18}! That difficulty disappears if we assume that all the energy which has been supposed to be spread out over a centimeter is concentrated in an area of cross-section of atomic dimensions and arrives in bursts, that it is discontinuous in time and space.

So the photon has the dimensions of an atom? Not so fast. We do not know. In various recent publications by some of our foremost physicists it is stated that a photon may have a volume of the order of one cubic meter.[1] The argument underlying that statement is based upon the interference of light.

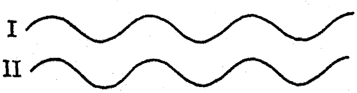


FIG. 5–8. Two trains of waves in the same phase; if they can be brought together in this relation they will produce a maximum motion.

In Fig. 5–8 are represented two trains of waves of the same wave length, crest accurately agreeing with crest—or, as we say, in the same phase. If we add together the ordinates, we get crests of twice the amplitude of one. We can imagine these two wave trains brought together by a lens or being slightly inclined

[1] See for example W. F. G. Swann, *The Architecture of the Universe*, p. 108.

in directions so that they come together at a point. In either case a maximum wave motion, whether of water waves, sound, or light, would result. But if train I is shifted forward one-half a wave or any odd number of half-waves compared with II so that crest is opposite trough, the two trains added together cancel one another; they would produce no motion. This is true so long as *both* wave trains are nearly unlimited in length. But suppose they are limited. They will annul one another if in opposite phase only for that portion which is common to both. The

FIG. 5–9. Two wave trains limited in length can only partially annul one another.

two uncompensated ends both produce light. In order that there may be an appreciable minimum, the common portion must be an appreciable fraction of the total length. Now one essential condition in obtaining interference in light is that light from some definite source be divided into two parts and that these two after traversing different path lengths be brought together. Hence when two such light beams come together to produce a definite minimum, the wave trains must be at least longer than the difference in path of the two beams.

We Cannot Split a Photon. Interference Cannot Be Explained by Photons unless—.

Consider light reflected from a soap film. Part No. 1 is reflected at the first surface, part No. 2 goes through and is reflected at the second surface, then out to mingle with No. 1. With monochromatic light and a small angle between the surfaces, the two beams may produce a maximum or a minimum. Instead of a soap film we may take an interferometer which we may represent as in Fig. 5–10. Light entering along L is broken into two parts by the separating surface S (thinly silvered glass). The two beams 1 and 2 are reflected at M_1M_2 and emerge along the two arrows. When the adjustments are right, the reunited beams may produce maxima or minima. Now interference has been obtained for very narrowly monochromatic light (red cadmium line by Michelson) for a path of difference of 800,000 waves or 40 cm. This implies a wave train of the order of at least 50 cm.

length. For this reason it has been argued that a photon must have at least this length. Similarly at the focus of a lens we have interference effects which imply a fixed phase relation between all parts of the light entering the lens. Hence we say that a photon must have an area of cross-section equivalent to that of a very large lens (Yerkes, 40-inch diameter) or mirror (Mt. Wilson, 100-inch diameter). Or, since Michelson used mirrors at a distance of about 20 feet in measuring the diameter of Betelgeuze and obtained interference, we might give the diameter of a photon as 20 feet. But all of these arguments are, in the author's opinion, illusory. Let us look at Fig. 5–10. The entering light beam is split in two. That is entirely possible if light is a wave motion. It is entirely possible if light consists of photons, *but then it would be necessary for one photon to go along path 1 and another along path 2. We cannot split a photon.* To split it would mean to halve its energy—that would halve its frequency; deep blue light would become deep red by reflection. This does not happen. If the separating surface sent one photon along path 1 and allowed another to go along path 2, there would be no necessary phase relation between the two photons and interference would not result. Hence the argument which gives to a photon a possible volume of a few cubic meters is illusory.

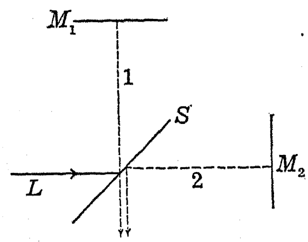


FIG. 5–10. An idealized interferometer. A beam of light L is split into 1 and 2 by a thin film of silver S. But photons cannot be split.

It is evident from what has been said that it is not possible to explain the interference of light on the photon basis unless we require a phase relation between all photons sent out. To have a discontinuity in energy emission but a continuity in the timing mechanism which expels the photons and to have this mechanism controlled as it would be by the wave theory—to ask all this of a simple atom like hydrogen—would be to put such an atom almost beyond the realm of physics.

We Cannot Fix the "Volume" of a Photon.

It has been shown that we cannot attach importance to computations which give a superior limit to the "volume" of a

photon. As to limits of smallness, experiments have failed to give definite results. Light will pass through holes as small as they can be made; it passes through arrangements of atoms which make it appear continuous; it passes through rapidly opening and closing shutters which would cut off the head or tail of a photon if it had a length of a few centimeters. Altogether we have no knowledge of its smallness. There is one point of view, however, that makes it appear that it is of atomic dimensions. For suppose we have a monochromatic radiation falling on 1 $cm.^2$ of a photoelectric surface. We can measure the energy per second; knowing the frequency of the light we know the number of photons striking the surface per second and the number there absorbed. We can measure the resulting current and therefore know the number of electrons which come from the area per second. These may come not only from the first layer of atoms but from layers a few atoms deep. However, we can find the total number of atoms which may send out electrons and we find that only one in about one thousand million of all the atoms upon which light falls does so. This result would again oppose the idea that light is continuous, but if we compute the ratio of number of photons falling on the square centimeter per second to the number of electrons which emerge, it comes out to be of the order of ten to one. In no case is the number of electrons greater than the number of photons absorbed. There may be, then, depending greatly on the surface and the frequency of the light, a fair efficiency on the part of absorbed photons in ejecting electrons.

Light Must Consist of Both Waves and Photons.

So far as we have discussed these matters we have shown that the wave theory of light is entirely satisfactory for interference but fails completely to account for the photoelectric phenomenon; exactly the opposite for the photon idea. But there is one photoelectric result which requires that a photon be endowed with another property taken from the wave theory—another property besides that of frequency. To make clear this point we must call attention to an important quality of light.

Light which has been reflected from glass at 57° incidence will not be reflected from a second glass surface for the same angle of incidence if the plane of incidence of the second surface is normal to that of the first. This experiment shows that light which has

been reflected from glass as above has a special property in the plane of incidence—or in the plane normal to that plane. This phenomenon of polarization shows that the vibrations constituting light are normal to the direction of propagation.

Consider two metal rods in vertical line a few millimeters apart, charged up and discharging. The electrical field all around the rods on the horizontal plane of symmetry is vertical and the electric wave which travels out along that plane has a vertical electric vector; we say that it is plane polarized. If it falls upon a grid of vertical copper wires, very little of it will pass through; most of it will be reflected. If the grid be turned so that the wires are horizontal, practically all of it passes through. A similar phenomenon was discovered for light two hundred years ago. Instead of a grid of wires we use a crystal, iceland spar or tourmaline, with molecules specially arranged. It was not until after the time of Hertz and Maxwell that the idea of an electric vector was associated with light, but now we say of light which has been reflected from glass at 57° incidence that it is plane polarized and that its electric vector is parallel to the surface or perpendicular to the plane of incidence. Now the phenomenon connected with photoelectricity to which reference has been made is this—when plane-polarized light falls upon a sodium-potassium surface and the electric vector is in the plane of incidence, the photoelectric current is about sixty times as great as it is if the electric vector is parallel to the surface. On the wave theory, this effect was qualitatively explained, after its discovery forty years ago (1894), thus—when the electric vector is in the plane of incidence the electrons are easily "jerked out" by the electric field; when the vector is parallel to the surface they are not jerked out. However, when later it was shown that the amount of light which was absorbed varied also as 60 to 1, it was seen that for that case the number of electrons "jerked out" was proportional to the amount of light absorbed. So that phenomenon merely emphasized the point that photons could be polarized—that they had planes of symmetry.

Must We Abandon the Idea of a Model when We Think of Light?

We now call attention to the fact that if we are to have a picture or model of a photon, we must not liken it to a bullet but to a

sunfish, possessing head, tail (though these may be similar), a meridional plane of symmetry, the frequency of the bones in that plane representing the "frequency" or energy of the photon. But physicists are not fond of sunfish.

It is clear from what has been written above that we have no single picture, nor model based on ordinary phenomena, which can be used to illustrate all the phenomena of light. Shall we then take a lofty view of this matter and announce that we are superior to pictures and models? It seems hardly probable that this will be the course that physicists will long pursue. We may be content for a time to describe some phenomena in terms of x, y, and z during part of the week and in terms of a, b, and c for the other part, but we shall constantly be striving to find a unified picture which will be understandable in everyday terms. This has been the history of all science. This striving for a model is illustrated in later chapters, in radioactivity, in transmutation of the elements, in cosmic rays, in the deBroglie atom, even when we form and solve the Schroedinger equation, as shown by our picture of the various states of the hydrogen atom. But a complete model constantly eludes us.

Some Experimental Details.

(*a*) *Contact Difference of Potential.* Let us think of two cylindrical vessels standing on a table and connected at the bottom by a cross-tube containing water. Into one vessel we pour metal filings, into the other fine soil. The water will moisten the soil and rise in that vessel. There will be a *height of water* in the soil vessel as compared with the metal-filing one.

An action somewhat similar to the above takes place when two metals are placed in contact. Let us picture two metal plates of different metals facing one another and momentarily connected by a wire. Apparently electrons favor one as compared with the other. For if one of the plates, A, had been connected to a sensitive electrometer needle and the other, B, had been connected to the case and to earth, then when A is drawn away from B the needle will move. This would not have happened had A and B been at the same potential. But by means of a variable voltage we may change the voltage of A in such a way that when the separation takes place there is no alteration in the position of the needle. The voltage to which

A has been changed then compensates the *contact difference of potential.* Hence this quantity can be found and Einstein's important equation becomes

$$(V + K)e = hf - hf_0,$$

where V is the *apparent* or applied stopping potential and K is the contact difference of potential.

Thus for the determination of the *threshold frequency*, f_0, it is necessary to measure K as well as V. And this enables us to clear up one rather curious fact. When two clean metals A and B are brought in rapid succession before a Faraday cylinder C (Fig. 5–6), the stopping potentials V and V^1 for photoelectrons due to light of the same frequency are the same. Hence it follows from

$$(V + K)e = hf - hf_0$$

and

$$(V^1 + K^1)e = hf - hf_0^1$$

that

$$K^1 - K = \frac{h}{e}(f_0 - f_0^1);$$

or that the difference between the contact potentials is equal to h/e times the difference between the threshold frequencies.

Thus we see that a rather obscure property, contact difference of potential, enters when we attempt to test one of the most important relations of the quantum theory—the Einstein photoelectric equation.

(*b*) *Variation of Sensitivity of Alkali Surfaces by Oxidation, Hydration, and Other Chemical Treatment.* A great amount of data has been gathered on this topic. Here we summarize some results. When alkali surfaces are sensitized by oxygen, water vapor, sulphur vapor, there is an increase in the photoelectric activity due chiefly to increased sensitivity for long waves. Indeed a new maximum of sensitivity for the long waves may appear, separated from the original maximum sensitivity for the short waves. Samples of the behavior of photocells are given in the tables on the next page.[1]

The data show the hundredfold increase in sensitivity of one cell due to the treatment of potassium by sulphur and argon.

For another cell the data show the sensitivity for wave length changes. A potassium surface for which the threshold frequency

[1] Data given by A. R. Olpin, *Phys. Rev.*, Vol. 36, p. 251, 1930.

Cathode History	Currents for Cathode Voltages	
	−8	−50
Freshly distilled K	34	28
After admitting a trace of S	179	217
After admitting more S	905	424
After admitting still more S	840	386
Sulphur tube sealed off	846	418
Argon admitted	1360	2280
Argon admitted to 0.1 mm.	1470	4120

was 68.8×10^{13} (or $\lambda = 4360$ Å in the deep blue) was treated by hydrogen (K H); a similar cell by sulphur (K S) and the latter surface was again treated by hydrogen (K S H). The sensitivities are given below in relative values.

Color	K H	K S	K S H
Violet	20	11	11
Blue	132	101	93
Green	44	85	59
Yellow	9	72	120
Red	1	11	58

The data show the great increase in sensitivity for the long waves when potassium receives the sulphur hydrogen treatment. Such a cell might be quite sensitive for the infra red.

Caesium however is the material used for long wave detection. Its threshold wave length is 6800 Å, but treated as above the threshold is in the infra red.

The variation of the sensitivity of the different alkali metals prepared with ordinary care for light of different wave lengths is

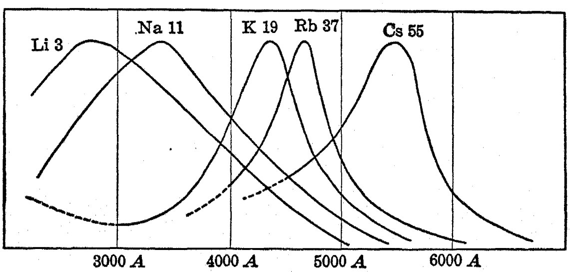


Fig. 5–11. Maximum of photoelectric sensitivity shifts towards the red with increasing atomic weights of the alkali metals.

shown in Fig. 5–11. The atomic number of the element is given in each case and it is seen that, as the atomic number increases, the maximum of sensitivity shifts toward long waves. In the figure the maximum photoelectric currents for the different elements are represented by equal ordinates—though for equal absorbed energies they may differ by very large ratios. However, the performance of these surfaces depends greatly upon the degree of outgassing.

(c) *Variations in Photoelectric Actions.*

(1) The photoelectric cells we have been considering may be called photoelectronic, in that the electrons emerge from one metal surface and pass over to that of another, either through a vacuum (high vacuum cell) or through a rare gas (gas-filled cell). The latter form may be made five or ten times as sensitive as the former. As has been pointed out, they may vary enormously in sensitivity as we change the wave length of the incident light. The effect produced in either the vacuum or the gas-filled cell may be greatly increased by a modern electron tube amplifier (Chap. 8). They have numerous and very important applications in both science and industry.

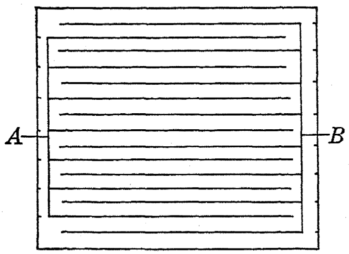


FIG. 5–12. A thin film of selenium, baked on a surface on which had been laid two interlocking combs of conducting wires, has a high electrical resistance in darkness, a lesser resistance when exposed to light. A selenium cell.

(2) Photo-resistant cells. Fine copper wires (Fig. 5–12) in the form of two combs are fastened to an insulating plate by a nonconducting adhesive. The resistance between A and B is then very great. The wires and plate are covered by a thin layer of powdered selenium which is then baked and heat-treated so that it forms a uniform thin sheet. In the dark the resistance between A and B is large, but when the cell is exposed to light the resistance is greatly decreased. Thus the cell may be made sensitive but it has a time lag and it does not lend itself to precision in manufacture. It has not found favor in applications in either science or industry.

(3) Photovoltaic. We have two electrodes in a transparent electrolyte, the whole forming an electric cell. When light falls

on one of the electrodes, the electromotive force (E.M.F.) may be changed. Its sensitivity is of the order of that of the vacuum photoelectric cell.

(4) Rectifier cells. Grondahl discovered that cuprous oxide, CuO_2, on copper was a rectifier for electric currents. In Fig. 5–13

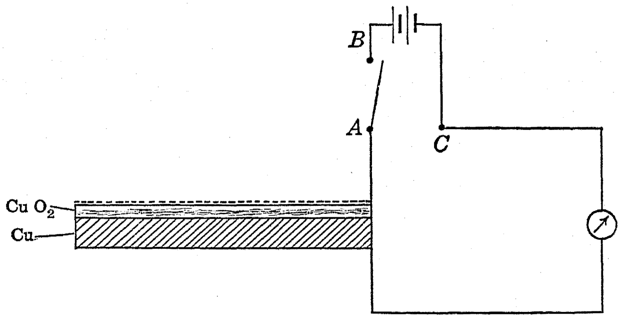


FIG. 5–13. Under the action of a battery, electrons go from copper to cuprous oxide; under the action of light they go in the opposite direction.

we picture a layer of cuprous oxide, perhaps 0.01 mm. thick, on a plate of copper. Above and in contact with the cuprous oxide is a grid of fine wires. When a battery is connected as shown with the positive terminal connected to B (switch AB connected) a current will flow in the circuit. This shows that under the action of an E.M.F. electrons will flow from A to B, from the copper to the cuprous oxide. If the battery is reversed, only a very feeble current will flow, showing that electrons are blocked in an attempt to flow from the cuprous oxide to the copper. The combination CuO_2 on Cu is a rectifier. Now if the switch AB is opened and AC closed and if light is allowed to fall upon the cuprous oxide, a current flows *but in the opposite direction to that produced by the battery in the first case.* It appears that light not only sets free electrons from one surface but drives them from the cuprous oxide to the copper—the direction in which they were blocked when a battery was connected.

Important facts regarding this cell are: (1) that its threshold frequency is in the long infra red region; (2) that some forms have a sensitivity curve almost identical with that of the human eye; (3) that the area of the plate surface may be large and thus it may give large currents. A special form of such a cell is discussed in Chapter 17.

CHAPTER 6

ATOMIC SPECTRA AND THE BOHR THEORY

Spectra.——The Balmer formula for hydrogen lines.——The Rydberg constant; its variation with the mass of the atom.——The Bohr Theory; the first great success; then failure.——The spectroscopic atom.

The First Ideas Regarding "Series" in Spectra. Balmer and Rydberg.

As has been said in Chapter 4, bright line spectra originate in a gas or vapor excited generally by a flame, an electric arc, a spark, or an electric discharge. As might be expected, hydrogen has a very simple spectrum with four lines in the visible. Ångström in 1868 with an improved grating on glass measured these lines with fair accuracy. Then frequent attempts were made to find some relation between the wave lengths. Finally Balmer in 1885 found that these were connected thus:[1]

$$\lambda = C \frac{n^2}{n^2 - 4}$$

where C was a constant and n was 3 for the red, 4 for the blue, 5 and 6 for the two violet lines. Rydberg altered the form of this equation. He saw that it was better to write the equation as a frequency relation or, more conveniently, the wave number relation which is the reciprocal of the wave length. For example, λ for green light is 5×10^{-5} cm. The wave number ν is 20,000. His relation then became

$$\nu = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

for the principal series of hydrogen. He generalized this relation for other elements and brought out the very important fact that the wave number of any line was equal to the *difference* between two terms of the form $CR/(n + a + b)^2$ where the n's were whole numbers increasing for the successive lines by unity towards

[1] In this chapter λ indicates wave length; f, frequency; $\nu = 1/\lambda$ = number of waves in 1 cm. = wave number.

short waves, and the a's and b's were constants for the lines of a series, C a constant for an element, and R the Rydberg constant.

Enter Two Great Americans, Rowland and Michelson. Extraordinary Precision.

The invention of the concave grating by Rowland in 1885 and the accurate measurement of certain standard lines by Michelson in 1895, using his interferometer, gave to spectrum analysis the ability to measure wave lengths with astonishing accuracy. As evidence of this accuracy Michelson's value for the red cadmium line is $\lambda = 6438.4722$ Å in air at 15° C. and 760 mm. Note that the last digit is in 10^{-12} or one million-millionth of a centimeter. That is about the one ten-thousandth of the diameter of the hydrogen atom or about the diameter of the nucleus of an atom! Precision of this order uncovered a number of variations in the a's and b's, and spectrum analysis became a vast realm of data still appearing boundless.

Evidence of Order—and Complexity—in Spectra.

Years ago, in the days when the atom was still regarded as a uniform elastic sphere, Lord Kelvin stated that spectrum analysis showed that it must be at least as complex as a grand piano! Now we know that he greatly underestimated its complexity. Yet when the lines belonging to one series are sorted out from the great maze of lines of a spectrum, we see some evidence of order—the order that is expressed in algebraical symbols in Rydberg's formula. In Fig. 6–1 are a few of the lines of the

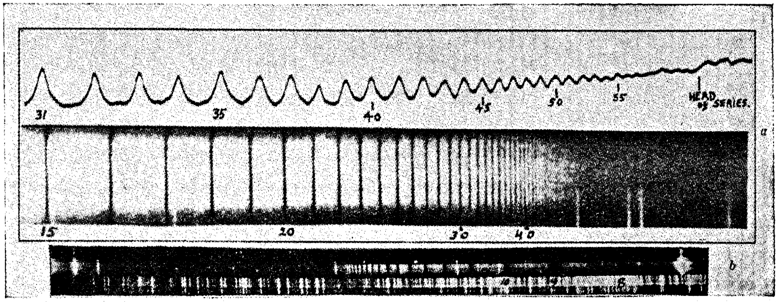


FIG. 6–1. Part of the absorption spectrum of sodium vapor. By means of the microphotometer trace above we can follow the lines nearly to the head of the series. (Wood and Straub. From Wood's *Physical Optics*.)

sodium absorption spectrum which we owe to the ingenuity of R. W. Wood. Here n runs from 15 out to about 60. Clearly here is order in the spacing!

The Electromagnetic View of the Cause of Radiation. But Niels Bohr Dissents.

How do we account for this order? How do we account for a spectral line? After the time of Maxwell, light became an electrical phenomenon. After the discovery of the electron it was supposed to be due to electrons vibrating in atoms. J. J. Thomson with his "pumpkin" atom, the sphere of positive electricity and electrons on shells inside it, labored long on this problem. He would allow an electron to be displaced from its position of equilibrium in the sphere, then find its frequency. But he was unable to obtain the simple Balmer [1] formula. Then came Rutherford's nuclear atom. It will be recalled that by measuring the number of alpha particles scattered from gold foil in various directions Rutherford concluded that a heavy atom must contain a concentrated positive charge. There must then be negative electrons distributed in rings outside this central positively charged nucleus. But these electrons could not be at rest for then they would be in unstable equilibrium, and they could not be describing orbits about the nucleus because then they would radiate energy and spiral into the nucleus. But Niels Bohr, accepting the new views regarding the quantum nature of light and denying one of the accepted results of Maxwell's theory, the necessity of radiation by an electron describing certain orbits, obtained for the first time the Balmer relation. He opened a new chapter in physics.

Bohr Uses His Knowledge of Orbits and Planck's Unit of Action *h* to Bring Forth a Great Theory.

It had been an accepted view that light would necessarily be produced when an electron described an orbit about an atom, but that due to the loss of energy by radiation the electron would spiral into the atom. The frequency of the radiation would

[1] The Balmer formula giving the wave lengths of the principal hydrogen lines. In the Rydberg form it is

$$\nu_n = 109{,}677\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where $n = 3, 4, \ldots$, and ν_n is the *wave number*.

equal that of the electron in its orbit or declining orbit. Bohr assumed that there might be orbits for which there would be no radiation. So he pictured an electron describing an orbit around a positive nucleus under the ordinary law of force $F = eE/r^2$ as given in Chapter 3.

Now Kepler (about the time that the Pilgrims were landing in America) set forth his famous laws, the second of which states that the radius vector from sun to planet sweeps over equal areas in equal times (Appendix 3–3). Newton showed that this would be true whenever there was a force of any sort along the radius vector (and no other force) and therefore it was true for attraction or repulsion according to the inverse square of the distance. In elementary physics this law is stated in another form and is illustrated by a man standing on a rotating table and holding in his hands rather heavy masses (Appendix 3–3). With his arms extended his rotational speed is slow, with arms close to his body it is fast. No moment of force is acting about the vertical axis, hence there is no change in angular momentum or in mvp, or mass $\times$ velocity $\times$ perpendicular distance from center to direction of motion. If we are dealing with a mass m describing an orbit about a large mass then the *angular momentum* is constant. Now it has been shown that h, Planck's constant, has the dimensions of energy $\times$ time or momentum $\times$ distance. The latter is angular momentum. So Bohr assumed that $2\,\pi$ times the angular momentum of the electron moving about the nucleus was equal to h, $2\,h$, $\cdots$ nh, where n is a whole number. He used $2\,\pi$ merely to bring the result he wanted. (Recently de Broglie has given a "reason" for this factor. That will be presented later in Chapter 15.) He dealt only with circular orbits where v is always perpendicular to r. Then in symbols

$$2\,\pi mvr = nh$$

for the n^{th} orbit. Making this assumption, we are choosing as orbits out of all the infinite number of possible orbits those for which the angular momentum satisfies the above values.

Bohr Derives the Balmer Formula and Computes the Rydberg Constant.

Now it has been shown in Chapter 3 that the total energy W for any orbit of radius r is given by $W = -\,Ee/2r$, the energy

at infinity having been put equal to zero. And since the force of attraction, eE/r^2, must equal the force holding the electron in its orbit

$$\frac{Ee}{r^2} = \frac{mv^2}{r}.$$

From these relations we may derive values for r, v, and W. It follows that the energy in the n^{th} orbit

$$W_n = -\frac{2\,\pi^2 m e^2 E^2}{h^2 n^2}.$$

Attention was called in Chapter 3 to the fact that if the earth were to go to another orbit closer to the sun, some of its energy would have to be lost. In our case, if the electron goes from the n^{th} to the 2nd orbit, we have

$$\text{energy lost} = \frac{2\,\pi^2 m e^2 E^2}{h^2}\left(\frac{1}{2^2} - \frac{1}{n^2}\right).$$

Bohr assumed that this radiation was lost in a quantum which would be equal to hf where f is the frequency of the radiation emitted. Now the frequency is the number of waves sent out per second. This number would extend a distance of 186,000 miles or 3×10^{10} cm. ($= c$). Hence the number of waves in 1 cm. would be $1/\lambda = f/c =$ wave number ν. Hence (since for hydrogen $E = e =$ electron charge)

$$\nu_n = \frac{2\,\pi^2 m e^4}{ch^3}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = B\left(\frac{1}{2^2} - \frac{1}{n^2}\right).$$

Substituting the known values of m, e, and h [1] we find the value of B (which we may call the Bohr constant) = 109,737. Now the Rydberg (spectroscopic) constant had been measured and was found to vary from 109,678 for hydrogen to 109,730 for heavy atoms. Here then was an amazing result. Bohr had not only given an "explanation" of the Balmer formula; he had by a combination of values of electron mass, charge, Planck's constant, and velocity of light obtained a quantity long known in spectroscopy—the Rydberg constant. The student ought now to

[1] The unit of e in these formulae is the electrostatic unit, since the force is given by $F = eE/r^2$. This is the basis of the e.s.u. The value of the electron charge is 4.77×10^{-10} e.s.u.; the mass of the electron $m = 0.9035 \times 10^{-28}$ gm.; $h = 6.54 \times 10^{-27}$ erg second.

survey and contrast all the methods used in the measurement of these various quantities. The Rydberg constant depends on the measurement of wave length and this upon a grating space and angle of deviation; the electron charge on the motion of an oil drop; its mass upon the magnetic deflection of an electron stream; h upon the photoelectric effect or black body radiation; the velocity of light upon the rotation of a mirror. To bring all of these quantities together into a mutual relation was to show a unity in the processes of nature never before in evidence.

Bohr, then, gave us this picture, that in a hydrogen-like atom in which a single electron describes an orbit about a positive nucleus there are only certain possible orbits, that if the electron "falls" from one orbit to an inner orbit the energy which must be lost takes the form of radiation, that the frequency of this radiation is determined by this energy change according to the quantum view.[1] A spectral line merely announces that a change has taken place in the energy state of an atom (or a molecule).

First Refinement. Allowance Is Made for Different Nuclear Masses. Different Rydberg Constants Result.

Had Bohr done nothing more than what has been told above, his name would have been for all time associated with this part of physics. But his theory accomplished much more. There was the difference between the spectroscopic value of R, 109,678 for hydrogen and his B, 109,737. Could that be accounted for? It has been pointed out in Chapter 3 that the focus of an orbit must be the center of mass of the two bodies. The relations that have been given above have been obtained on the assumption that the focus was the center of M, the large mass. When the correct assumption is made, the value of B must be multiplied by $\frac{M}{M+m}$ $(= \mu)$ where m is the electron mass and M is the mass of the nucleus. For hydrogen μ is less than 1 by 1/1845. Making this correction, the computed value of B_{H} differs from the spectroscopic value of R_{H} by an extremely small quantity. Moreover the corresponding values for ionized helium of B_{He} and R_{He} (109,722) agree. Here then is another triumph.

[1] Contrast this point of view with that which had previously been held—that the frequency of the light was equal to that of the electron in its vibration or in its orbit.

New Series Prophesied by the Bohr Theory Are Discovered.

We obtained the Balmer series for hydrogen by letting the electron fall from an outer to the second orbit. If to the first, then we would have had

$$\nu = B_{\mathrm{H}}\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

—larger frequencies, smaller wave lengths than before. Lyman found the very short ultraviolet lines predicted. Similarly, had the receiving orbit been the third, fourth, $\cdots$, we would have had

$$\nu = B_{\mathrm{H}}\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad \text{or} \quad \nu = B_{\mathrm{H}}\left(\frac{1}{4^2} - \frac{1}{n^2}\right)\cdots.$$

These series in the infra red have been found.

A Series Attributed to Hydrogen Is Found to Belong to Helium.

According to the Rutherford view the helium atom consists of a nucleus of nearly four times the mass of the hydrogen nucleus with two (electron) units of positive charge together with two outer electrons. If this atom should become ionized there would be only one outer electron. It would become a hydrogen-like atom. But E would now equal $2\,e$ and the Bohr formula would give

$$\nu = 4\,B_{\mathrm{He}}\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right).$$

Consequently Bohr predicted that ionized helium should have one of its series given by

$$\nu = 4\,B_{\mathrm{He}}\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$

where $n = 5, 6, 7$, etc. This can be put in the form

$$\nu = B_{\mathrm{He}}\left(\frac{1}{2^2} - \frac{1}{\left(\frac{n}{2}\right)^2}\right).$$

For $n = 6, 8$, etc., the lines should be very near the Balmer series for hydrogen. But there should be intervening lines for $n = 5, 7, 9, \cdots$. Now Pickering nearly twenty years earlier had found a series of lines in the spectrum of ζ Puppis which he attributed to

hydrogen and until Bohr's prediction they were so ascribed, but investigation showed that Bohr was right. Here (Table I) are a

TABLE I

n	H	n	He
3	6563	6 7	6560 5411
4	4861	8 9	4859 4542
5	4340	10 11	4338 4200
6	4102	12	4100

few of the approximate wave lengths of the two series in Å. It is seen that alternate lines in helium are a little shorter than the corresponding lines in hydrogen. The difference amounts to about 1 part in 2000, which is accounted for by the difference between B_{H} and B_{He}. Considering the difficulty of measuring accurately the lines in a star spectrum, it is not to be wondered at that Pickering ascribed these lines to hydrogen.

The Bohr Theory Assists in the Discovery of Heavy Hydrogen.

We anticipate the discussion in Chapter 12 to state that *heavy hydrogen* was identified by the difference between its Balmer lines [1] and those of ordinary hydrogen. If we carry through the computation for the R's for the two hydrogens, we find that the difference [2] is about 1/3700 of either R. In other words, the Balmer lines for H^2 are shorter than those for H^1 by about 1.5 Å.

When an Atom of an Element Is Ionized Its Spectrum Resembles That of the Element below It in the Chemical Order.

The point that has been made above holds through the whole of spectroscopy. Neutral helium has 2 outer electrons. When ionized it has one and its spectrum resembles that of hydrogen. Lithium has 3 outer electrons. When ionized (once or singly) its

[1] See also Appendix 12–1 for photographs of the Lyman series of H^1 and H^2.

[2] The R's for the first four elements are ${}_1H^1$, 109677.76; ${}_1H^2$, 109707.56; ${}_2He^4$ (ionized), 109722.4; ${}_3Li^7$ (doubly ionized), 109728.9.

spectrum resembles that of ordinary helium. Doubly ionized its spectrum resembles that of hydrogen. Neutral oxygen with 8 electrons, having been ionized in intense electric fields so as to lose 5 electrons, has a spectrum somewhat similar to that of lithium. All of this is in accord with the Bohr theory.

Radii and Energies.

If we proceed to find the radius of the circular orbits, we have $r_n = n^2h^2/4\,\pi^2meE$ for the n^{th} orbit. For hydrogen this is $n^2h^2/4\,\pi^2me^2$. Again substituting known values, $r_n = (0.528 \times 10^{-8})n^2$ cm. Thus the radii are proportional to the squares of the natural numbers and the radius of the smallest orbit is 0.528×10^{-8} cm. which is of the order of magnitude (10^{-8} cm.) of that derived from kinetic theory. But it should be remembered that the latter radius was that of the sphere of influence and hence should be greater than the computed Bohr diameter. Now the Bohr idea is that when energy is absorbed by an atom an electron goes to an outer orbit. Its kinetic energy decreases but its total energy increases. It can go only to orbits 9, 16 $\cdots$ times the radius of the smallest orbit. The farther out it goes, the more energy it can lose coming in. Hence the shortest wave lengths are likely to be obtained when the gas or vapor is excited by intense electric fields.

Extension to Elliptic Orbits.

In the first presentation of his theory Bohr confined his attention to circular orbits. But it has been shown in Chapter 3 that an electron describing any ellipse of semi-major axis a has the same energy as one describing a circle of that radius. So far as energy is concerned, therefore, there are an infinite number of orbits giving the same energy as the circular orbit. But let us restrict them in the following manner.[1] Let us take for example the fourth circular orbit of radius r. For that orbit $2\,\pi \times$ angular momentum $= 4\,h$. If we would restrict the ellipses by requiring that $2\,\pi \times$ angular momentum $= 3\,h$, $2\,h$, h, then b, the semi-

[1] This mode of arriving at the "allowed" elliptic orbits is the author's. It is not the original way due to Sommerfeld. The orthodox way is to make the "radial momentum" satisfy the quantum law. By radial momentum is meant the product of the component of the linear momentum along r, the radius vector, by the component of the arc along r, the total integrated around the curve. This is put equal to n_1h where n_1 is a whole number. In a circle this is zero but in an ellipse, since generally a component of the momentum is along the radius, it is not zero.

minor axis of the ellipses, would be $3\,r/4$, $2\,r/4$, $r/4$, and they would look as in Fig. 6–2. This can be very simply proved as follows. The period for a particle describing an ellipse depends on a, the semi-major axis, not at all on b. Kepler stated this as his third law.[1] The area of an ellipse is πab. Since the times of describing the whole areas are equal, the rate for the different ellipses is proportional to b. And we have shown in Chapter 3 that angular momentum is twice the rate of describing an area $\times$ mass of particle in motion. Hence the angular momenta in the three ellipses and circle are proportional to 1, 2, 3, 4.

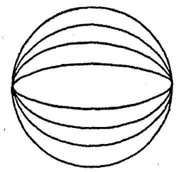


FIG. 6–2. The energy of an electron describing a circular orbit (about the center) is equal to that for an elliptic orbit about the focus; but the angular momenta in these orbits are as 1, 2, 3, 4.

Now let us picture all possible orbits thus restricted, from the first to the fourth, as in Fig. 6–3. The first orbit is a circle of radius a_1. For the second circle the angular momentum $\times\ 2\,\pi = 2\,h$. For the ellipse of the same energy it is h. We arrange

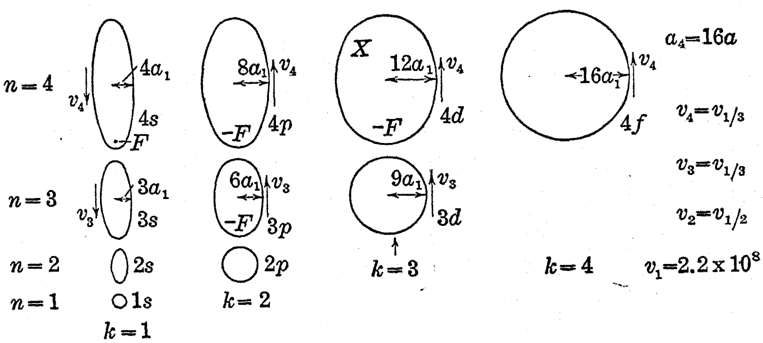


FIG. 6–3. Bohr-Sommerfeld orbits, for hydrogen, of equal energy (proportional to n) on the same row; of equal angular momenta (proportional to k) in the same column. $a_1 = 0.528 \times 10^{-8}$ cm. $v_1 = c/137$.

[1] Kepler's third law is generally stated thus—the squares of the periodic times are proportional to the cubes of the *mean* distances (from sun to planets). The *mean* distance from a focus to the periphery of an ellipse with angle as a variable is equal to the semi-*minor* axis. This would be the ordinary meaning of *mean* distance and would make the statement entirely incorrect. If *mean* be taken with the major axis as variable, it equals the semi-major axis; that interpretation of *mean* would state the law correctly, but it is highly probable that that interpretation is almost never placed on *mean* as here used. But the interpretation to be placed upon *mean* as used by Kepler is simply the average of the greatest and least distances to the planet.

these in horizontal rows of equal energy and vertical columns of equal angular momenta. We may designate the ellipse marked X by 4.3. (In cartesian coordinates y, x.) But too many numbers would lead to confusion. Let us designate the columns by letters *spdf*.[1] A frequency is radiated when an electron goes from the 2 p to the 1 s orbit. It can change from any n to any lower n, but it is found necessary by the experimental results to restrict the changes in k to $+1$ or -1. Thus we may have changes represented by $3\,s - 2\,p$, $4\,s - 2\,p$, or $4\,d - 2\,p$, but not $4\,f - 2\,p$ or $4\,p - 2\,p$. These look like the arbitrary rules of a game. Is there any "reason" behind them? If frequency radiated depends only on energy change and if 4 s and 4 d have the same energy, what difference is there between $4\,s - 2\,p$ and $4\,d - 2\,p$?

Let Us Try to Find a Reason for the Rules of This Game.

In Chapter 2 it was shown that the "mass" of a particle depends on its velocity, very critically as the velocity approaches that of light. If we solve for velocity in the Bohr circular orbits, we find

$$v_n = \frac{2\,eE}{nh} = \frac{2\,Ze^2}{nh}$$

where Z is the atomic number. For hydrogen this is nearly 2.2×10^8 cm. per sec. in the first orbit; for helium twice this, etc. As we go to larger orbits the velocity decreases, but for elliptical orbits as the electron rounds the curve near the nucleus (perihelion) the velocity may be large. (We can show [Appendix 3–4] that when $a_n = na_1$ and $b = a_1$, the velocity at perihelion is v_1.) Hence the "mass" of an electron changes slightly as it goes round an elliptical orbit and this influences the energy. Therefore the energy in an elliptical orbit is not quite equal to that of the circle which just circumscribes it. *The energy depends chiefly on the major axis, slightly on the eccentricity.*

[1] Why these letters? In the early days of spectroscopy it was seen that certain lines in the spectrum of an element belonged together; some were *sharp*, some seemed to belong to a principal series, some were diffuse; hence s, p, d. Later other lines which belonged together according to their physical characteristics and which did not fall in with the s, p, d above were called fundamental—though they were not especially fundamental. But f follows d (e has been appropriated for *electron*), and the later letters follow in alphabetical order, s, p, d, f, g, h, k, . . . z.

Elliptical Orbits Selected by Other Rules.

The scheme of orbits shown above is called the (B.S.) Bohr-Sommerfeld. But various other schemes have been tried and Fig. 6–4 shows the above scheme in comparison with three

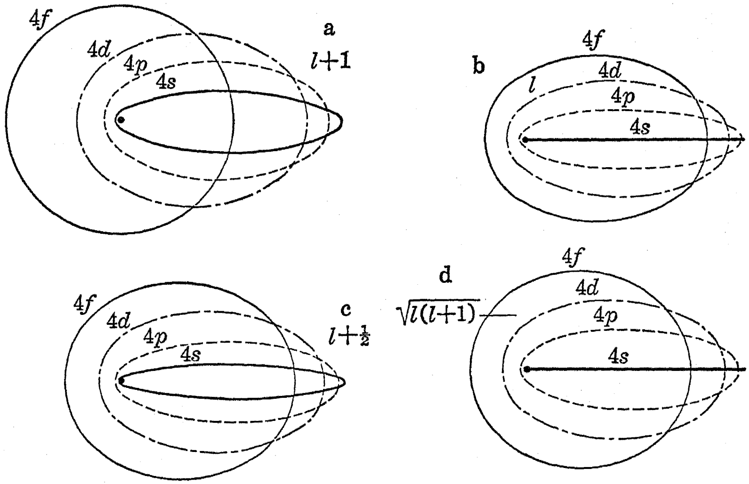


FIG. 6–4. The evolution of orbit models in attempts to suit spectroscopic data.

others.[1] In two of these the "orbit" degenerates into a straight line. The B.S. model would not permit this case. It is similar to our illustration in Chapter 3 of firing a body vertically upward with a speed of 6 miles/sec., then quickly shrinking the earth to a small nucleus so that the particle would come back and pass the position of the original earth surface on its way, to the nucleus—which we hoped it wouldn't hit, for that would produce a tremendous smash. Two of the models appear to allow that performance, but without producing any significant disturbance.

It will be noted that, though these ellipses all have the same major axis, no two have the same minor axis except when $b = 0$. Hence all have their own characteristic energies.

The orbits we have been discussing, the B.S. orbits (Fig. 6–4 a), are those for which the angular momentum is equal to $h/2\pi$, $2h/2\pi$, etc. For these $k = 1, 2, 3$, etc. For the other sets of

[1] See also Fig. 15–6 for pictures of the hydrogen atom.

orbits we have the numbers given in the table below. Frequent reference will be made in Chapter 10 to the last two models.

TABLE II

Electron	(a) B.S. $l = k$	(b) Sommerfeld $l = k - 1$	(c) Landé $l = k - \frac{1}{2}$	(d) Schroedinger $\sqrt{l(l+1)}$
s	1	0	$\frac{1}{2}$	0
p	2	1	3/2	$\sqrt{2}$
d	3	2	5/2	$\sqrt{6}$
f	4	3	7/2	$\sqrt{12}$

Convergence Limits for Hydrogen.

In the original Bohr theory, the largest wave numbers or shortest lines were given by

$$\nu = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

where $n = 2, 3$, etc. This represents transitions from outer orbits to the smallest orbit. The greatest value of ν, when $n = \infty$, is R or 109,678. In other words, the length of the shortest line due to hydrogen is 1/109,677 cm. This is the "convergence limit" of the Lyman series, and the orbit which gives the energy of the limiting state is 1 s. And now since all the lines of this series must be due to transitions from the P column to 1 s they are single, not double. This is in accord with fact.

Bohr's Theory for Complex Atoms Fails but the Idea of Energy States Is Established.

The above paragraphs show how the simple Bohr theory, extended to elliptical orbits, fits the facts for hydrogen and hydrogen-like atoms. But when we go to more complex atoms, what happens? In the first place, we would meet the difficulty that we could not solve the equations of motion. The problem of three bodies in astronomy has occupied the attention of mathematicians for over two centuries. It has not been completely solved. The problem involving the path of an electron moving under a force of attraction towards a positive nucleus and also

under a force of repulsion from a number of other electrons can be solved only approximately for special cases. *But one clear idea came from the Bohr theory which is now dominant in every part of spectroscopy—the idea that the frequency of the light coming from an atom or molecule is due to a change in the energy state and that it equals the energy loss divided by h.* Consequently the spectrum of a heavy atom is examined for series. The differences in the wave numbers of these lines give differences in "terms," these are proportional to "energy states." Thus a model of the atom is constructed based on those stationary states. As an illustration we might give the case of sodium. The most common line known to students is the yellow line of sodium—the D line. Its simplest representation is $3\,s - 3\,p$. It is really double of wave lengths 5890 Å and 5896 Å with wave number difference = 17 cm.$^{-1}$. A similar line is 3303 Å represented by $3\,s - 4\,p$. It is a doublet 3302.94 Å and 3302.34 Å with wave number difference of 5.5. Another doublet 2853 Å or $3\,s - 5\,p$ has a wave number difference of 2.5. All of these differences could be accounted for if the s levels were single and the p levels were double, p_1, p_2. In this way the energy level diagram (Fig. 6–5) for sodium is worked out without regard to theory. However, we retain quantum numbers and angular momenta similar to n and k in Fig. 6–3.

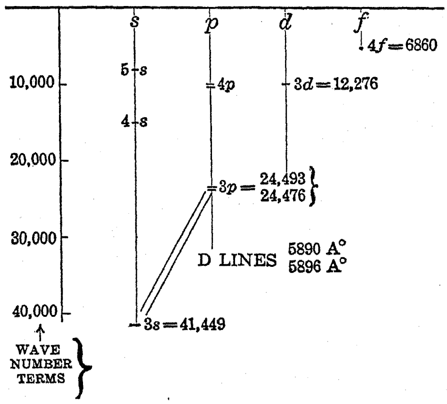


FIG. 6–5. A few energy levels of the sodium atom as determined from its spectrum. The 3 s, 3 p, 3 d now do not have the same energy.

More Complexity Is Required, so We Make the Electron Spin.

It was found that two variables were not at all sufficient to give the complexity demanded by the data furnished by spectroscopy. In 1925 it was proposed that it was necessary not only to have angular momentum of an electron in an orbit—it was also necessary that this electron spin about its own axis and that the

angular momentum of this spin should be one-half of that of the electron in its smallest orbit and therefore equal to

$$\frac{h}{4\,\pi}\left(\text{later changed to}\sqrt{\frac{1\cdot 3}{2\cdot 2}}\,\frac{h}{2\,\pi}\right).$$

This spin might be in the same direction as, or opposed to, that of the electron about the nucleus. Now it is probable that most physicists of the world had come to regard an electron as a minute accumulation of electricity, probably spherical in form, of a diameter of about 10^{-12} cm. There did not seem to be any way in which such a "particle" could be put into rotation, or stopped if in rotation. And its rotational speed would have to be very great in order that its energy would be appreciable even in spectroscopy. But if it had an appreciable angular momentum, it would also necessarily have a magnetic moment and that would have to be considered. So an electron becomes complex! It has a charge of 4.77×10^{-10} e.s.u., a mass of 0.9035×10^{-27} grams, a mechanical moment or angular momentum of $\sqrt{3}\,h/4\,\pi$ or 0.902×10^{-27} erg seconds, and a magnetic moment of

$$\frac{\sqrt{3}\,h}{4\,\pi m}\,\frac{e}{c}$$

or 1.59×10^{-20} erg gauss (Appendix 6–2). If it has mass, electric charge, magnetic properties, it is a far more complex unit than the old atom of kinetic theory!

New Energy Levels Due to Electron Spin.

Let us now include the spinning electron as a member of the family of an atom and revise our views of energy levels. We cannot picture these now as we did in Fig. 6–3. Let us indicate them by letters. We must still have the principal quantum number n, as in Fig. 6–3, which roughly divides the energy states, and we must have variations of these levels. Let us now take

$$l = 0, 1, 2, 3, 4, 5, \cdots$$

and symbols

$$s, p, d, f, g, h, \text{etc.}$$

Since the electron contributes $\frac{1}{2}$ or $-\frac{1}{2}$ of the angular momentum unit, we may have a $p\,\frac{1}{2}$ or a $p\,1\frac{1}{2}$, a $d\,1\frac{1}{2}$ or a $d\,2\frac{1}{2}$, etc., but only an $s\,\frac{1}{2}$. But this is only the beginning of the matter. We would

have to consider the effect of the electron spin in causing the orbital motion to change—to precess as does the axis of a top. We would have to consider the interaction of this spin on the motion of the other electron. In fact, the "picture" now becomes so confused that its details become meaningless. The new treatment by wave mechanics avoids these details.

Spectroscopic Terminology.

Neglecting small variations in energy levels, let us picture [1] all the electrons in a boron atom. The atomic number of boron is 5. It has 5 electrons outside the nucleus. The normal atom, call it B I, has 2 electrons in the innermost ring or lowest level—this is the *s* level of quantum number 1. We write this $1\ s^2$. Spectroscopic evidence shows that it has 2 electrons on a level of (rough) quantum number 2 but of the single lower branch of that level. We write this $2\ s^2$. It has one electron of a higher branch of the same level 2, or $2\ p^1$ (omit the 1). Hence we represent the normal boron atom as $1\ s^2\ 2\ s^2\ 2\ p$. If an electron is pulled away from this atom, a certain voltage is required, 8.28 volts. We say that the ionization potential is 8.28 volts. It would lose the $2\ p$ electron. The atom would then become B II and would be written $1\ s^2\ 2\ s^2$. Having only four electrons outside the nucleus, it would not be a neutral atom as the nucleus has a positive charge of $5\ e$. It would require a larger voltage to remove another electron. The ionization potential is now 25.0 volts. If again ionized, it would lose one of the $2\ s$ electrons and would become B III or $1\ s^2\ 2\ s$. The ionization potential is now 37.7 volts. Similarly B IV ($1\ s^2$) has only two electrons left and requires now 258 volts to remove one of the $2\ s$ electrons. It becomes B V with only one electron left, $1\ s$. It is similar to a hydrogen atom. The voltage required to ionize it is 338 volts. The Bohr value of the wave number of the shortest line for hydrogen corrected for mass ratio of nucleus and electron was 109,678. According to the same theory it is [2] for B V $109{,}731 \times 5^2$ (leaving out relativity correction). This is a wave length of about 36 angstrom units—which places it in the X-ray region. Thus it is seen that the removal of an electron increases the electric force on any electron

[1] Here we adopt the proposals of Russell, Shenstone, and Turner in their "Report on Notations for Atomic Spectra," *Physical Review*, Vol. 33, p. 900, 1929.

[2] Giving the wave number for the series of shortest lines.

which is changing its position or energy state and thus causes greater energy differences, larger wave numbers, shorter wave lengths, than are found in the normal atom. The similarity in the energy levels or wave number values for lithium-like atoms is shown in Fig. 6–6.

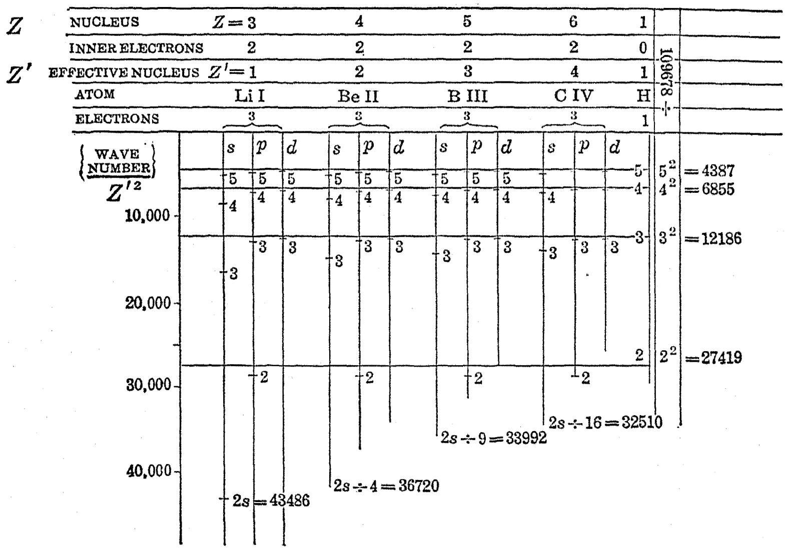


FIG. 6–6. Carbon atoms which have lost 3 electrons, boron which has lost 2, and beryllium which has lost 1 have spectra similar to that of lithium.

Comparison of Similar Spectra.

In the simple Bohr theory the energy levels expressed in wave numbers equal Z^2B/n^2, where Ze is the total positive charge inside the orbit which is being described by the electron, B is Bohr's constant, and $n = 1, 2, 3, \cdots$. For hydrogen $Z = 1$, $B = 109{,}678$. Now the normal lithium atom has a nucleus of 3 positive units, then 2 electrons outside this nucleus, then one valence electron outside of all. For such an atom the *effective* charge inside the valence electron is 3–2 or one positive unit. Beryllium has a nucleus of 4 positive units, 2 electrons outside, then 2 outside of all. If beryllium is singly ionized, it becomes Be II and the outer electron has only 4–2 or 2 positive electrons inside. Hence the *effective charge* inside is 2. Similarly for B III and C IV the effective charges inside are 3 and 4 respectively.

Hence if we divide the wave numbers in the corresponding terms of these atoms or excited atoms by 1^2, 2^2, 3^2, 4^2, we might expect to find some similarity. Figure 6–6 shows roughly the points of fair agreement—and the rather large discrepancies. The second level for hydrogen is $109{,}678/2^2 = 27{,}419$. The $2\,s$ level for lithium is 43,486; the $2\,p$ is 28,582. The 4th level for hydrogen is $109{,}678/4^2 = 6855$. The $4\,s$ level for lithium is 8475, the $4\,p$ is 7018, the $4\,d$ is 6863; the $4\,f$ is 6856. Here they are coming together. The $2\,s$ number for Be II is 146,881. If we divide this by 4 we have 36,720. This is to be compared with 43,486 for lithium and 27,419 for hydrogen. In these levels there are large discrepancies, but as the electron gets out to higher energies the agreements become closer. On the whole, while the Bohr theory is a fair general guide, it is entirely unsatisfactory as a quantitative leader.

The Main Features of a Spectroscopic Atom.

Let us now give a picture, leaving out many details, of the normal barium atom. We call it Ba I. Its atomic number is $Z = 56$. It has 56 electrons outside [1] the nucleus. Spectroscopy represents it thus:

$$1\,s^2\,2\,s^2\,2\,p^6\,3\,s^2\,3\,p^6\,3\,d^{10}\,4\,s^2\,4\,p^6\,4\,d^{10}\,5\,s^2\,5\,p^6\,6\,s^2.$$

(The sum of the exponents equals 56.) This is the normal unexcited atom! But only a partial view is given of the electrons outside the nucleus. For example, the part $3\,p^6$ indicates that there are six electrons in the p sub shell of the main shell of quantum number 3. But every one of those electrons has an energy slightly different from that of each of the other five.

These symbols indicating only partially the energy states of the electrons and ignoring completely the possibility of complex energy states in the nucleus show that we have gone far from the *uniform sphere* of the kinetic theory. The atom has become an exceedingly complex grand piano!

[1] The old view, 1913–1933, was that in the case of barium, for example, there were 56 electrons outside the nucleus and that the latter consisted of 137 protons and 81 electrons. The 137 protons would account for the atomic weight of barium, and the excess positive charge of the nucleus, 137 minus 81, would neutralize the electrons outside. But now after the discovery of the neutron we have the view that the nucleus consists of 56 protons and 81 neutrons. There are no electrons in the nucleus. See Chapter 14.

Success and Failure of the Bohr Theory.

And now let us leave spectroscopic data and return to consider the Bohr theory itself. It was satisfactory in its derivation of the value of the Rydberg constant and in accounting for some of the simple facts of spectroscopy. It was unsatisfactory in that it offered no "reason" for the "allowed" orbits except the Planck relation: it presented no mechanism for producing oscillations; the frequencies of the radiation had no significance except that they satisfied the quantum law; an electron must know before it leaves one orbit that another to which, by our spectrographic information, it is "allowed" to go, is unoccupied—otherwise it cannot absorb radiation readily available. These are a few of the criticisms, perhaps primitive and tinker-toyish.

The ingenious model devised by Bohr, delightfully satisfactory upon first examination, finally was found not to be in accord with experimental fact. Atomic spectra are extraordinarily complex; the Bohr formula was very simple. Vain attempts were made to add complexity to it by adding the spinning electron, the variation of mass due to speed, the variation of energy due to interpenetrating orbits, space quantization of orbits. Finally the picture toppled over, loaded down by these adornments. Heisenberg, Schroedinger, Dirac formulated new theories in which algebraical symbols replace geometrical forms. Their formulae for energy states have numerous variables; these are in accord with numerous spectroscopic results. The Bohr picture has passed, except in the most elementary presentation, but its terminology and many of its ideas remain. Outstanding is the concept of *energy states* and the view that the frequency radiated or absorbed depends only on the differences of energies in these states.

CHAPTER 7

X–RAYS

The story of X-rays briefly told.——The story of a phenomenon which has profoundly influenced all science, which has extended the realm of nature manyfold.

A Startling Announcement—X-Rays Are Discovered.

In Christmas week 1895 word was flashed across the Atlantic that Roentgen in Wurtzburg had discovered a new kind of radiation which would penetrate the flesh of one's hand and give a shadow of the bones on a photographic plate enclosed in a light-tight box. Journalists, perhaps for the first time, became scientifically minded and the news was brought to the attention of the man on the street. Since then X-rays have become of vast importance in hospitals, in industrial plants, in all kinds of scientific laboratories.

X-Ray Tubes—Old and New.

The kind of tube that was developed and used during the first twenty years of X-ray history is shown in Fig. 7–1 a: A glass tube as shown is exhausted to a vacuum of about 1/100 mm. The aluminum cup-shaped cathode C and the heavy (tungsten) target A are connected to a steady potential of from 10,000 to 200,000 volts. The cathode stream from C striking the target causes X-rays to emerge. These produce a fluorescence in the glass hemisphere in front of the target. Some gas is necessary in the tube. After the discharge has been passing for some time, the gas in the tube may be practically "used up." This is made evident by the fact that a higher voltage is necessary for the discharge and also by the greater penetration of the X-rays. But some gas may be supplied to the tube by causing a discharge through a side tube, thus heating up a special electrode in which there is occluded gas. The X-ray tube then becomes "softer."

Another way in which gas may be introduced into the bulb without opening any side tube is of very great scientific interest. In Fig. 7–1 b, T is a *thimble* of platinum about 5 cm. long, 2 mm.

in outer diameter, of quite thin wall. It is sealed into the glass bulb with the closed end outside. When T is heated by a bunsen or alcohol flame, the hydrogen molecules in the flame having great

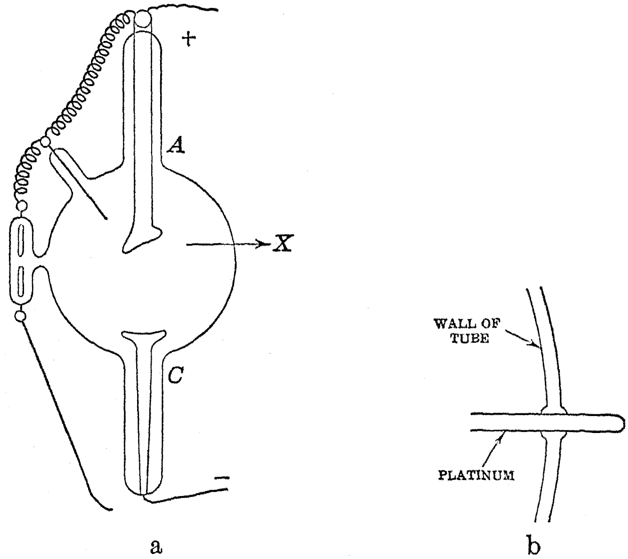


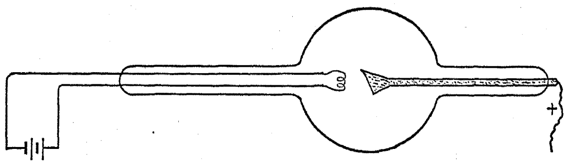
FIG. 7–1. a, excellent X-ray tubes of the 1900–1925 period. The amount of gas in the tube could be altered during use. b, hydrogen from the hot gases around a flame in which a thin platinum thimble is being heated may pass through the thin wall into the X-ray tube.

speeds can penetrate the walls of the hot platinum. Thus hydrogen alone is admitted to the bulb. When the hydrogen is cooled down inside the molecules cannot escape. (Compare Chapter 11, the Rutherford and Royds experiment.)

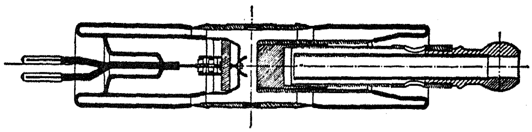
The other kind of tube, which is known in America as the Coolidge tube, is shown in Fig. 7–2 a. Here the electrons come from a wire electrically heated as in the ordinary radio tubes. The vacuum is very high. No gas is necessary to supply electrons.

A variation in the *form* of this tube is shown in Fig. 7–2 b and c. Here the target is a heavy block of copper which can be water cooled. Consequently the tube may be one of high power. It is completely metal shielded except for two windows of glass through which the X-rays emerge. Photographs taken with this

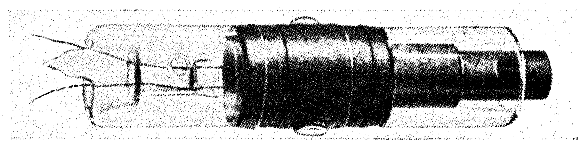
powerful tube as source of X-rays are shown at the end of this chapter.



a



b



c

FIG. 7-2. In the Coolidge tube a and in more modern tubes b and c, electrons are supplied by a hot filament; in b and c the heavy metal anode is water cooled.

Early Ideas Regarding the Nature of X-Rays. Ether Pulses.

In the very beginning it was shown that X-rays could not be deflected by either magnetic or electric fields. Therefore they were not electrified particles. But they were not reflected, refracted, polarized, nor diffracted. (So it appeared at first.) Therefore they were not like light! Roentgen suggested that they might be due to *longitudinal waves in the ether;* Stokes, that they might be *transverse ether pulses.* The latter point of view was developed by J. J. Thomson at considerable length. By means of it he was able to account for, and even to prophesy, some of the properties of the rays. In this he held to the accepted result of Maxwell's theory that when an electric charge is accelerated radiation takes place. (Bohr's theory ignored this view.) The electron was discovered (1897) very soon after the X-ray. Its velocity under definite conditions could be measured.

It was possible to make this velocity very large. Electrons could not penetrate metals very far; hence must be stopped quickly. So J. J. Thomson had at hand all the elements of the pulse theory.

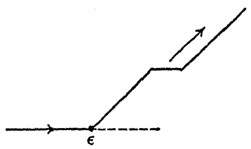


FIG. 7-3. Illustrating the pulse theory of X-rays. When an electron ϵ is suddenly stopped by the target in an X-ray tube, a pulse runs out along a line of force as it does along a stretched spring when one point of it has suddenly been jerked sideways.

Figure 7-3 represents an electron moving with great speed, then suddenly slowed down. The electric line of force (only one of them shown) which had been a straight line moving with the electron continues to move forward at points distant from the electron, while the electron is being stopped. Consequently a kink or bend in the line runs out with the velocity of light nearly at right angles to it. It may be evident from this theory there should be no radiation in the direction in which the electron is going nor in the opposite direction since no longitudinal disturbance can be propagated through the ether. Moreover, the X-rays ought to be partially polarized since the electric vector is in the plane containing the direction of motion of the electron and the direction of observing the X-rays. This view was experimentally verified. This is the *ether pulse* which sufficed to picture an X-ray for about eighteen years. (It was these considerations that caused Thomson, about the time that Einstein was elaborating the quantum theory of light, to propose the *ether string* theory, that light was due to waves along strings.)

Some Ordinary Properties.

The ordinary facts concerning X-rays are well known—that they affect photographic plates; that they cause certain crystals to fluoresce (hence the fluoroscope); that they penetrate light substances, wood, paper, aluminum, easily; that thick lead shuts them out; that penetration increases with voltage. That they ionize a gas through which they pass, that they make a gas conducting, is easily shown. The photographic plate, the fluorescent screen, and the ionization chamber are the devices used for detecting and measuring X-rays.

After Seventeen Years a Great Discovery Is Made. X-Rays Are Wavelike.

All attempts to observe the diffraction of X-rays and so to measure their wave length failed until Laue in Munich in 1912 proposed that a crystal be used for that purpose. The idea was that the atoms in a crystal would be regularly spaced and be very close together. He and his co-workers, allowing a fine pencil of X-rays to pass through a thin (about 2 mm.) plate of zinc blende and to fall on a photographic plate, obtained an effect as illustrated[1] in Fig. 7–4. The large central spot is due to the direct X-ray pencil. The various small spots are due to rays scattered from special many-atom planes in the crystal. Bragg of Manchester immediately adopted another arrangement. His method has since been in universal use both for the measurement of wave lengths and for the study of crystal structure.

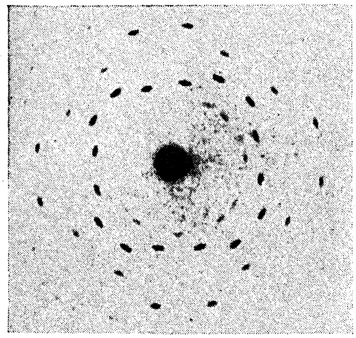


FIG. 7–4. X-rays passing through zinc blende, ZnS, parallel to the (100) axis give this pattern; the historic experiment proving that X-rays are wave-like in character.

The Distance between Atom Planes in a Crystal. Bragg's Method.

In optics, in order to measure wave lengths of the order of 0.0006 mm., gratings are used in which the grating space is of the order of 0.002 or 0.001 mm. In other words, it is advantageous to have a grating space two or three times the wave length. What is the grating space in a crystal? What is the distance between atoms, or between planes in which atoms are abundant? The problem is like that of finding the diameter of steel balls, all equal, when 1728 fill a cubic foot. The answer is 1 inch. Twelve would make a linear foot, 144 would cover 1 ft.^{2} and there would be 12^3 in 1 ft.^{3} In common salt[2] (NaCl) there are 6.06×10^{23}

[1] See also Fig. 15–3.

[2] This illustration might be quite misleading. For the volume of a small sphere is $(\pi/6)d^3$ and the actual volume of the spheres themselves is only slightly more than one-half a cubic foot. If we have a great number n of small spheres in 1 cm.^{3},

atoms each of sodium and of chlorine in 1 mole or gram molecule; i.e., in 23 + 35.46 grams. The density of salt is 2.17 gm./cm.[3] The student can find that d, the distance between atoms, supposed equally spaced as on the corners of a cube, is 2.81×10^{-8} cm. For calcite ($CaCO_3$) it is 3.029×10^{-8} cm. (Compare with the diameter of the hydrogen atom as found from kinetic theory and from the Bohr theory.)

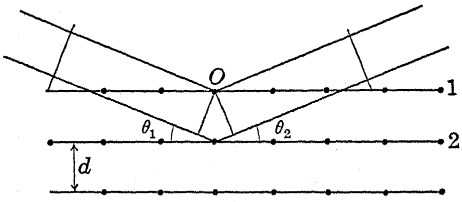


FIG. 7–5. X-rays may be scattered from atom planes of a crystal so as to give a maximum intensity in a certain direction.

Let us picture a salt crystal (Fig. 7–5) as sodium and chlorine atoms alternating and equally spaced and let lines 1, 2, 3 represent such planes at right angles to the paper. X-rays if due to continuous waves falling on plane 1 would be scattered in all directions, but as in the case of light and for the same reason there would be a maximum in the direction where $\theta_1 = \theta_2$. The amount reflected from one plane of atoms is extremely small. We want the rays from planes 2, 3, $\cdots$ to amplify the effect. In other words, we want all the reflected parts to be in *the same phase.* Since a line at right angles to the incident rays is a wave front or locus of points in the same phase, so also is one from 0. We want a line from 0 normal to the emerging rays also to be a wave front. This will be true if $2\,d \sin \theta = \lambda$ (or $2\,\lambda$, $3\,\lambda$, etc.). Hence knowing d and measuring θ we can find λ. A check is obtained by computing d for another crystal and, using it and the same radiation, test for λ. Or if the crystal be a cubic, as is rock salt, we may orient the crystal so as to make the distance between the planes $= d/\sqrt{2}$, or $2\,d/\sqrt{3}$. (A student with a

---

the diameter of a sphere for tetrahedron or closest packing is

$$\frac{\sqrt[6]{2}}{\sqrt[3]{n}} = \frac{1.12}{\sqrt[3]{n}}.$$

When the spheres are on corners of cubes as pictured above, the distance between centers $= (1/\sqrt[3]{n}) = \sqrt[3]{(M/\rho N)}$ where ρ = density, M = molecular weight, N = Avogadro's number. But the distance from Na to Cl $= \sqrt[3]{(M/2\rho N)}$. In a liquid the "spheres" might be arranged so as to produce "closest packing" but in a solid crystal they would have definite geometrical positions not necessarily in closest arrangement.

model of a rock salt crystal, Appendix 7–3, before him can easily derive these values for the distances between its crystal planes.) Using the same wave length we can verify these values—thus proving that the crystal is a cubic.

X-Ray Spectra Depend on the Target. Moseley Establishes Atomic Numbers.

Immediately after Bragg established the validity of his method of measuring λ and of analyzing an X-ray spectrum into its various wave lengths, Moseley tested the X-rays obtained from 38 different elements as target, with aluminum as lowest in atomic weight. It was found that there was a general radiation depending upon the voltage but, very specially, *definite spectral lines depending upon the target.* It was the latter *characteristic radiation* that engaged Moseley's attention. Rutherford had already proposed that the elements should be characterized by atomic numbers approximately half the atomic weights. Moseley found that by choosing the atomic number of aluminum as 13 (approximate weight 27) and by increasing this number by unity as one passed to the next greater atomic weight (making allowance for omissions from the periodic table), the square roots of the frequencies increased also in arithmetical progression. In other words, the square root of the frequencies of corresponding lines plotted against *atomic numbers* as proposed above gave a straight line. *It was this experiment that established atomic number as an essential characteristic of a chemical element,* thus profoundly influencing the whole fields of chemistry and physics.

Identification or Discovery of Elements by Their X-Ray Spectra.

Moseley found that every element gave two quite different series of lines, a very short and a longer series. Barkla some years earlier, not knowing anything about wave lengths, had found that, judged by penetration power, there were two types. He called them the K and L radiations (M and N, longer wave length series, are also in evidence, chiefly for heavy atoms). Moseley's photograph (Fig. 7–6) in which he arranged the series of shortest lines, the K series, of the elements from calcium to copper (with scandium left out) in order of atomic number vertically and frequencies horizontally will always be a historic land-

mark. It is evident from Fig. 7–6 that calcium is out of place; it should be one step higher. The K spectrum for scandium could be computed and inserted. Similarly the K spectrum of any as yet

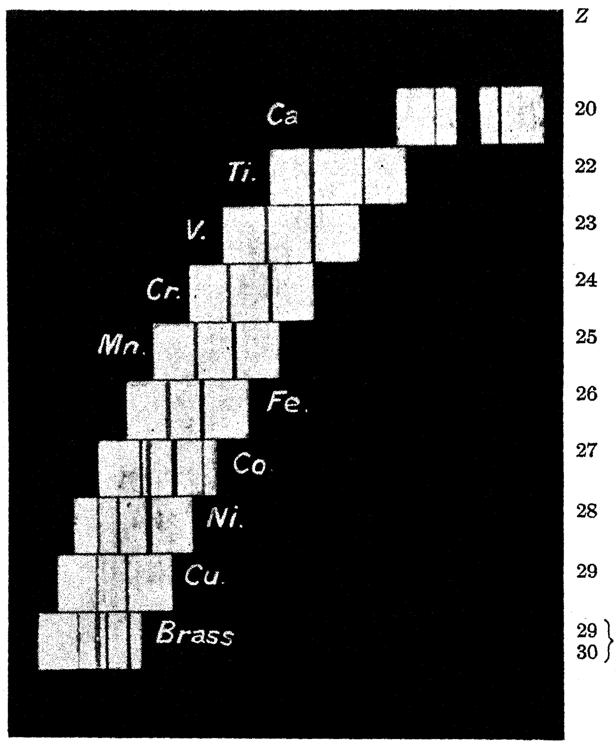


FIG. 7–6. Moseley's historic photograph. Electrons fired against calcium, titanium, etc., produce X-rays, the shortest spectral lines of which, the K series, are shown. It is evident that calcium should be a step higher.

undiscovered element is known with rather high accuracy. New elements[1] have been identified in this way.

Moseley also photographed a longer series, the L series. A relation similar to that for the K series holds when the photographs are placed in order of atomic numbers. Siegbahn's excellent photographs for elements from gold to bismuth are shown in Fig. 7–7.

[1] In 1926 in the University of Illinois element no. 61 was so discovered by Hopkins, Ynterma, and Harris. It was named illinium.

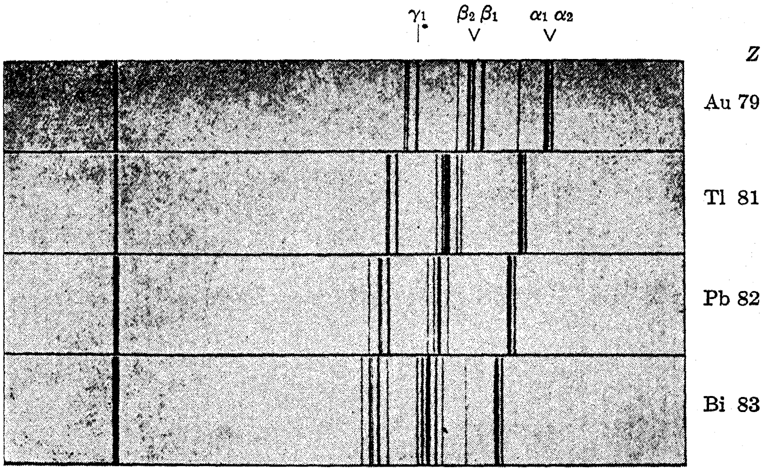


FIG. 7–7. *L* Series Spectra (Siegbahn). The spectrum of atomic number 80 (mercury) is missing.

X-Ray Spectra Fit into the Bohr Picture.

The formula fitting Moseley's experimental results is $f = K^2(Z - k)^2$ where K and k are constants for any series for all the elements and Z is the number assigned by Moseley to designate the element but now established as the atomic number. Compare this with the Bohr formula for the shortest series of an element,

$$f = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right).$$

Moseley's formula is only approximate and Bohr's theory can be extended to take account of the repelling action of electrons surrounding the nucleus. Both corrections bring the formulae towards one another so that we now regard the X-ray spectra as falling in with the Bohr idea.

General X-Radiation Depends on Voltage. Minimum Wave Length Given by the Quantum Idea.

The nature of the general X-radiation for a tungsten target[1] is shown in Fig. 7–8. There is a suggestion of similarity between these intensity curves and those giving the distribution of in-

[1] The distribution of the intensity in general radiation depends on the target, the voltage, and the direction of observing.

tensity in black body radiation (Fig. 4–5). In Fig. 7–8 the voltage is constant for one curve, in Fig. 4–5 the temperature is constant. In the black body curves the wave length for the maximum ordinate $\times$ absolute temperature is constant. The corresponding quantity in Fig. 7–8, $\lambda_m V$, is not constant. Moreover, it ought to be pointed out that the form of these curves depends partly on the direction the X-rays make with the original cathode rays. If that angle is small, the intensity maximum is pushed towards short waves. Curve IV in Fig. 7–8 shows that the maximum of the energy radiated is shifted towards short waves when the rays are examined (nearly) along the original direction of the electron stream. *But the curve for any voltage cuts abruptly the line of wave lengths at a critical minimum wave length* λ_x. For that voltage there is no radiation shorter than λ_x. It will be found that $V\lambda_x$ is a constant. The quantum theory gives a clear explanation of this result. For the electrons which leave the cathode are driven into the target with an energy $\frac{1}{2} mv^2 = Ve$ where V is the potential and e is the electron charge, both in absolute units. The energy of an electron goes into a quantum $hf(= hc/\lambda)$ and this $= Ve$. Hence $\lambda_x V$ is constant where λ_x is the minimum wave length; more definitely $\lambda_{\min} = hc/Ve$ (c is the velocity of light). Thus[1] $\lambda_{\min} = 12{,}345/V$ where $\lambda_{\min}$ is in angstroms and V is in volts.

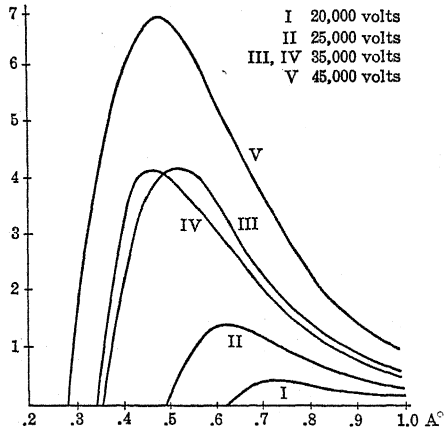


FIG. 7–8. Intensity curves for general X-radiation. The higher the voltage, the shorter the wave length that may be obtained.

To Bring Out a Series the Voltage Must Be Large Enough to Bring Out the Shortest Line.

Now the characteristic radiations are sharp high horns on these general radiation curves. Looking up tables one finds that the

[1] More accurately $\lambda_{\min} = 12{,}336/V$, but 12,345 is easily remembered.

shortest of the K lines from molybdenum is 0.618 Å. Hence the voltage necessary to bring out that radiation must be greater than 12,345/0.618 = 20,000 (nearly). From the curves of Fig. 7–8 it will be seen that a voltage of 25,000 would bring the K lines for molybdenum at the maximum of the general radiation curve.

The computation above illustrates a general rule—if we want to bring out one line of a series we must use a voltage high enough to bring out the shortest line of that series. A reasonably satisfactory explanation of this result may be given by using the Bohr picture. Let us consider the K series. If by means of a sufficiently high voltage we are able to give enough energy to an atom so that an electron from the K shell is ejected, it will be thrown completely out of the atom—there is no resting place in other orbits on the way out. Then an electron from an energy level higher than the K may go in to the place left vacant. On account of this transition, energy must be radiated; we would have one of the K lines. In another atom an electron of a different energy might move in; that would give another line of the K series. As there would be a great number of atoms energized by the impressed voltage, there would be chances for all the electrons which can take part in the generation of the various lines of the K series to do so. The intensity of a line would be determined in part by the number of electrons of the appropriate energy available for the transition. For example, we say that the $K\alpha_1$ line is due to a transition from the L_{III} level to the K shell; the $K\alpha_2$ from the L_{II}. The former line has twice the intensity of the latter; the L_{III} level has 4 and the L_{II} has 2 electrons; hence the probability that the $K\alpha_1$ transition is twice that of the $K\alpha_2$. Thus we account for the intensities of the lines as well as for the fact that all the lines of a series are apt to appear when the voltage is high enough to bring out them all—and not otherwise.

Scattered X-Rays.

When X-rays fall upon a plate of solid material, there are scattered rays which in general are of two kinds: (1) rays which are nearly of the same wave length as the incident rays; these are almost completely polarized as prophesied by the pulse theory and are of maximum intensity in the plane containing the direction of the electrons and the primary X-rays; (2) rays which depend upon the material of the target; they are characteristic rays of this

material, homogeneous, perhaps monochromatic; they are not polarized. (In no case is the wave length of the scattered ray shorter than that of the incident.) The latter kind of scattered rays is called fluorescent radiation, since the phenomenon is similar to that in light.

The Pulse Theory Accounted (Partly) for Scattering.

Now it may appear to the student that the topic of scattered X-rays is not an exciting one, but it was on account of work connected with scattered radiation, including scattering from crystals and scattering of light, that five Nobel prizes were awarded—to Barkla, Laue, Bragg,[1] A. H. Compton, Raman. To begin with, the pulse theory not only "explained" the polarization of the general scattered rays; it gave (through J. J. Thomson) the total fraction scattered. The formula deduced is $\sigma = 8\,\pi ne^4/3\,m^2c^4$ where σ = total fraction scattered per cm. path of scattering material, and n = number of scattering electrons per cm.^{3}. This was experimentally determined for carbon; then n was computed. But the number of atoms of carbon per gram is $(6.06 \times 10^{23})/12$. It was found that n was 6 times this number; hence every carbon atom had 6 electrons which scattered X-rays. *But 6 is the atomic number of carbon.* Thus the pulse theory sets forth the importance of *electrons* as scattering centers for X-rays, and—astonishing result—the *quantum theory* (the Compton Effect) fits in (so far) completely with this view. Note again that it is the electron outside of the nucleus that does the scattering. Why not the protons and neutrons in the nucleus?

But though the pulse theory was very successful in the points above stated, it failed completely to account for the fluorescent radiation. There the quantum theory and the structure of the atom, to the building of which various lines of reasoning contributed, was sufficient. But the explanation of this phenomenon is so bound up with that of absorption that we pass on to that topic.

Absorption of X-Rays.

In all problems concerning the absorption of light, therefore of X-rays, there is an important principle, viz., that if I_1 is the intensity of homogeneous radiation falling on a plate of absorbing

[1] W. H. and W. L. Bragg.

material of thickness x and I_2 is the amount which passes through, the ratio I_1/I_2 is constant whatever I_1 happens to be. It results from this that $I_2 = I_1e^{-\mu x}$ (Appendix 1-1) where μ is called the coefficient of absorption. This brings out the law of fractional decrease with thickness. It is represented by the exponential curve.

If we now set up a Bragg X-ray spectrometer so as to be able to obtain radiations of homogeneous wave lengths, variable at will, and test the absorption of a thin plate of platinum, we shall

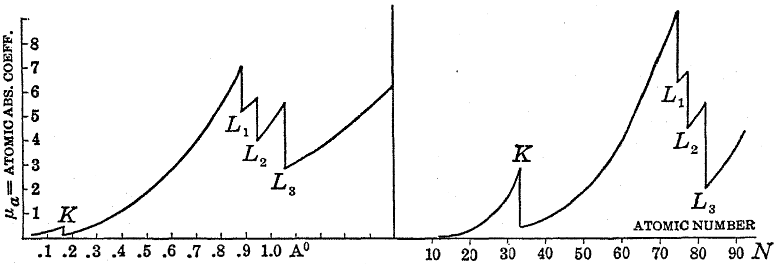


FIG. 7-9. As the wave length of the X-rays increases the absorption increases, then suddenly decreases. The steps correspond to energy levels in the atom.

FIG. 7-10. The greater the atomic number of the atom, the greater the absorption of X-rays of a certain wave length (λ = 1 Å)—except for breaks.

get a curve as in Fig. 7-9. For a long wave length 1.5 Å the absorption is large; it decreases until a wave length of about 1.07 Å is used, then there is a sudden increase, again a decrease until wave length 0.93 Å is reached, again an abrupt increase; just below 0.88 Å there is a continued decrease until wave length 0.16 Å is reached; then a small abrupt rise, then a decrease.

Energy Levels in the Atom and the Quantum Idea Necessary to Explain Absorption.

Why do we have these abrupt changes in absorption? Let us recall the conditions in the original X-ray tube. There, electrons are driven by a difference in potential V against a target. The energy given to an electron is Ve and that equals hf of the radiation. Although both V and e ought to be measured in absolute units to give precise energy units (ergs), we may measure the potential in volts if we wish and we may speak of the electron as

having an energy of so many electron-volts. Similarly for the X-ray generated, it has an energy of so many electron-volts. Now we may measure accurately the wave lengths of the "breaks" in the absorption curve and we may speak of these "breaks" in the terms of electron-volts. We now come to the explanation of these abrupt increases in absorption. Starting on the right of Fig. 7–9 we proceed towards the left—towards short waves, high energies. At the first break the energy in the quantum is sufficient to eject an electron from an L level in the atom. Before this, the energy was not sufficient for this purpose and the absorption was small. From the curve then it is seen that there are 3 L levels, only one K level. In order to eject an electron from the K level in platinum we must have radiation of about 84,000 electron-volts. Now if an electron is ejected from the K ring, one from one of the L rings may go in to take its place and we shall have the emission of a line of the K series. Similarly if one of the L electrons is ejected or removed, an electron from one of the M rings will go in to take its place and we will have one of the L series emitted. Thus it is clear that if a radiation just shorter than the K critical absorption wave length is thrown into an element, the entire X-ray spectrum of that element will be excited; but if the incident radiation is just longer, the K lines will not be excited but the rest of the spectrum will be. So it follows that the frequencies of the X-ray lines are equal to the differences between the frequencies of the absorption limits or breaks in the absorption curves. Also it follows that an element is quite transparent for one of its K lines but quite opaque for a line just shorter than its shortest K.

The data taken from absorption curves show that there is one K or innermost level, 3 L levels, 5 M levels, etc. Moreover, they make clear why a voltage high enough to bring out the K line—the shortest line in the entire spectrum—is necessary in order that the $K\alpha$ line, a much longer line, one of a smaller number of electron-volts, may be radiated.

Let us return now to the scattering of X-rays. In order that there may be scattered characteristic or fluorescent radiation, there must be a high enough voltage in the X-ray tube to produce the quantum necessary for that radiation. This is the radiation which is produced when an electron goes from the L to the K level or from one of the higher to one of the lower levels, as pro-

posed in the preceding paragraphs. Another way of stating the matter is that *fluorescent* or *characteristic radiation* is *emitted when the absorption suddenly increases.*

Another way of demonstrating abrupt changes in absorption is shown by Fig. 7–10. Here a definite wave length 1 Å is used and the absorbing element is changed. Starting with elements of small atomic number the absorption is small; only a little energy is required to eject an electron. As the atomic number is increased the absorption increases until we come to atomic number 32, then there is a sudden decrease. We have come to atoms requiring for the ejection of the K electron a greater energy than that in a 1 Å photon, and as the electron is not ejected the absorption is small. Similarly we interpret the breaks in the curve at L_1, L_2, L_3. It is to be noted that the quantity represented by the ordinate is the atomic absorption coefficient. Hence the density of the material and its atomic weight must be taken into consideration.

Phenomena of Absorption Can Be Explained on the Quantum Idea but Not on the Wave Idea.

Let us further bring out the philosophic importance of this phenomenon connected with fluorescent radiation and absorption. In the original X-ray tube an electron acquired a certain energy, nearly all of which was passed over to an X-ray; this goes out (theoretically) any distance and in passing through matter ejects an electron, this ejection requiring nearly all the energy of the original electron. We might put the matter thus; it is as if we dropped a block of wood into a pond of water, waves spread out in widening circles and at a great distance throw a block of wood on the surface of the water to a height equivalent to that of the original drop. This of course is an impossibility from the point of view of energy spreading out uniformly and continuously. But if the energy could be localized in a bullet which ejected another bullet with (nearly) equal energy, the explanation would be satisfactory.

The phenomenon above is, of course, identical with that of the photoelectric effect. Light of a certain wave length from the feeblest star will cause electrons to emerge from a certain material with the speed that would be given to them by light of the same wave length from the most intense arc lamp.

Electrons Give Rise to X-Rays and Then to Electrons. Bragg's Doublets.

It appears from what has been said that an electron smashing into a target produces an X-ray; this falling on a plate may eject an electron—and the process may continue. Before the discovery of the wave nature of X-rays, Bragg was bold enough to publish a view, playfully put forth probably by various physicists, certainly by the author, that an electron striking an atom picked up something which neutralized it electrically and it turned into an X-ray; that this, encountering an atom, reversed the process and became an electron, etc. The X-ray would then be a neutral particle and as it contained at least an electron it must be a kind of positive and negative doublet. But about the time that Bragg's name had made this idea respectable, Laue and his co-workers found that X-rays were diffracted. The doublet theory of X-rays dropped out of sight. Recently a new particle, the neutron, has been discovered which has some of the properties of the doublet.

The Pulse Theory Drops Out.

Laue's experiment also smashed the pulse theory. For it has been shown in our discussion of the interference of light (Chapter V) that in order to have destructive interference we must have a wave train of many wave lengths. A pulse does not satisfy that requirement. However, it is difficult to see how an electron of about 10^{-12} cm. diameter smashing into an atom can produce a long train of waves, each of wave length say 10^{-8} cm. This would mean a wave train of length of perhaps 10^{-6} cm. or one million times the diameter of the electron. The only possible reason (so we used to think) for this to happen would be that something in the atom was set vibrating and continued to vibrate for the necessary length of time. The student may care to assemble arguments to prove that this can or cannot happen.

The Compton Effect.

There is one other phenomenon connected with the scattering of X-rays that must now be discussed. When a monochromatic beam of X-rays is scattered from material like carbon, the scattered radiation consists partly of rays of the original wave length but chiefly of rays of a longer wave length. The change in wave

length or frequency depends on the angle of scattering but in any case is small. At first A. H. Compton, the discoverer, had some difficulty in persuading physicists that the change was genuine, but his great precision in measurement and the theory which he worked out to account for the change completely won out in the controversy (1924).

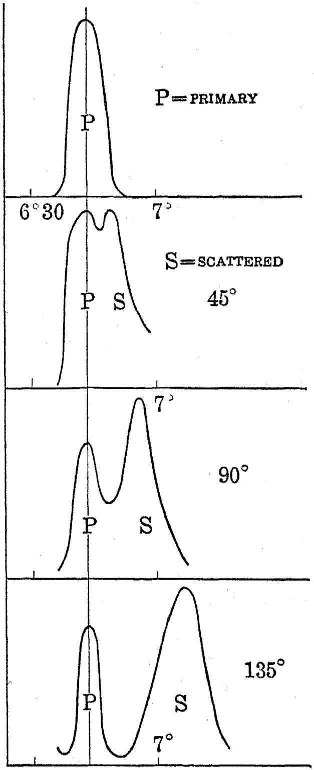


FIG. 7–11. X-rays scattered from carbon show the primary at P, and at S a displaced line, the displacement increasing with the angle of scattering. The Compton effect.

Let us picture a photon of energy hf, having of course the velocity of light, c, striking an electron of mass m at rest. Every photon has energy, it also has momentum equal to hf/c, as can be simply proved as follows. It has been experimentally proved that the pressure of a light-beam is equal to the energy density of the beam. Consider a photon beam 1 cm.^{2} in cross-section, of n photons per cm.^{3}. The energy density is nhf. Let M equal the momentum of a photon. Then the momentum loss per second (assume absorption) is nMc. This is the pressure and it also equals nhf. Hence $M = hf/c$. The problem now is similar to those discussed in Chapter I, only here it is always true that both energy and momentum are conserved. Thus the energy (and therefore the frequency) of the scattered quantum is less than that of the original quantum on account of the energy given to the particle. We might consider the case of a head-on collision, in which case the electron would be driven forward and the photon would be "scattered" straight back. Since the energy of a photon is hf and its momentum hf/c, we have from the conservation of energy

$$hf = hf^1 + \tfrac{1}{2}\, mv^2;$$

from the momentum law

$$\frac{hf}{c} = mv - \frac{hf^1}{c}.$$

In this case (see Appendix 7–2) the wave length of the photon thrown back is less than the original by 0.048 angstrom. In the general case $\lambda^1 - \lambda = 0.024 \ (1 - \cos \phi)$ Å. The intensity and displacement of the scattered X-ray in comparison with the primary are shown in Fig. 7–11. That both intensity and displacement (change of wave length) increase with the angle of scattering is clearly seen. It is to be noted that the *change* of wave length is not dependent upon the wave length of the original photon but upon m, the mass of the particle, and ϕ, the angle of scattering.

An X-Ray Photon Bounces Off a Heavy Atom without Giving (Losing) Much Energy.

The result obtained above for the change in wave length was for the case of a "free" electron, or at least one which could be separated from its atom without the expenditure of much energy. In other words, the energy required to separate it should be small compared with that of the incident photon. It might be supposed then that as the mass of the atom increased this work of separation might become appreciable. Moreover, if the electron were not separated it would return to its original condition in the atom with the result that the incident photon in that case would lose no energy.

All of these considerations are illustrated by the curves of Fig. 7–12. The unmodified line from the silver target is shown at P. With lithium as scattering material, the intensity of the unmodified line is very small; nearly all the energy is in the modified line M. As we go to scattering-atoms of increasing atomic number, the unmodified grows stronger and the modified weaker. If however we were to use a very much shorter wave length, therefore a photon of much greater energy (say a γ radiation from radium), all the scattered radiation would be changed in wave length. If we went in the other direction and used visible light as our primary, there would be no change in wave length. If the process were similar to that of the Compton effect, the photon would be too feeble to eject an electron from an atom (see Appendix and

page 125). Experiments bear out these conclusions. Let us recall the simple experiments discussed in Chapter I, the rifle bullet being fired into the block of wood or a small steel sphere

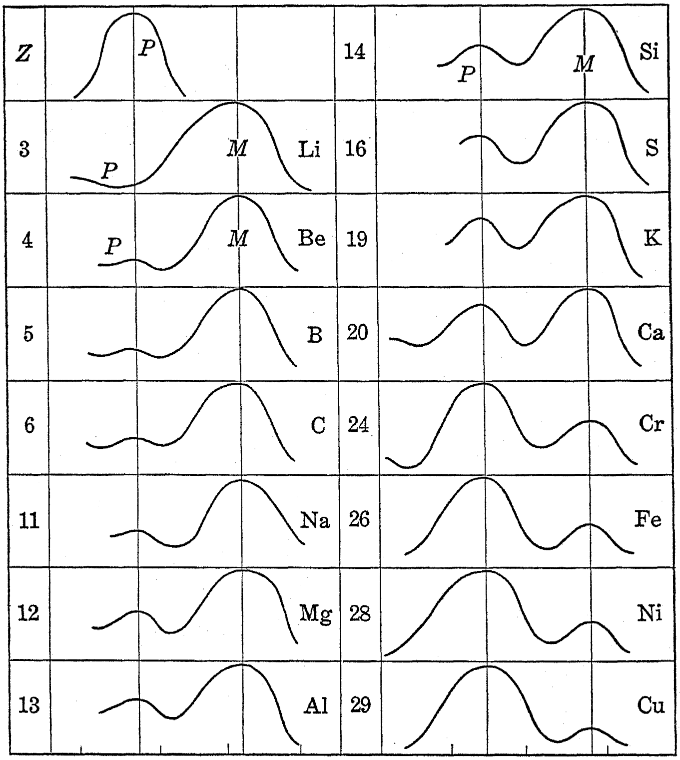


FIG. 7–12. The heavier the atom, the less the intensity of the scattered ray. The photon bounces off heavy atoms without losing much energy.

impinging upon a large steel sphere; in either case the energy given to the large body is small. So the energy which would be given to an atom by a photon (of X-ray range) would be inappreciable. Indeed, the theoretical result we have obtained for the change of wave length contains m in the denominator, so that if it were possible for a photon to communicate motion to an entire atom without tearing it into pieces, the change in wave length would only be 1/1850, or less, of the small change found above. The energy communicated to the atom would be correspondingly

small. In such a case, with energy conserved, practically all the energy remains in the (scattered) photon.

The "Recoil" of an Electron Made Visible.

The experiments enumerated above show therefore that an X-ray "acts upon" a single electron. Indeed a photon, judged by its performance in these experiments, behaves very much as if it were a perfectly elastic ball which could impart energy to another elastic ball, an electron. But can we detect motion in an electron struck by a photon? The electron so struck is called a "recoil" electron. An experiment by Bothe and Geiger shows that when an electron (a recoil electron) is fired out from the path of an X-ray beam, a photon goes out at the same time more or less in the opposite direction. This experiment serves to confirm Compton's explanation of the effect he discovered—that we are dealing with *discrete units* of *radiant energy*, *quanta*, and that these can *bombard single units* of *electricity*, *electrons*. But though it has been suggested that photons might be thought of as elastic balls, their interactions with matter are quite different from those due to "particles." It was the keen appreciation of this difference that enabled Chadwick to make one of the most important discoveries of recent years—the discovery of the neutron. That matter will be discussed in Chapter 12.

Another method of showing the existence of "recoil electrons" is the "cloud track" method devised by C. T. R. Wilson. That method will be presented in Chapters 9, 12, 13, and 14.

Are Photoelectrons Identical with Recoil Electrons?

In discussing photoelectric phenomena we pictured a photon of energy hf entering an atom and imparting all its energy to an electron which escaped with a velocity v given by $\frac{1}{2}mv^2 = hf - w$ where w is the energy lost in getting free of the surface. But we can easily show that if both momentum and energy are conserved and if we confine our attention to the photon and electron, it is impossible for an electron to absorb all the energy of a photon (unless the photon has the energy of a cosmic ray, i.e., of 10^9 electron volts). Neglecting the relativity concept of mass, we have, for a "head on" collision, the energy relation $hf = \frac{1}{2}mv^2$ and the momentum relation $hf/c = mv$. But this would require the electron to acquire a velocity of 2 c *whatever* hf *might be*. This is a

clear indication that the above relations are incorrect. Certainly we have a strong conviction that an electron cannot have a velocity greater than that of light. (For the solution using the relativity correction see Appendix 7–1.) Hence we *state that a free electron cannot absorb all the energy of a photon.* Or a photoelectron, as ordinarily understood, is not a scattered electron.

If a *free* electron cannot absorb all the energy of a photon, how then can a *bound* electron do so? Or is a photoelectron *bound?* In any event, in the case of the photoelectron, the *atom* must be considered. It may be given a small velocity, therefore an appreciable momentum, but a negligible energy. Just how the electron can take to itself practically all of the energy of the photon is not evident. But we can easily show that some process differing from the Compton effect is involved. Let us illustrate this by a computation.

A free electron can obtain the most energy from a photon when the collision is "head on." In this case we can easily show (Appendix 7–2 and Appendix 12–2) that if E is the energy given to the electron by a quantum hf, then

$$\frac{E}{hf} = \frac{1}{1 + mc^2/2\,hf}\,.$$

Suppose that the incident light has a wave length of 5000 Å. Then V in the relation $V = 12{,}345/\lambda = 2.47$ volts. Therefore $hf = 2.47$ electron volts $= 2.5$ electron volts (nearly). Now for an electron

$$mc^2 = 9 \times 10^{-28} \times 9 \times 10^{20} = 81 \times 10^{-8} \text{ ergs}$$

and

$$1 \text{ e.v.} = 1.59 \times 10^{-20} \times 10^8 = 1.6 \times 10^{-12} \text{ ergs.}$$

Hence mc^2 for an electron $= 500{,}000$ e.v.

(As this relation will be wanted many times in later work, it should be marked for reference. Another way of viewing this is that if an electron were changed into energy it would produce a photon of 500,000 e.v. or a wave length of $12{,}345/5 \times 10^5 = 0.0247$ Å.)

It results that the energy given to the electron is

$$E = \frac{1}{1 + 100{,}000} \times 2.5 \text{ e.v.} = 0.000{,}025 \text{ e.v.}$$

This from the bombardment idea. But a wave length of 5000 angstroms may cause a photoelectron to emerge with an energy of possibly 2 e.v. Hence we see that in this region *the recoil process is of a different order from that of a photoelectric process.*

In contrast to the above problem, let us consider what happens to an electron struck by a photon of 1 angstrom or 12,345 electron volts. The computed energy goes up (nearly) with the frequency squared. Or the electron energy now will be 25×10^6 times what it was before, or 625 e.v. This is sufficient to separate an electron from an atom—except for heavy atoms and rather deep layers.

X-Rays Are Polarized.

In the beginning of this chapter it was stated that in the very early days of X-rays, physicists could not find that the rays obeyed any of the laws of light except that of straight line propagation. On the other hand, the pulse theory predicted that they should be polarized, and this was the first of the properties in common with light discovered. Barkla found this in 1904. His apparatus is shown in Fig. 7–13. An X-ray tube is mounted so that it can be rotated about an axis OX passing through the target and through openings of lead screens as shown. The X-rays passing through these openings are scattered at right angles by a paper or carbon block, O, into two ionization chambers E and F. The currents produced in these chambers are read simultaneously and the ratio of the readings found. Then the X-ray tube is rotated about OX so that the cathode stream is at right angles to its first direction and again this ratio found. In the first position of the tube the chamber E has the larger current, in the second position the chamber F. This result is in accord

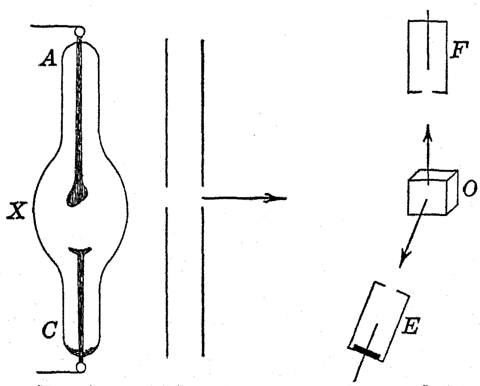


FIG. 7–13. More X-rays are scattered into E than into F from a carbon block O when the tube is as shown. X-rays are polarized.

with the pulse theory. For in the first position of the tube the electrons are decelerated, when they strike the target, along the line OF. Hence the electric vector pulse is along that line. When that pulse reaches O it cannot be scattered along the line of the pulse OF but can along OE. However, experiment shows that there is a scattered portion along OF but a greater fraction along OE. If a second scattering block be placed at E, it will be found that the once-scattered rays are almost completely polarized.

Can this phenomenon of polarization be accounted for on the photon idea? Certainly photons proceeding along the line XO and possessing no structure associated with any plane passing through that line could produce no phenomenon similar to the above. All planes passing through O would have the same properties. A *photon must possess structure.* The sunfish photon earlier pictured (page 80) is still in demand.

Interference.

The phenomenon of interference was the next property possessed by light to be discovered in X-rays, by Laue and Bragg in 1912. This has been discussed at length. But the same phenomenon showed the reflection of *homogeneous* X-rays from planes of atoms—or regularly spaced units lying in a plane. But the question regarding the *intensity* of the rays reflected from various planes is a complicated one and is not here presented. The phenomenon of the total reflection of the rays will be taken up in connection with refraction.

Refraction. Lorentz's Theory.

The topic of the refraction of light is a large one. In Newton's time, it will be remembered, there were the two points of view: Newton's, that corpuscles would be attracted towards the surface of a *dense* medium (compared with air) like glass or water and so be bent towards the normal; Huyghens', that waves would move slower in the dense medium and also be bent towards the normal. After Young and Fresnel seemingly had established the wave theory and after Foucault (1850) had shown that light traveled slower in water than in air, the wave theory prevailed. But why should light waves move slower in water than in air? About the time of the discovery of the electron (1897) and for years after,

various physicists, notably H. A. Lorentz of Leiden, attacked the problem in the following way. Picture a train of waves in the ether passing into a *dense* medium, Fig. 7-14. The ordinate in the wave is, in accord with Maxwell's theory, an electric vector or line of force. It is this electric vector that is propagated. When the wave enters a dense medium, the electric vector operates on electric charges (now electrons) and sets them in motion. But they are restrained by connections with the atoms and by frictional forces. All of these actions are put into the proper differential equation, some care being exercised to keep the equation in a form in which it can be solved. Then, when solved, the equation gives the velocity of the waves in the new medium. The ratio of the velocity in "free ether" to that in the medium is, on the wave theory, the index of refraction μ. Now it is extraordinary that this theory accounts for a vast number of facts connected with light, and by light we mean all waves from the minutest of X-ray wave lengths to those of radio. Let us get a view of this range. The frequency of the waves sent out by an ordinary broadcast station is, as the average small boy knows, say, 1000 kilocycles or one million per second. The frequency of an X-ray of 1 angstrom is $3 \times 10^{10}/10^{-8} = 3 \times 10^{18}$ per second. Hence the ratio of these frequencies is 3×10^{12} or three million million. Yet the formula obtained by Lorentz covers, with appropriate approximations, this range. In our case, for X-rays, the Lorentz formula[1] is $\mu = 1 - ne^2/2\pi mf^2$ where n = number of electrons/cm.^{3}; e, m are charge and mass of electron, f frequency of wave motion. Now when Lorentz developed this formula the *atomic number* idea was unknown, nor was it known that X-rays had frequencies of any sort. But after the picture of the nuclear atom arrived (Rutherford, 1912) with its outer electrons and after X-rays were shown to have wave lengths (Laue, 1912) and there-

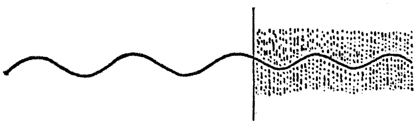


FIG. 7-14. The velocity of light (X-rays) may change when it enters a medium of large electron density.

[1] The more complete relation is

$$\mu - 1 = \Sigma \frac{n_1 e^2}{2\pi m(f_1^2 - f^2)}$$

where Σ indicates the sum of a number of similar terms and n_1 is the number of electrons per cm.^{3} of natural frequency f_1.

fore known frequencies (deduced from the old wave theory presently to be viewed with suspicion), the data were at hand for substituting in the formula.

The Index of Refraction for X-Rays. Total Reflection.

Let us suppose that the dense medium is quartz. (This is nearly the same as, but more definite than, glass.) The formula is SiO_2. The molecular weight is $28 + 32 = 60$. The number of electrons (atomic numbers) per molecule is $14 + 16 = 30$. The density[1] is 2.5. Hence there are 6×10^{23} molecules in 60 gms. or in 24 cm.^{3}. Therefore $n = 6 \times 10^{23} \times 30/24 = 75 \times 10^{22}$. Let us take the wave length of the X-ray as 1 Å or 10^{-8} cm. Then $f = 3 \times 10^{18}$. Since $e = 4.77 \times 10^{-10}$ and $m = 9 \times 10^{-28}$, $\mu = 1 - (3.4 \times 10^{-6})$ or 0.9999966. For a wave length of 1.5 Å $\mu = 1 - (7.5 \times 10^{-6})$. It is seen from this theory that the index of refraction is very slightly less than 1 and that the discrepancy increases with square of the wave length. An index less than 1 means, according to the wave theory, that X-rays travel faster in glass than light in a vacuum. (The index of refraction of light in a metal is less than 1.) It ought to be possible then to observe the phenomenon of total reflection for X-rays. In ordinary light this is observed when light, traveling in glass, strikes the surface at an angle of grazing incidence less than 48°. In X-rays the incidence giving the "critical" angle would have to be nearly grazing, as in Fig. 7–15. Figure 7–16 taken from Compton

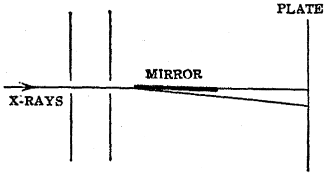


FIG. 7–15. X-rays are totally reflected when the grazing angle of incidence is less than a critical angle.

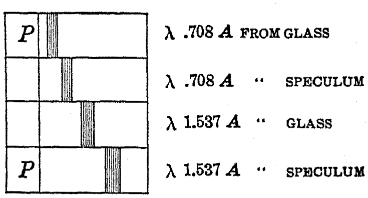


FIG. 7–16. The critical angle depends on the wave length of the rays and the material.

(*X-Rays and Electrons*) shows strong X-rays for wave lengths 0.708 and 1.537 A totally reflected from glass and speculum

[1] The density of crystal quartz is 2.65; of fused quartz, 2.20.

metal. P, P is the original direction of the rays. It is seen that the deflection increases with wave length and with the molecular weight of the reflector. Moreover, the quantitative values of μ thus experimentally determined are of the same order as those computed from Lorentz's theory.

Total reflection is a special case of refraction. The ordinary refraction of X-rays is directly observable in the case when X-rays pass through a glass or crystal prism; they are bent slightly away

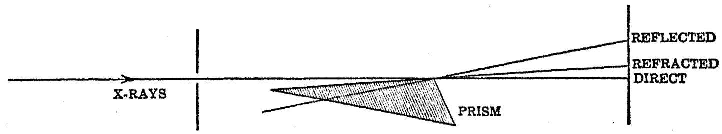


FIG. 7–17. X-rays are also refracted.

from the base. Figure 7–17 shows the positions of the various rays. *So it can at last be said thirty years after the discovery of X-rays that these rays possess all the properties of light!*

In a later chapter there will be a brief discussion of electric waves. There it will be shown that they too, though perhaps a million million times as long as X-rays, possess all the properties of light. And attention will be called to the different ways in which the frequencies of the waves are determined.

Although there will later be a discussion of experimental operations and applications of X-rays, the student may now review their extraordinary history; the theories which have been proposed; the vast domain opened up by the discovery of the wave character; the new sciences in which X-rays are the chief sources of knowledge—and the reaction of all this development on human progress.

Properties of a Photon.

We may now gather together the properties which a photon must possess: frequency $= f$; energy $= hf$; momentum $= hf/c$; moment of momentum about the direction of propagation $= h/2\pi$ (see Appendix 6–4);

$$\text{electric vector} = 4\sqrt{\frac{\pi hf}{c\,(\text{volume})}};$$

$$\text{magnetic vector} = 4\sqrt{\frac{\pi hf}{c\,(\text{volume})}}.$$

What is its length? diameter? volume?

Some Experimental Details.

Bragg's X-ray spectrometer is shown schematically in Fig. 7–18. A crystal C is mounted on the small central table and an ionization chamber I is mounted on an arm which can be rotated about the common axis of the small and large tables. By optical tests, if necessary, the fine openings in the lead screens, L, are lined up radially on the circles. When a definite beam of X-rays is passing through these openings, we move I slowly away from A, the zero position, at the same time rocking the crystal about the vertical axis. Then for some (small) angle ϕ we shall have a maximum ionization current in I. The angle θ in the relation $\lambda = 2\,d \sin\theta$ is then equal to $\phi/2$. For a crystal of a definite material d has a definite set of values. (For rock salt we may have d, $d/\sqrt{2}$, $d/\sqrt{3}$, $2\,d/\sqrt{3}$, where $d = 2.81 \times 10^{-8}$ cm.)

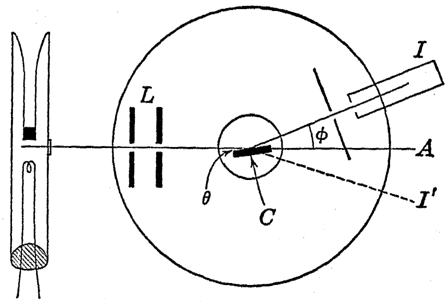


FIG. 7–18. Bragg's spectrometer for measuring wave lengths of X-rays.

When the ionization current is plotted against angle θ we have a figure similar to that of Fig. 7–19. We have certain spectral

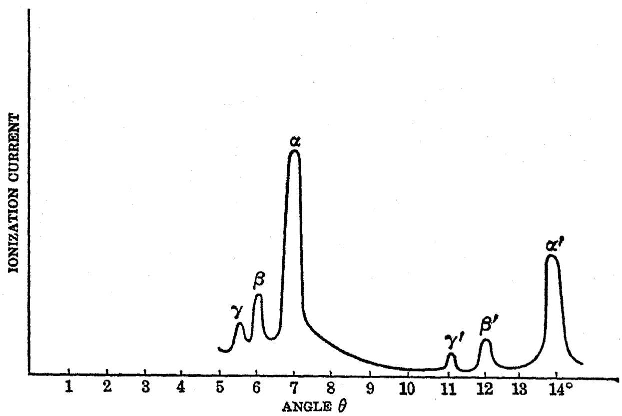


FIG. 7–19. For a certain metal target and with high enough voltage in the tube, the ionization current has definite maxima depending on the angle of incidence with the crystal.

lines in the X-rays α, β, γ. With angles (roughly) twice as great, we get similar lines α', β', γ'. But these are the second orders of the α, β, γ lines, as can be seen from the relation $2\lambda = 2d \sin \theta_2$.

The angle θ is not necessarily the angle between the plane of the crystal (even if it has a polished or smooth plane) and the direction of the incident rays; rather it is the angle between the *planes* of *atoms* and those rays. The crystal may split so as to give one of its surfaces parallel to the atom planes, but it may not. For this reason we must measure the angle θ by measuring ϕ, and we may measure ϕ rather accurately by locating the maxima on the other side of the direct beam. Then one-quarter of the total angle between I^1 and I is equal to θ. But we may have only a number of fragments of crystals. For this reason there has been developed (1916) by Debye and Scherrer (then in Switzerland) and by A. W. Hull, research physicist in the General Electric Company in America, a very important method of obtaining the structure of any crystal which can be obtained in fragments or which can be powdered. It is known as the *powdered crystal* method.

Instead of slits in the lead screens of Fig. 7–18, there are fine holes in line with the X-ray source and the crystal, and instead of a crystal plate there is a very small tube of thin glass, paper, or cellophane, which contains the crystal in powdered form. Or there may be a fine wire of the substance to be analyzed at the

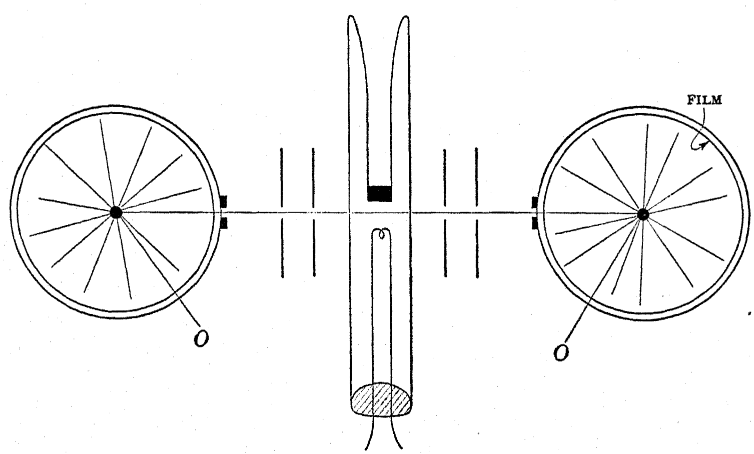
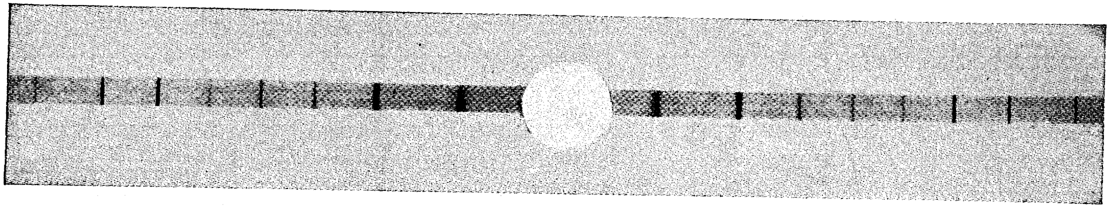
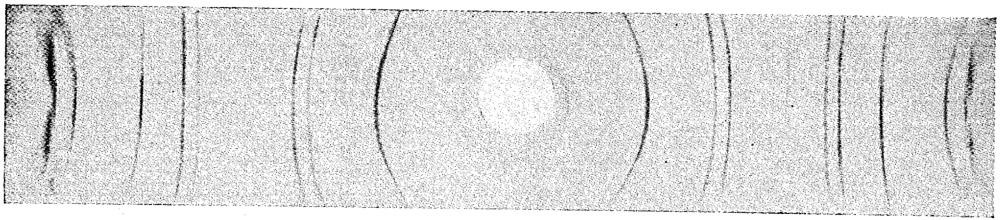


FIG. 7–20. A number of cameras, here only two shown, may be placed around the X-ray tube.



a



b

FIG. 7–21. When the films in the cameras are developed we have (a) the spectrum of an aluminum wire, (b) of tungsten.

center O. Then since the small crystals will present all angles and will be oriented in all directions so that we will have all possible d's between the planes, the emerging maxima directions, for one particular wave length falling on the crystals, will show all the possible d's and therefore the structure of the crystal. The emerging directions for maxima will lie along cones, all having a common axis along the original direction of the X-rays. If a photographic film is mounted perpendicular to this direction, there will be circles of maximum intensity where the cones intersect the film. Generally the photographic film (perhaps an inch high and a foot long) is a cylinder around the powder, Fig. 7–20. A hole is generally cut through the film and so placed that the direct rays will pass through this hole—and therefore the film will not be completely darkened in the vicinity of the direct rays.

Figure 7–20 shows two cameras arranged to receive the X-ray beams emerging through the two windows of a modern X-ray tube. Obviously the cameras, including the tube, must be completely light-tight. Figure 7–21 a shows the spectrum for an aluminum wire 2.5 mm. in diameter. The X-radiation was the copper $K\alpha$ line; exposure time with an excellent tube, 3 minutes. The spectrum for tungsten is shown in b. Here only a narrow strip of film was exposed to the X-rays, but the curvature of the lines near the opening in the film can be detected. In both of these photographs the X-rays have been scattered or reflected back towards the source.

The form of the crystals of aluminum or tungsten and the distances between the atom planes can be determined from these photographs.

In a later chapter (see Fig. 15–3) other X-ray photographs will be discussed.

CHAPTER 8

ELECTRON TUBES

Their Importance in Science and Industry.

The electron tube in its many forms has become a very important article of commerce; it has become a very important device in science. In a comparatively few years the business connected with electron tubes has grown so that in one year the sales in the United States of radio sets alone (of which the electron tube is an absolutely essential part) has amounted to nearly five hundred million dollars. But a radio set is only one of many devices in which an electron tube can be used. There are hundreds of different kinds of tubes, each kind having a special function. And in all the scientific laboratories of the world electron tubes are being put to work in various ways, some of which will be described here.

The present generation of students cannot appreciate the change which has come over human affairs due to the introduction of the electron tube. It is not now a matter of amazement that we can, in our own homes, listen to an important address delivered hundreds or even thousands of miles away: that, under similar conditions, we can enjoy the performance of a great orchestra in a distant city. That ships at sea can communicate with one another and with the main land; that they can be guided through fog into harbor, that airplanes can be similarly guided to a landing field—these results now belong to the commonplace. It will be impossible in this text to deal with these matters in any detail, but it will be the object of this chapter to discuss briefly the performance of electrons as they disport themselves in electron tubes and to give a summary of the functions of some of the different kinds of tubes—high and low voltage rectifiers, detectors, amplifiers, oscillators, modulators, transmitters, grid glow tubes or thyratrons, cathode ray oscillographs.

Apart from the economic importance of the electron tube and apart from the extraordinary rôle it plays as the essential device in the transmission of speech and music from one part of the

world to another, there is still another reason for its consideration in this place. Probably no other man-made device has ever been introduced into science which offers to the amateur as well as to the skilled experimenter such opportunities for experimentation. Given a few coils of wire, a few condensers, electron tubes, batteries, and proper measuring instruments, and observing certain precautions, an experimenter might spend many interesting hours merely recording the different results which could be obtained with different arrangements of the various parts. An amateur might be content to record the results: the trained scientist would know that he has been driving electrons about from one terminal to another, or from one circuit to another, and he would want to know the why and wherefore of these motions. We start with this as our viewpoint—the why of an electron tube.

THERMIONS—EARLY EXPERIMENTS AND THEORY

When an electric current passes through an electrolyte, charged atoms or molecules move from one terminal to the other. But when a current passes through a conductor—a copper wire—the particles of the conductor cannot move; the current must be carried by electrons. When one ampere flows, the number of electrons which must pass across the cross-section of the wire per second is $1/1.59 \times 10^{-19}$ or about 6 million million million.[1] Consequently we must picture a *conductor* as a medium in which there are many electrons free to be moved by any imposed electrical field, however small—the number of electrons being perhaps as few as one electron for every ten atoms or as many as two or three electrons for every atom of the conductor.

When a conductor is heated, energy is given to the atoms of the material—and, to a limited extent, to the electrons. There are restraining forces at the boundary of the medium but some electrons may have energy enough to break through this restraint and emerge into the surrounding space. The older method of considering this phenomenon (that due to O. W. Richardson, 1901–1905) was to think of these electrons as if they were gas particles. Their speeds would be distributed according to

[1] The most sensitive galvanometer made can detect about the one million-millionth of an ampere. Consequently more than six million electrons would have to pass through a galvanometer per second before that instrument would be conscious of the fact, or at least would indicate that anything was happening.

Maxwell's law (Chapter 1). The number of electrons per cubic centimeter moving along any direction with velocities between u and $u + 1$ normal to the surface of the metal would be proportional to e^{-u^2/a^2} or $e^{-W/kT}$ where W is the kinetic energy of the electrons of velocity u. The number which would cross unit area per second would be ue^{-u^2/a^2}, if there were no limiting energy required. Now in discussing the operation of X-ray tubes—and elsewhere—we have shown that if an electron leaves a cathode with zero energy, it would arrive at the anode with an energy $= \frac{1}{2} mu^2 = \phi e$ where ϕ is the voltage or potential difference and e is the electron charge, both in absolute units. Similarly if an electron is moving towards the boundary of a medium it would require an energy $\frac{1}{2} mu^2$ which would have to be greater than ϕe in order to drive it against a potential barrier ϕ. All electrons having velocities greater than $\sqrt{2 \phi e/m}$ should get through this barrier or over this wall and should appear outside the medium as "free" electrons. When this summation is carried out it is found that the number of electrons passing outward through one square centimeter per second is proportional to $T^{1/2}e^{-e\phi/kT}$. If all electrons are accounted for, the current should be given by $i = AT^{1/2}e^{-e\phi/kT}$. The quantity $e\phi$ is known as the work function and represents the energy which an electron must have in order to break through the boundary. The above law was the first one derived by Richardson. Shortly after Richardson derived this law, H. A. Wilson (1905), then he and Richardson, not using the idea of an electron gas, derived a slightly different law, $i = AT^2e^{-e\phi/kT}$. Now it may appear as an astonishing fact that it has been extremely difficult to prove experimentally that one of these laws was right and the other wrong. It has been found that within ordinary ranges of temperature both laws are substantially correct. And the reason is that the exponential factor $e^{-e\phi/kT}$ is the dominant one.

But though Richardson, assuming that Maxwell's law of distribution of energies held for electrons in metals, deduced a law for thermionic emission ($i = AT^{1/2}e^{-e\phi/kT}$) which has been found to be experimentally correct, there is one feature in his proof to which we do not now subscribe. For we now know that electrons in metals do not share in the thermal energy as atoms do, that Maxwell's law does not apply to those electrons. The facts regarding specific heats and conduction of electricity in

metals are not in accord with the view that any appreciable percentage of the electrons in metals have increased energies due to increased temperatures. The Fermi-Dirac law of distribution of energies (1926) rather than Maxwell's law appears to hold. However, using that law (1928) a similar relation for the emission current is obtained, $i = AT^2e^{-e\phi/kT}$. According to the Fermi-Dirac view, only the higher energy electrons experience an increase in energy due to temperature increase. And $e\phi$ is now the increase above the maximum energy at 0° absolute, an increase necessary to overcome the restraining action of the surface.

Now the student may ask this question: If electrons escape from hot wires, why do they not escape from the white hot tungsten filaments of ordinary lamps? The answer is, they do. But they don't escape very far—and to a great extent they go back again. Their escape leaves the wire positively electrified or, perhaps better, the region just outside the filament becomes negatively electrified—therefore the electrons return to the wire.

The Edison Effect. Its Explanation.

But any metal filament which has not been completely outgassed in a high vacuum might be, when hot, a source of positive ions. More surely this would be true of the old-style carbon filament. However, it seems that Edison in 1883, in the so-called Edison effect, was dealing with electron emission from a filament. Let a plate be sealed into a lamp, its position being shown in Fig. 8–1 a. The positive end of the filament would attract electrons, the negative end repel; the plate being positive would attract and intercept electrons driven out from the negative half of the filament. These would flow from the plate to the positive end of the generator through the milliammeter MA. Had MA been connected between the plate and the other end of the filament the current through it would have been (nearly) zero, as Edison found. At that time, however, the electron was unknown and to Edison the phenomenon was obscure. Though he patented the device, he did not use it. Fleming, in England, about the time of the discovery of the electron (1897), saw the possibility of the commercial application of this effect and, after years of experimentation, patented (1905) this device for rectifying high frequency (radio) currents. Thus, as a very special case, came into use the detector tube in all radio sets.

Can the Edison Effect Be Observed in Ordinary Electron Tubes?

Let us return to discuss the phenomenon of Fig. 8–1 a. The large difference in the two currents in this case is due to the favorable position of P between the filament terminals, and the high potential (50 or 100 volts) between them. That high

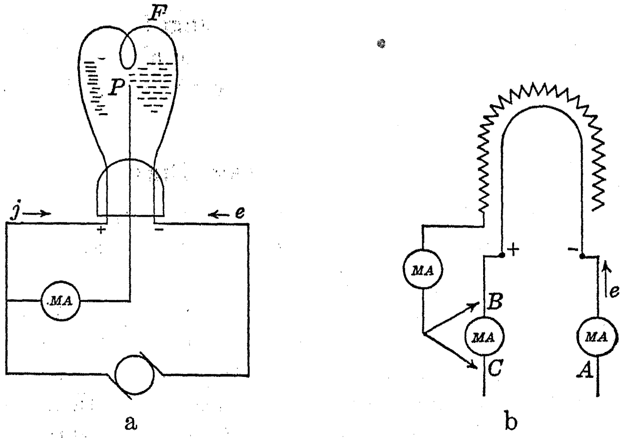


FIG. 8–1. In a, the plate P being positive with respect to the negative end of the filament attracts electrons from that part; in b, the current into the filament may appear to differ from that out of it.

potential would draw the electrons away from the negative end of the filament towards the positive. The plate would intercept them. If one tries this experiment with any ordinary electron tube, he will find that the current between either end of the filament and the plate is nearly zero.

The ordinary ammeter tells us that a certain positive current is flowing from the positive to the negative terminal through the instrument. That is the convention regarding direction of current. But it would be better to follow the *electron* current, in which case we reverse the arrows (but not the current) and indicate that we are dealing with the electron current by attaching an e to the arrows. It is seen then that, of the electrons that enter the filament at $F -$, some escape to the plate and go out of the tube by the MA route. The others continue on through $F +$. So it is clear that there is a larger current in the negative part of the filament than in the positive part. With the plate connected to the filament the difference is small.

But if the grid is substituted for the plate, Fig. 8–1 b, there will be a current (about 2 ma.) when *MA* is between the positive end of the filament and grid; a smaller current for the negative end.

With two milliammeters (*MA*) in the filament circuit as shown, the two read the same if the grid *MA* is not connected. They still read the same but both greater if the grid *MA* is connected to *B*. But if the grid *MA* connection is changed from *B* to *C*, the *BC MA* decreases. The student should be able to account for this performance. See the discussion in connection with Fig. 8–5.

We turn to a brief theoretical discussion.

Modern Theory—The Fermi-Dirac Law

It was O. W. Richardson in the early years of the century who, by his experimental and theoretical contributions, called attention to the important properties of thermions.

We may pause here to note that Richardson's derivation of a relation which was correct according to experimental tests seemed to establish the validity of the view that electrons in metals behaved like gas molecules, at least that their energies were similar. His work was regarded as of outstanding importance. (For this and his other scientific work he received the Nobel prize.) Twenty years passed before a different view was put forth—one which we now see is in accord with numerous facts with which the Maxwell law could not be made to harmonize. The moral is—there may be a number of ways of deriving theoretically a certain relation. That theory is most acceptable which is in accord with the most facts.

The experimental physicist might emphasize the statement above—there may be a number of ways of deriving theoretically a certain relation—and doing so might seem to disparage theory. But the theoretical physicist may take great comfort in the history of the evolution of our ideas regarding electrons in metals and may support Sir Arthur Eddington in his statement that "experimental facts are of little value without a good theory to support them." For, after Richardson derived the law above, he showed, as may easily be done, that if the electrons inside the metal had a velocity distribution which followed Maxwell's law, those outside the metal would follow the same law. He proceeded to find the law of distribution for electrons in "free space." This can be done by placing retarding potentials on the anode and

measuring the negative current, the number of electrons per second, which passes from the cathode to the anode. His measurements showed that the distribution of velocities followed Maxwell's law. Many similar tests have been made during these twenty and more years. Perhaps the most precise have been made by Germer and by Davisson (1925) in the Bell laboratories. Those tests were carried out with all the refinements of modern technique and with the full utilization of the resources of a great laboratory. Germer states the results of his experiment thus: "At 2475° K the assumed Maxwell distribution was verified up to a retarding potential so great that only one electron in 10^{10} emitted electrons was able to reach the collector." [1] This seemed to settle the matter. The question arises then—if the electrons outside of the metal have this velocity distribution, do those inside have the same? If so (Richardson proved the converse), have the electrons inside a Maxwellian distribution? If they do, they must share with atoms the changes in energy due to temperature which are implied in Maxwell's law. They would have zero kinetic energy at absolute zero temperature. But, as has been stated, the facts connected with specific heats and electric conductivity do not support these views. How may we explain this contradiction? Fermi in Rome and Dirac in Cambridge arrived at another law of distribution which seems to satisfy both sets of facts.

It will be noted that in our discussion of the law of distributions of speeds in Chapter 1 it was pointed out that as the temperature of the gas decreased, the curve of Fig. 1–4 shifted towards the left, the number of particles of small speeds increasing and those of large speeds decreasing. Finally it can be seen that at absolute zero—if the law holds to that limit—all speeds, therefore all energies, become zero. The Fermi-Dirac law of distribution, however, has in it a term which depends on the *concentration* of the particles, with the result that the energies at absolute zero are distributed as shown in the parabolic curve Fig. 8–2 a. The number of particles having energy E increases for increase of energy up to a maximum

$$E_0 = \frac{h^2}{2\,m}\left(\frac{3\,n\nu}{8\,\pi}\right)^{3/2}$$

[1] *Phys. Rev.*, Vol. 25, p. 795, 1925.

where h = Planck's constant, m = mass of electrons, n = number of atoms per cm.^{3}, ν = number of free electrons per atom. Then it suddenly drops to zero. The value of E_0 for tungsten (since $n = 6.06 \times 10^{23} \times 19.3/184 = 6.36 \times 10^{22}$, $\nu = 1$) is 5.77 electron volts corresponding to a speed of 1.47×10^8 cm./sec. or 910 miles/sec. This is very far from being zero speed. The

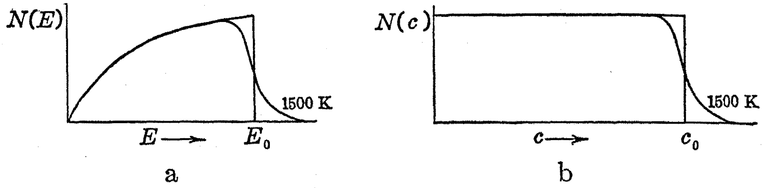


FIG. 8–2. The Fermi-Dirac law. a, the distribution of energies of electrons in metals at absolute zero. The number increases up to a critical energy. b, the distribution according to velocities. The dotted lines are for 1500 K. The electrons of small energy are not disturbed by an increase in temperature.

curves giving the distribution of particles according to their speeds and therefore the curves to be compared with those of Fig. 1–4 are shown in Fig. 8–2 b. Speeds are plotted in terms of c_0 where $\frac{1}{2}mc_0^2 = E_0$. For $T = 0$ the figure is a rectangle. In other words, all speeds are equally probable up to the critical value c_0; then the number drops to zero.

For higher temperatures some particles of the highest energies have their speeds increased as shown by the dotted curves. Thus the electrons of low energy are unchanged by rise of temperature. As it is only the high energy electrons that are emitted, we are concerned only with those electrons. Now the distribution of speeds among these high energy electrons is, so far as it can be tested experimentally, the same as that of the Maxwell law. Hence any experiment which confirms one law for electrons *outside* the metal confirms the other.

Electron Sources in Ordinary Tubes.

We turn now to consider the play of electrons in electron tubes. In a modern electron tube the cathode may be a wire filament directly heated by an electric current as in Fig. 8–3 a, b, c, or it may be a cylinder indirectly heated by a current in a filament inside it, as in d, e. Form c gives a large surface so that large

emission currents can be obtained; forms d and e can be used with alternating current in the filament. In the last two forms, since the cathode or electron source K is not electrically connected

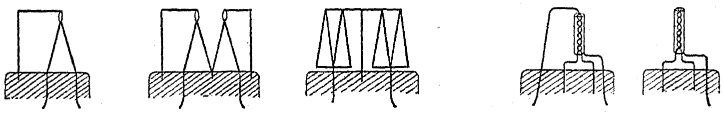


FIG. 8-3. In a, b, and c, heated filaments are electron sources; in d and e cathodes indirectly heated by filaments.

with the a.c. circuit (except to a mid tap or null point) the variations of voltage on the filament do not disturb the tube's performance.

Electron Emission Greatly Dependent upon Temperature—and Surface.

The surface of the cathode, the thermion or electron source (filaments in a, b, c; cylinders in d, e) may be that of a pure element, like platinum or *tungsten*, or it may be *thoriated tungsten* (tungsten with thorium embedded in it), or it may be an *oxide-coated film* formerly on a platinum or tungsten core, now on nickel, or better, on Konel metal.[1] To give an appreciable electron supply the pure platinum or tungsten filament must be heated to a white heat, the thoriated tungsten to a yellow color, and the oxide-coated surfaces to a medium or dull red. To see the great effect of temperature on the electron emission from tungsten, these data may be given. I is current in amperes per square centimeter of filament and T is absolute temperature.

T	I	T	I
1500	0.58×10^{-6}	2400	0.365
1800	214.0×10^{-6}	2700	4.35
2100	0.15×10^{-3}	3000	31.7

Thus by increasing the temperature from 1500 to 3000 absolute, the electron emission has been increased 55,000,000 times.

[1] Konel metal, an alloy of cobalt, nickel, iron, titanium, has recently come into large use as a *core* for filaments. The emission of oxide-coated Konel filament at a dull red heat (750° C.) equals that of oxide-coated platinum at yellow heat (950° C.). This emphasizes the fact that emission depends not only on the atom layer on the surface but also on the atom layers below.

Thorium is said to be "rich in electrons." At least electrons easily leave it. When a thoriated tungsten filament has been properly prepared it will have an outer film of thorium one atom thick. The "work function" above referred to is small and electrons easily escape. However, when too high a potential is put on the plate of the tube, this thorium film may be pulled away from the filament, in which case the filament will lose almost all its power of emission at that temperature. However, it can be restored to its original condition by removing the high plate potential and heating the filament for some time. Thorium in the body of the filament then diffuses out and gives another surface one atom in thickness.

The Extraordinary Influence of a Surface Film upon Emission.

The oxide-coated cathodes usually have a barium oxide surface. The extraordinary influence of the nature and extent of this surface on the emission is seen in the following statement. Let us suppose that we have an electron tube (high vacuum) with a pure platinum filament and are able to deposit on it barium atoms. The electron emission from the filament when it is heated to a bright red color by an electric current is very small; indeed it cannot be measured except by a sensitive galvanometer until about 80 per cent of the surface of the filament has been covered by barium atoms. But as the percentage of the surface covered by barium atoms increases from 80 to 100 per cent, the emission increases at an exceedingly rapid rate until at 100 per cent—that is, when the entire surface is covered with barium one atom thick—*the emission is more than one hundred million times that of the pure platinum surface.*[1] If we continue to deposit barium on the filament so that part of the surface has now a film two atoms thick of barium, the emission drops off until, when it is completely covered with this double layer, the emission becomes equal to that of pure barium. Now the amazing fact is that the particular combination above referred to, viz., a layer of barium one atom thick on a platinum core, gives an electron emission several million times that which would be given by a filament of either pure platinum or of pure barium. In view of this ex-

[1] See *The Rôle of Barium in Vacuum Tubes*, by J. A. Becker, Bell Laboratories Records, Vol. 9, p. 54, 1930.

traordinary fact, what becomes of the precise laws of emission, $I = AT^{1/2}e^{-e\phi/kT}$ or $I = BT^2e^{-e\phi/kT}$ given above? Clearly a layer of barium one atom thick completely upsets our elaborate mathematical analysis which gives the above value of the current. Such a layer must make easy the escape of electrons from the composite surface—platinum-barium. The work function $e\phi$ or the energy which an electron must have in order to escape must be small. And since the dominant factor in I is $e^{-e\phi/kT}$, to make $e\phi$ small becomes as important as to make kT large. Consequently we see that this kind of surface may give a greater emission at a low temperature than a pure platinum surface may give at a high temperature.

In the data given in Table 1 it was shown that when the temperature increased from 1500 to 3000 absolute, the emission of pure tungsten increased 55,000,000 times. Here we see that we could have brought about a similar increase by altering the surface of platinum.

The discussion above has been presented to show the profound influence of surface films in thermionic (and photoelectric) emission. Now, we cannot "see" a layer of atoms of barium on a platinum surface or of thorium on tungsten, and the greatest care must be exercised in the experimental operations to ensure that the surfaces are those of definite materials. It results that great care must be exercised in the use of the oxide-coated filaments. Too great heat and too much voltage on the plate may strip the oxide coating off the filament and completely alter its emission. In the case of the thoriated tungsten a similar stripping may take place but in this case the surface film may be re-formed in the manner already given. (But note below how easily the surface layer of thorium may be stripped off even at ordinary voltages if a very small amount of gas is admitted to the tube.) Even the pure metals, platinum and tungsten, must be completely out-gassed before their behavior becomes definite.

With What Speed Do Thermions Emerge? Some Experimental Tests.

We can easily devise an experiment which will show that electrons emerge from a hot metal with sufficient energy to carry

them some distance away from the hot source. In Fig. 8–4 suppose that we have a two-element high vacuum tube with both plate and filament of pure tungsten and suppose we have means of varying the voltages. With current in the filament large enough to heat it to a white heat, no battery in the plate circuit, and the plate connected to the negative end of the filament, we get a small current through the galvanometer G. Since the other end of the filament is positive, the filament on the whole is positive (perhaps three volts) with respect to the plate and therefore should hold back electrons which have evaporated from the filament. Still some have had enough energy, after escaping from the filament, to go over to the plate, if it is near enough, and then to complete the circuit through the galvanometer to the filament. If the plate P be moved closer to the filament (as can be done by means of the magnet $N - S$) the galvanometer reading increases; if farther away, it decreases. If the filament current is increased, more electrons go to the plate. By putting small negative voltages on the plate we can stop the electrons from reaching it and we can get a measure of the energy of the electrons as they emerge from the filament.

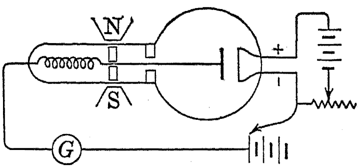


FIG. 8–4. The electron current between filament and plate depends on both the distance and voltage between them, also on the temperature of the filament.

Electron Currents without Batteries.

If we are not concerned with contact difference of potential between filament and plate (or grid), the phenomenon above can be easily shown with any ordinary electron tube. Using the three-element tube 201 A, with the filament heated to normal temperature, grid floating and a microammeter connected between positive filament terminal and plate, the current is about 10^{-6} ampere, connected between negative terminal and plate, nearly zero. But with plate floating and grid substituted for plate, the currents in the two cases are about 1.5 ma. and 0.1 ma. There are twelve different ways in which we can connect these three elements, and the currents vary from about 2.5 ma. to

zero. A triple grid tube (Fig. 8–5) is also very satisfactory for this purpose. In this tube the cathode C is indirectly heated by the current in the filament. Three grids surround the cathode. A milliammeter connected to cathode and G_1 will show perhaps 0.15 milliampere flowing. If G_2 be substituted for G_1, the current will be less than one-third of the G_1 current. For G_3 it will be very small and for the plate too small to be measured in this way. With G_2 connected to G_1 the current is surprisingly large, perhaps 0.25 ma. (There are about 60 different ways of making these connections.) These are amazingly large currents to be passing through a circuit in which there is no battery. Had the cathode been cold, the resistance between cathode and grid would have been of the order of one thousand million ohms (10^9 ohms). Yet when the cathode is hot, the above currents flow even with no battery in the circuit. It ought to be noted that the electrons are driven out of, or escape from, the cathode due to heat, push over to the grid, are picked up there and return to the cathode by way of the wire. Thus the cathode is positive compared with the grid. The similarity between the above phenomenon and that in which a current flows when one junction of a thermoelement is heated must be evident. The relations between these phenomena will be reserved for later discussion.

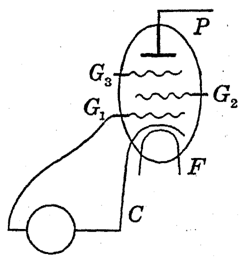


FIG. 8–5. The electrons emerge from the hot cathode with sufficient energy to carry them over to the nearest grids. An appreciable current may be obtained with no battery in the circuit. Tube No. 59.

For precision work it would be necessary to heat the filament in Fig. 8–4 with an intermittent current and to observe the electron current from the filament to P only when the current was not flowing in the former. Or if we are working with oxide-coated surfaces, we could use as cathode a cylinder surrounding the filament as in Fig. 8–5. By either expedient we can obtain a constant potential cathode and avoid the disturbing fact that the two ends of the filament are at different potentials.

Precision experiments lead to the conclusion that, for a tungsten surface at 2400 K, 90 per cent of the emitted electrons have speeds greater than 0.35×10^7 cm. per sec., 50 per cent

greater than 10^7 cm./sec., and 1 per cent greater than 4×10^7 cm./sec. Now referring back to Chapter 1 we see that the root-mean-square speed of hydrogen molecules at 20° C. is 1.9×10^5 cm./sec. and at 2400 K, if they are still molecules, is $1.9 \times 10^5 \times 2400/293 = 5.37 \times 10^5$ cm./sec. Assuming that electrons are like gas particles and remembering that if the temperatures are the same for two gases their molecular speeds are given by the relation $\frac{1}{2} m_1v_1^2 = \frac{1}{2} m_2v_2^2$, it results that the speeds of electrons at the above temperatures are 1.16×10^7 and 3.28×10^7. Thus the computed temperature is of the right order for electrons in "free space." But the discussion given on page 142 shows that Maxwell's law does not hold for electrons in metals and the speed 3.28×10^7 is entirely too small for electrons in a hot metal.

Electron Clouds Check Currents. Space Charge.

Let us now suppose that we have a two-element electron (high vacuum) tube with a pure tungsten filament and that we have means for varying filament current and plate voltage. Given a certain voltage on the plate (say 20 volts), the current from the filament to the plate outside the tube increases as the temperature of the filament increases (due to the filament current), as shown in curve I, Fig. 8–6, up to a certain point, then it bends over to become practically constant. But if the voltage on the plate is 40 or 60 instead of 20, it rises to a higher value before becoming constant. Why do we obtain this limiting constant current for a certain plate voltage? Surely more electrons are ejected from the filament at the higher temperature. Why do not more electrons go to the plate? The explanation is as follows: Electrons emerging from the filament find a cloud of electrons ahead of them. Repulsion by these electrons causes the later electrons to come to rest and perhaps to return into the filament. At any rate, the cloud of electrons prevents all the emitted electrons from advancing towards the plate and so contributing to the electron current. We say that the current is limited by *space charge* and in Appendix 8–1 we show that the current is pro-

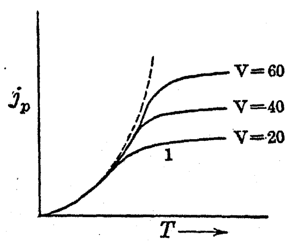


FIG. 8–6. The electron current to the plate increases with the filament temperature and in part with the plate voltage.

portional to $V^{3/2}/x^2$ where V is the voltage and x is the distance between plate and cathode (supposed rectangular and parallel to the plate). The full law for tungsten is $i = 2.33 \times 10^{-6} V^{3/2}/x^2$ amperes per square centimeter of surface of the anode. For a cylindrical anode and a filament along its axis the current per unit length of anode is $1.46 \times 10^{-5} V^{3/2}/r$ where r is the internal radius of the anode.

Saturation Currents.

There is still another way of showing the relation of the plate current and plate voltage. Given a certain current through the filament and therefore a definite filament temperature, the plate current increases with voltage up to a certain point, then becomes constant (Fig. 8–7). It becomes a saturation current. All the electrons emerging from the filament are carried to the plate by the electric field. A further increase in plate voltage within ordinary limits does not increase the current since there are no more electrons to be driven over. But if the filament current (and therefore the filament temperature) is increased, a larger supply of electrons is available and larger plate currents are possible. The dotted curve represents $i = KV^{3/2}$. Curve (b) shows the electron emission from a thoriated tungsten filament under unusual conditions. If a *very small* pressure of argon is in the tube, the emission current suddenly decreases as the plate potential is increased above 30 volts. The explanation is that the argon is ionized at that voltage, the positively charged argon atoms are then driven into the filament with sufficient energy to strip off the atom layer of thorium, and the emission consequently falls.

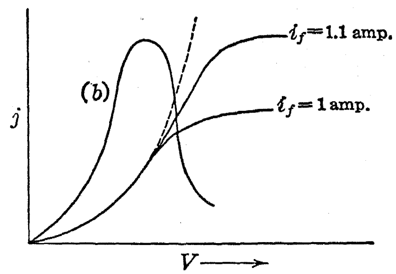


FIG. 8–7. The electron current depends on filament current and plate voltage. In (b) the surface has been altered by the discharge.

Electron Tubes as Rectifiers. High Vacuum.

Since electrons are supplied by the filament and must proceed from the filament to the plate and not in the opposite direction (except when the plate becomes hot under conditions later noted),

an electron tube with a filament and one plate, or a filament and two plates, becomes a rectifier. A few illustrations of such devices will now be considered.

Figure 8–8 represents the "hook-up" for a "280" tube. A 110-volt 60-cycle primary in a transformer produces a rather high voltage (say 700 volts) in one secondary and a low voltage, 5

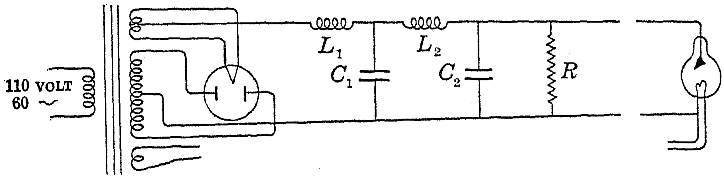


FIG. 8–8. An alternating current is rectified by a "full wave" rectifier and the rectified current is smoothed out by coils and condensers.

volts, in another secondary. The two terminals of the high voltage are connected to the plates of the tube and of the low voltage to the filament. Both secondaries have mid taps which are connected to the "load" R through inductances L_1L_2. These inductances together with condensers C_1C_2 constitute a filter for smoothing out the variations in the rectified current.

The variation of the transformer voltage is shown in Fig. 8–9 a; the current through the tube is shown by the continuous curve in b, and the smoothed-out current c is shown in milliamperes (ma.). The voltage available in the plate circuit is about 350 and the current may be 115 ma. R_p is the internal resistance of the tube from one plate to the filament. But of the 350 volts in the plate circuit only the fraction $R/(R_p + R')$ can be made use of in the output circuit. Here R_p is the resistance of the tube, and R' is the "equivalent" resistance of the rest of the circuit.

If there is no filter in the circuit, the current in the tube and circuit is shown by the dotted curves in b. This would be a pulsating current with a minimum of zero and with a maximum more than twice that of the smoothed-out current.

Rectifiers Containing Gas.

In the high vacuum type of tube the resistance R_p is large—of the order of 2000 ohms. Consequently unless the external resistance is large compared with 2000 ohms, an appreciable fraction of the

voltage is not available for external use. In other words, there is a rather large voltage drop in the tube. In gas-filled tubes (mercury vapor No. 82) this unfavorable characteristic is eliminated, for in those tubes the potential drop is only about 20 volts. When such tubes are used, the resistance in the secondary winding of the power transformer becomes a matter of importance. It should not be too high.

When the resistance R is tapped (and condensers are connected across the taps) we have various voltages, which may be used, for example, in a radio set for the proper plate voltages for the amplifier, detecting and power tubes.

Rectifier tubes may be used, as above indicated, to supply constant DC voltages for amplifier and oscillator tubes; or to provide direct current for charging batteries; to provide rather high voltages (1000–2000 volts) for the oil drop and similar experiments; quite high (200,000 volts) voltages for X-ray tubes. The last may be shown in some detail.

High Voltage Rectifiers.

The modern X-ray tube (the Coolidge tube in America) is a very high vacuum tube with a heated filament supplying electrons. It itself is a rectifier unless the discharge has been great enough to heat the target to a bright red heat. Then the target may be a feeble source of electrons. But in any case, if the X-ray tube is connected directly to the terminals of the secondary of a high potential transformer, the current could not pass through the tube in the right direction except during one-half of every cycle. But if an X-ray tube is inserted in the circuit of Fig. 8–8 in place of R (in this case the L^1s and C^1s are generally unnecessary), current could pass through it as shown in curve b of Fig. 8–9. It is obvious that in such a circuit only one-half of the total voltage of the high tension transformer could be used for the X-ray tube.

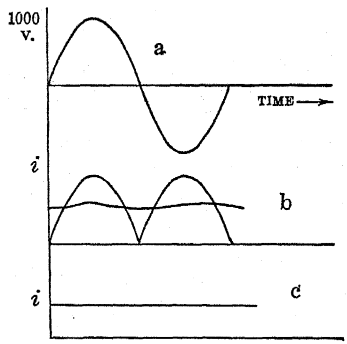


FIG. 8–9. a, voltage in an alternating circuit; b, current rectified; c, smoothed-out current.

Consequently if a voltage as high as 200,000 were wanted in this tube, the secondary of the transformer would be required to deliver more than 400,000 volts. With the arrangement of Fig. 8–10, however, nearly the entire voltage of the transformer, except for small drops in the rectifying tubes, is available for the X-ray tube. During one-half of the cycle the electron current could pass from the filament to the target of the X-ray tube by way of rectifiers 2 and 3 and by way of 4 and 1 during the other half, etc. If the rectifiers 1, 2, 3, 4 are of the modern mercury vapor type, the potential drop in each tube when current is passing is only of the order of 20 volts, whereas each tube if properly chosen could resist a voltage of 20,000 or more in the opposite direction.

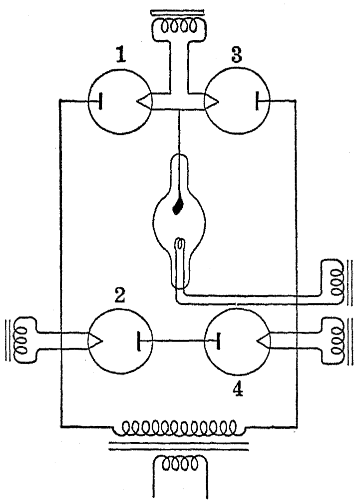


FIG. 8–10. High voltage direct current may be obtained for an X-ray tube. When so operated an X-ray set will not disturb radio reception over a large area, as is the case for mechanical rectification.

It is evident that the secondaries of the transformer supplying the filament current for the various tubes would have to be very well insulated from one another, from the high voltage secondary and from the primary windings.

The Three-Element Tube. A Grid Is Introduced. Amplification Results.

Of the many ways in which two-element tubes (sometimes called *diodes*) may be used as rectifiers, we have given two illustrations. We turn now to another type, a tube containing a grid surrounding the filament and nearer the filament than the plate. Let us recall that unless the voltage of the plate is quite high there is apt to be a cloud of electrons surrounding the filament. Now the grid may be made to modify that cloud very greatly. If charged negatively, it will hold the cloud back, thus decreasing the electron current to the plate. If charged posi-

tively, it may draw off the cloud towards the plate, thus increasing the current. In general in such tubes, one volt applied to the grid may produce as great an increase of plate current as 10, 20, 100 volts applied to the plate. This ratio is called the *amplification factor* of the tube.

The Tube as a Voltage Amplifier or as an Oscillator.

It appears from the statement above that if a "signal," a varying voltage, is applied to the terminals A, B (Fig. 8–11), a varying current will be produced in the plate circuit and consequently there will be an amplified voltage difference at the terminals C, D. This may be utilized in a mechanical device, a relay, or a telephone or loud speaker, or it may be passed on to the grid filament circuit of another tube for greater amplification.

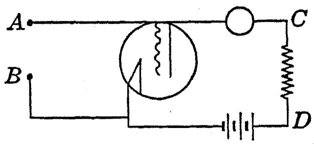


Fig. 8–11. A voltage impressed between A and B gives an amplified voltage between C and D.

Now it frequently happens that the signal to the grid is brought in by the circuit in Fig. 8–12. Here a signal from an antenna comes to a very small, air-core transformer, thence to the grid. By means of the variable condenser C the circuit CL may be tuned[1] to the incoming frequency and thus a relatively high potential may be impressed on the terminals of C and therefore on the grid. This amplified signal is passed on to the plate circuit and again amplified. But the large swings of voltage in the plate circuit will cause the plate itself to vary up and down in potential. This variation will induce

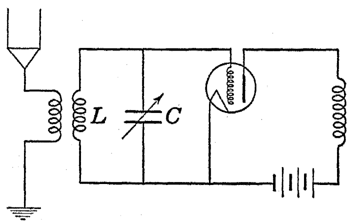


Fig. 8–12. A circuit containing a coil L and a variable condenser C may be tuned to an incoming signal.

[1] The resonance frequency of such a circuit is given by $f = 1/(2\pi\sqrt{LC})$ where f is the frequency in cycles per second (c.p.s.), L is the inductance of the coil in henries, and C is the capacity of the condenser in farads. Thus we "tune" the circuit by varying the capacity of the condenser. For example, the inductance of a coil of 45 turns wound in a single layer on a cylinder of 10 cm. diameter, 5 cm. long, is nearly 0.2 millihenries or 2×10^{-4} henries. The capacity of a small variable condenser such as is used in a radio receiver may be 200 micromicrofarads or 2×10^{-10} farads. The frequency of the circuit formed by these would be nearly 8×10^5 c.p.s. or 800 kilocycles. The wave length would be 375 meters.

variations in the grid and so the grid circuit, in combination with the plate circuit, may be set in continuous oscillation. This is exactly the effect desired when we are using a tube as a source of oscillations in circuits connected to it, but it is highly undesirable in an amplifier circuit such as we are considering. To obviate this result another grid is inserted in the tube; this time it may surround the plate and shield the grid from the influence of the swings of voltage of the plate. So we have the screen grid tube—a four-element tube (a tetrode).

The Screen Grid Tube—the Tetrode.

The manner of construction of a screen grid tube (No. 24) may be seen in Fig. 8–13 a. A cylindrical plate is surrounded inside and out by a cylindrical screen grid. The control grid connection is usually brought out at the top of the tube. The other five connections are in the base pins. The amplification factor of the screen grid tube may be large, two or three hundred, but only a fraction of this ratio, perhaps one-half, can be utilized in practice. Figure 8–13 b shows one circuit for a screen grid tube. A signal is coming to the tube from the transformer T. It passes voltage to the control grid which can be "biased" to any desired average negative voltage by the variable contact D [1] (a potentiometer device). A large variation in the current in the plate circuit results, but the voltage difference between the terminals EF is not equal to fe where f is the

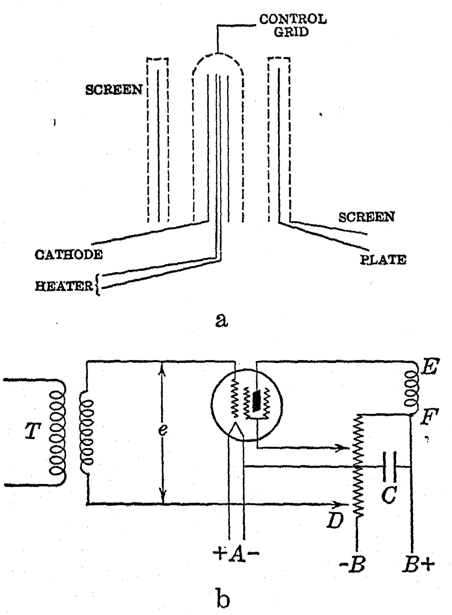


FIG. 8–13. a, the cross-section of one form of screen grid tube; b, this tube as used in a radio set for controlling volume.

[1] "Volume" in a radio set may thus be controlled.

amplification factor but only a fraction of this quantity, depending on the resistance (or reactance) of EF and the other resistances in the complex filament-plate circuit. However, if the resultant voltage amplification is of the order of 100, three such tubes in series would give an amplification of one million. But unless the initial voltage variations were very small, an amplification of one million would probably be unnecessary and undesirable. This conclusion may be justified by a study of the curves in Fig. 8–14, for a three-element or single grid tube. Curve 40 shows the plate current (vertical distance) when there are 40 volts on the plate and the grid voltage is changed from − 1 to + 2 volts. For the other curves the plate voltages are 60 and 80. It is seen from the figure that we get the same plate current for the two cases—grid voltage 0 plate 40, and grid voltage − 1 plate 60. In other words, one volt negative on the grid offsets 20 volts positive on the plate; the amplification factor is 20.

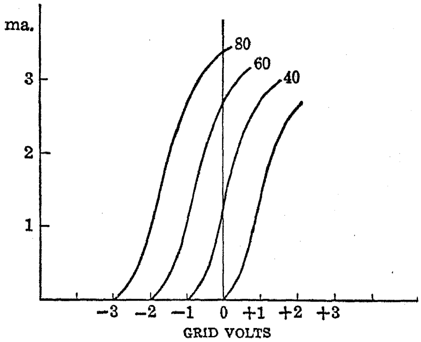


FIG. 8–14. One volt negative on the grid offsets twenty positive on the plate. The amplification factor is 20.

Again if the plate voltage is 80 and the grid voltage swings from − 2 to − 1, the current increases nearly uniformly with variation of grid voltage. We are using the straight line part of this curve. If the grid voltage swing were greater than two volts, the plate current would be limited and the output signal would not be a faithful reproduction of the input. The performance of the tube would be characterized by distortion or a lack of fidelity. Thus there are limits to desirable amplification unless the greatest care is taken to ensure a linear relation between input and output.

The characteristic curves above are the simplest that we know—those of the filament, grid, plate, tube. When we add another grid either as a screen grid or of any other nature, we increase considerably the number of ways in which the various elements may be used. As connected in Fig. 8–13 b, the screen grid tube will not only render improbable the setting up of

electrical oscillations in parts of the circuit but it will also operate to control automatically the volume of a signal. This results from the fact that an increased current in the plate circuit will cause a voltage drop in the screen grid, and this in part checks the rise of current in the plate circuit. However, this characteristic of automatic or super-control can be produced in other ways. One way is to change the structure of the control grid. In Fig. 8–13 a the control grid is uniformly spaced. If the spacing is coarse in the middle and fine at the ends, we have a new characteristic which results in a decrease in amplification for large volume. Thus super or automatic control.

Electric Oscillations May Be Produced by a Tube.

In the discussion of Fig. 8–12 it was stated that enough energy might be passed back from the plate to the grid to cause continuous oscillations in the circuits. If the capacity between grid and plate is not large enough to bring this about, a small variable condenser connected between grid and plate could be made to produce this result. But frequently the circuits are arranged in another way. Figure 8–15 shows a tuned plate circuit, the frequency of which is determined largely by the inductance and capacity in that circuit. (Frequency nearly equals $1/(2\pi\sqrt{LC})$.) Inductively coupled to L is a coil in the grid circuit. If there are oscillations in LC, these induced voltages are passed to the grid which under proper conditions will cause variations in voltages in the plate circuit which sustain those oscillations. So we have continuous oscillations, the necessary energy coming from the batteries. This is only one of very many methods of producing electric oscillations.

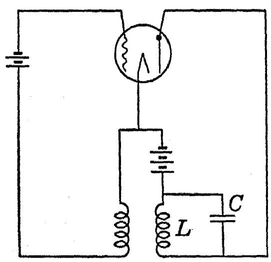


FIG. 8–15. Continuous oscillations are maintained in the circuit LC by induced impulses passed on to the grid.

The Oscillation of Electrons in the Tube. Barkhausen-Kurtz.

But there is one mode which deserves our special attention since it actually shows the *oscillation of electrons*. Suppose the

grid is made highly positive (Fig. 8–16) and the plate slightly positive, or even negative, compared with the filament. Then we can imagine an electron starting from the region of the filament speeding up towards the grid, passing through between the grid wires towards the plate, then being pulled back again—and back and forth with decreasing amplitude until it is pulled into the grid. We would expect to have then high frequency electron oscillations. And that is what we have. They are called Barkhausen-Kurtz oscillations. The higher the voltage of the grid, the higher the frequency of oscillation, or the shorter the wave length. It is left as an exercise for the student to find the velocity of an electron as it passes the grid, assuming that it leaves the filament with zero velocity; then knowing the velocity one may find the frequency of oscillation, hence the wave length. (What would happen if a magnet were held above the tube?) The experimental result is that $\lambda = 1000\, d/\sqrt{E}$ cm. (Appendix 8–2), where d = diameter of plate and E = voltage of grid. Electric waves of less than one meter length can be obtained in this way.

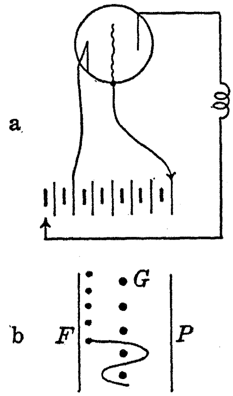


FIG. 8–16. a, with a high positive voltage on the grid and small negative on the plate very rapid oscillations of *electrons* to and fro past the grid are obtained as shown in b.

Push Pull Oscillators—Lecher Wires for Measuring Wave Lengths.

But the simplest and most definite way of producing very short waves (a few meters in length) is the "push pull" method of Fig. 8–17. Two similar tubes, 10's or 45's or 50's, are connected as shown. The grid circuit is tuned by sliding a cross-bar on the fine copper rods connected to the grids. Given a definite position of a similar bar on the rods connected to the plate, the milliammeter in the plate circuit will show a maximum response. Then it is known that the circuits are oscillating. The wave length or frequency can be measured in a variety of ways. One very important application of these very high frequencies, apart from radio and probably television, is in diathermacy and in the production of artificial fever. A person places his arm between

two metallic plates connected to a high frequency source. A sense of warmth will be experienced all through the arm, although the surface may be cool. Arthritis may be cured in this way.

The arrangement of Fig. 8–17 allows the measurement of the wave length to be made rather easily. A long bare copper wire

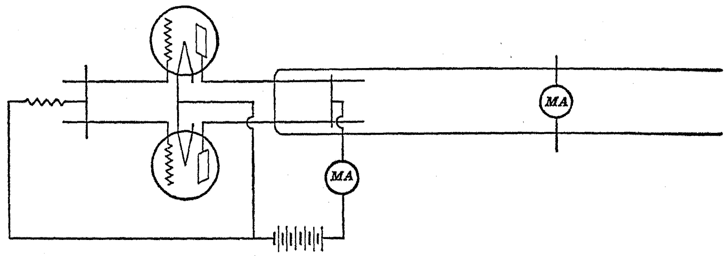


FIG. 8–17. The push-pull method of producing high frequencies, short waves (from 1 to 5 meters). The wave lengths can be measured on the "Lecher wires."

bent to form three sides of a long rectangle (known as Lecher wires) is placed above the plate leads. A sensitive thermo milliammeter is connected across these wires as shown, and its position is varied along the rectangle. At a certain point there will be a maximum current in the *MA*; farther out another and still another. The distance between two points of maximum response is half the wave length.

Many Different "Hook-Ups." Many Applications.

We cannot leave this part of the discussion of the "radio" tube without calling attention to the extraordinary number of ways in which a single tube can be connected up. For example, leaving out all batteries (except those for the heater if it is not heated by a transformer), there are 60 different ways in which an ammeter may be placed between the cathode and the other elements (grids and plate) of the tube of Fig. 8–5. When batteries are introduced and a few different voltages are allowed for every element, the number of ways becomes very great. Some of these ways are only of academic interest, but there are still many ways, in any one of which the tube would perform a useful function. The firm manufacturing the tube ordinarily supplies information concerning two or three arrangements in which the elements may be used.

The Beginning of Broadcasting.

The tube of three elements, filament, grid, plate, made its way rather slowly into the realms of science and industry. With its beginning in 1907, it had progressed sufficiently to become an important device in limited fields during the war. But in 1915 the engineers of the American Telephone Company making use of the great resources of that company and of the combined technical skill of many departments succeeded in transmitting speech from Arlington, Virginia, to the Eiffel Tower in Paris and at the same time to the Hawaiian Islands. That extremely expensive but successful experiment may be said to have inaugurated new human habits. For, soon after the war, stations transmitting speech or music sprang up and a considerable percentage of the people of the United States would spend many hours of the night trying to hear faint, uncertain voices emanating from California, Texas, or Cuba. During these years the three-element tube in America was still a necessary device of a radio set. But in the late twenties the screen grid tube came upon the scene, radio sets were made "all electric," [1] loud speakers were greatly improved, and the broadcasting activity became a very important part of national and international life.

Gas-Filled Tubes.

It is not possible here to go into detail regarding the characteristics and uses of the multi-electrode tubes, pentodes, triode-pentodes, duplex-diode pentodes, pentagrid converters, rectifier doublers, super-control radio-frequency amplifiers, duplex-diode triodes, triple grid detector amplifiers, and so on. There are many classes and there are many variations within classes. However, there is one general class to which brief reference has been made and which will now be discussed. This class is the gas-filled tube with either hot or cold cathodes.

In all the high vacuum tubes already discussed, the current is carried by electrons. When a small amount of inert gas—neon, argon, or helium—is admitted to the tube (perhaps two or three millimeters of mercury pressure), the current, if any flows, will be carried by electrons in one direction, positive ions in the other.

[1] An "all electric" set is one in which the filaments of all the tubes are heated directly or indirectly by an alternating current; and, by means of a rectifier tube, direct constant voltage is supplied to the plates and grids of all the tubes.

The performance of the tube has been very greatly changed as compared with the high vacuum tube. For now there may be only a small voltage drop in the tube (10 to 20 volts in the hot cathode type, perhaps a hundred volts in the cold cathode type), which does not change appreciably with considerable current change. There will be luminosity in the tube when the current flows; in the case where mercury vapor is the "inert gas" the color will be greenish due to the strong green mercury line; in the case where neon or helium is used, there will be a rosy color. The simplest form of tube is the two-element tube of plate and filament. If filled with argon vapor to a pressure of about 5 cm., the heavy tungsten filament can be heated to a high temperature without causing great evaporation of the filament. A dense electron emission is produced; but when the voltage on the plate (preferably a block of carbon, Fig. 8–18) is high enough, the argon is ionized and a large current may flow, of course in only one direction. This arrangement is limited to moderate voltages. Indeed, it generally is used for charging 6 to 12 volt batteries. In the circuit shown, current can flow through the battery only in alternate half-cycles. It is therefore quite low in efficiency, about 20 per cent.

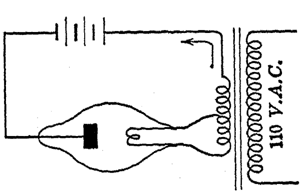


FIG. 8–18. A tungar rectifier; an electron tube containing argon used for charging batteries. Electrons are supplied by a hot filament.

Mercury Arc Rectifiers.

The mercury vapor arc is also used as a low voltage rectifier, Fig. 8–19. Here a pool of mercury, C, is the cathode and since this pool is not intentionally heated it may be called a cold cathode tube. Two carbon blocks, A_1A_2, serve as anodes and another carbon block, not shown, very close to the mercury pool, is used for starting the arc. After the current has been started the mercury surface is the source of electrons and mercury vapor furnishes the ions. Again a very large current can flow as compared with the current in ordinary high vacuum tubes. When the current is flowing, an intensely luminous spot can be seen dancing over the mercury surface. The current seems to be concentrated in this spot. It is estimated that the current

density right at the spot is a *few thousand amperes per square centimeter* and that the temperature of the vapor just at the surface is 2000° C. Yet we call it a cold cathode! The mercury itself only rises about 50 degrees but the temperature of the anode is about one hundred degrees higher than that of the mercury. The potential drop in the tube is from 18 to 25 volts. In the circuit shown in Fig. 8–19, if A_1 is positive at a certain instant, A_2 is negative. The electrons from C would flow from C to A_1 and if it were possible electrons would flow from A_2 to C. This would destroy the tube as a rectifier. But the "strike back" voltage in this form of tube may vary from a few hundred to over a thousand volts, depending on the current, the temperature, and the tube geometry.

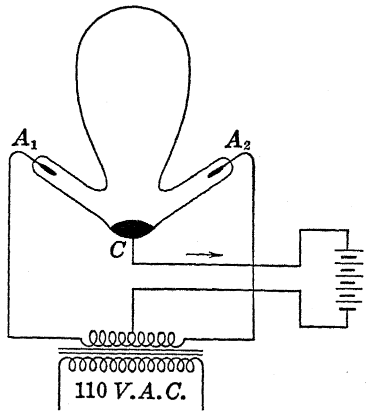


FIG. 8–19. A mercury arc rectifier. Electrons are supplied by the mercury vapor.

Ignitrons.

Certain precautions must be taken: (1) in the manufacture, that all atmospheric and occluded gases be pumped out; and (2) in the operation, that a pair of auxiliary electrodes placed very near the mercury pool shall be so electrically connected (separately excited) as to ensure a secondary supply of electrons. This secondary supply acts somewhat like a pilot light to ensure that the main discharge does not fail. This result may now be obtained in a different way. Dipping into a pool of mercury, Fig. 8–20, is a refractory rod R. When it is made positive and a

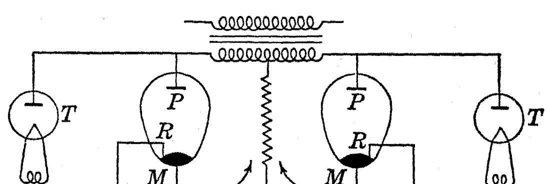


FIG. 8–20. A special form of mercury arc rectifier, the ignitron.

medium current is passed by T through it, an arc starts between it and the mercury surface. If the plate P is sufficiently positive at that time, the arc is almost instantly transferred to that electrode. In the figure the inert gas-filled rectifier will carry a current sufficient for the formation of the arc between P and M. But then, since the voltage between P and M is only 15 volts, the current in T ceases. The arrangement shown is a rectifier for very high currents—a thousand amperes for short intervals.

The trade name for these bulbs manufactured by the Westinghouse Company is ignitron.

A Rectifier for High Voltages.

If we want a rectifier for high voltages, we must use a tube pumped out to the highest possible vacuum. A coiled tungsten filament towards one end of the tube provides the electrons. The tube is made quite long (or if short must be placed in oil) so that the high inverse voltage will not jump the air gap between the electrodes (Fig. 8–21).

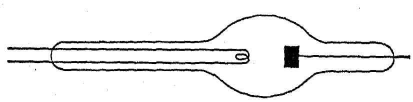


FIG. 8–21. A rectifier for a 200,000 volt circuit.

Thyratrons—Outstanding in Importance.

But the gas-filled tube which has come to have outstanding importance is the kind that contains an inert gas or mercury vapor, a hot cathode, a plate, and a grid or two or more grids. To understand the operation of this kind of tube, let us recall some facts about the high vacuum type. In the latter tube there is, all around the hot filament or cathode, a cloud of electrons. This cloud constitutes the space charge and it operates to limit the current to the plate. The grid in this tube can, if negative, intensify the restraining effect of the space charge or, if positive, it can hurry the electrons along towards the plate. But in the "thyratron"[1] after the filament has been heated sufficiently to ensure a supply of electrons and the plate voltage is then applied, no current will flow if the grid is sufficiently negative or sufficiently low if positive. But if the voltage of the grid, becoming less negative or more positive, rises above a certain critical point,

[1] For brevity the trade name used by the General Electric Company is adopted.

the current suddenly acquires a large value and the grid then loses all control over the plate current. A safety resistance must be included in the plate circuit or the current may reach a value which would destroy the tube. How do we account for this loss of control by the grid? In the high vacuum tube the grid is in a region of "pure electrons." But in the thyratron, when electrons can move fast enough, gas atoms are ionized—then there is no longer a cloud of pure electrons. Near the grid, positive ions nearly equal the electrons in number and, whatever action the grid would have on the electrons, it would have the opposite effect on the ions.[1] So it has no control. In order that the grid may gain control again, the plate voltage must be cut off long enough to allow the ions and electrons to recombine. This time is of the order of ten microseconds or 10^{-5} seconds. Then if the potential of the grid is sufficiently negative, no current will flow.

The behavior of one of these thyratrons may be illustrated by the curves of Fig. 8–22. After the filament has been heated for

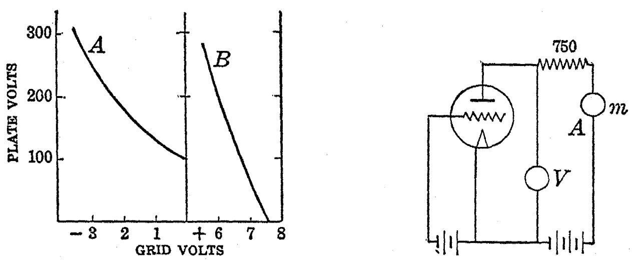


FIG. 8–22. The thyratron, an electron tube containing a grid and filled with gas or vapor, has a great number of important applications.

the time specified by the manufacturer, the grid is made negative by 1 volt, the plate positive by 10, 20, · · · 90 volts. Up to this time no current has been indicated by the milliammeter in the plate circuit. The reading of the electrostatic voltmeter V equals the voltage of the B battery. But a further increase to 95 volts causes a breakdown in the tube. The tube becomes luminous, a current of 0–1 ampere flows, and the voltmeter reading drops to 20 volts. If now the voltage on the grid be made more negative, there is no change in the readings of A and V. The grid has lost

[1] Another way of explaining this lack of control is to picture the grid with a positive sheath rather near and all round it. The field of the grid would then extend only to this positive sheath. The electrons and ions in the rest of the tube would move in their respective directions unhindered by the pressure of the grid.

control. If the plate circuit be broken and the potential on the grid be made − 2 volts, then the plate voltage would have to be increased to 195 before a breakdown occurs. Again V shows about 20 volts. After the breakdown occurs, the plate potential may be reduced to about 20 volts before the current suddenly ceases. The curves A and B show the *starting* conditions of two different kinds of thyratrons. In the tube to which A applies, for all points below and to the left of A, the current will not start; for all points above and to the right the current will not stop as one changes the plate and grid voltages down and towards the left.

The tube to which B applies requires a positive grid as well as a positive plate to bring about a breakdown. While tube A requires very small power for its operation, tube B requires a very small time for deionization. Tube B can therefore be used to record rapidly occurring signals.

If the tubes contain mercury vapor, the critical lines A and B will change with the temperature of the tube. As the temperature increases the lines move towards the left. An increase of 50° C. would cause the lines to shift about two grid volts.

If alternating voltages be applied to both plate and grid and in opposite phase as in Fig. 8–23 a, the negative voltage on the grid AGB may be adjusted to such a value as everywhere just to

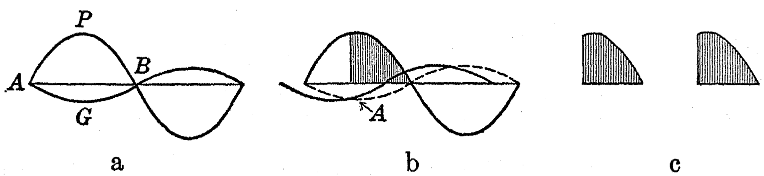


FIG. 8–23. In a, the grid voltage AGB is just sufficiently negative to offset the plate voltage APB so that no current starts. In b, the phase of the grid voltage has been changed; current flows for the shaded portion of the cycle. In c, the current flows in successive cycles. The distance or time between the shaded portions should be a wave length or period, twice the distance AB.

prevail over the positive voltage APB on the plate so that no current flows. During the next half-cycle, though the grid is positive, the plate is negative, consequently no current flows. But suppose the phase of the grid voltage is shifted as in Fig. 8–23 b; then during alternate half-cycles when the grid voltage is not sufficiently negative to offset the plate voltage, current will

flow and will continue for the remainder of that half-cycle. Thus at the point A (Fig. 8–23 b) the grid voltage curve cuts the critical voltage curve and on the right of those points to the end of the positive plate cycle current will flow. It ought to be noted that it is not necessary for the grid voltage line to cut the zero voltage line, that is, to become positive, in order that current may flow. It is only necessary that it become less negative.

Thus it is possible without breaking high voltage and high power circuits to reduce to zero or to regulate the power in those circuits by adjusting a low voltage, very low power, grid circuit. Moreover the power may jump to a maximum in a few microseconds or be broken as quickly, a result which could not possibly be brought about by mechanical devices.

It is obvious too that thyratrons may be used as rectifiers for large currents. They may be used in the inverse process—in changing *direct into alternating current.* Used in this way they are called "inverters." Now that generators of very high voltage direct current are possible, it seems altogether probable that power may be transmitted as high voltage D C, then by inverters changed to A C, and by transformers to usable voltages. On page 168 is shown (Fig. 8–28) an application of a very sensitive yet powerful screen grid thyratron.

The Cathode Ray Oscillograph.

One very important application of electron motion is found in the cathode ray oscillograph. Electrons are produced by a heated oxide-coated cathode in a high vacuum tube (Fig. 8–24).

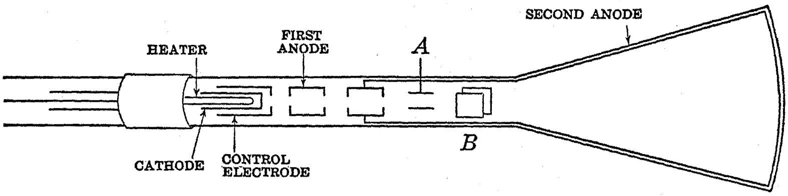


Fig. 8–24. The cathode ray oscillograph. A stream of electrons may be deflected up and down by the A plates, perhaps a million times per second; horizontally back and forth by the B plates with a chosen frequency.

A first anode at about 800 volts accelerates the electrons. A second anode, consisting of a small cylinder (with holes in its ends) and the entire inner surface of the flared end and fluorescent

screen, all conducting, at a high voltage—4000—gives further acceleration. Focusing of the beam to a fine spot is accomplished by adjustment of the voltages on the anodes and control electrode. One voltage under test is applied to one pair of deflecting plates. To another pair at right angles to the first is applied a voltage of known frequency or a voltage produced by a "sweep circuit." Suppose the voltage applied to the A pair has a frequency of one million per second and that the light spot is moved vertically with simple harmonic motion. Then if to the B pair is applied, at the same time, a voltage of the same frequency, an ellipse, a circle, or a straight line is produced. If the B frequency is twice or one-half that of A, the well-known 2 to 1 Lissajous figure is produced. But in general it is best to give to the B pair a voltage which increases uniformly with time up to a certain maximum—then very suddenly becomes zero to start again on its uniform increase. Such a voltage given to the B pair will cause the spot to move horizontally with uniform speed a certain distance, then to jump very quickly back to its zero position. If the time for this change from zero back to zero be synchronized with the period of the A voltage, there will be a constantly recurring figure on the fluorescent screen on the end of the tube. Hence the wave form of the A voltage is rendered visible and can be photographed.

A Sweep Circuit.

One way of producing a sweep circuit is by a combination of diode and thyratron tubes (Fig. 8–25). A constant current

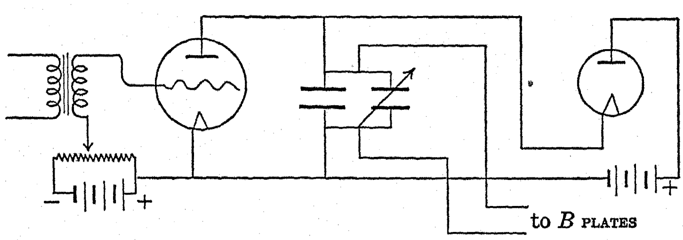


FIG. 8–25. A sweep circuit, one to make the electrons move across the screen with uniform speed, then suddenly to jerk back to the starting point.

through the diode charges up a condenser which is connected to a thyratron as shown. When the voltage of the condenser reaches a certain value, which may be determined by the voltage on the grid

of the thyratron, this tube breaks down and the condenser is almost instantly discharged. As there is (practically) no voltage then on the plate of the thyratron, it then ceases to conduct and the condenser starts charging again. Thus there is a periodic rise and fall in the voltage on the condenser, the rate of rise uniform, the fall very rapid. This is for a steady negative voltage on the grid. But if a signal voltage is also imposed on the grid (Fig. 8–25) the sweep circuit may be synchronized with the signal voltage. The filament current in the diode (Fig. 8–26), the voltage of the plate battery, the capacity of the condenser, and the bias of the thyratron grid—all of these control the period of the sweep circuit. Its frequency can vary from a few per second up to about a million per second.

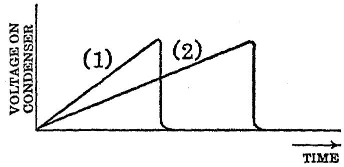


FIG. 8–26. By increasing the filament current in the sweep circuit tube, the time of sweep may be decreased.

An Amplifier for a Photoelectric Cell.

As is very well known, electron tubes are extensively used to amplify minute photoelectric effects. With the photoelectric cell *Ph* (Fig. 8–27) in darkness, the voltage E_g is adjusted so that practically no current flows through the relay R_y. Then when light falls on the cell, a current passes through it and through the resistance R (10^7 to 10^8). That will make the grid partially positive compared with the filament. Current then flows through T and R_y. The relay then opens or closes a circuit to give the desired signal or operates a counting mechanism. The circuit can, of course, be arranged to give a signal when light is cut off from the cell. Thus there are a vast number of applications of the photoelectric cell in the scientific laboratory and in industry.

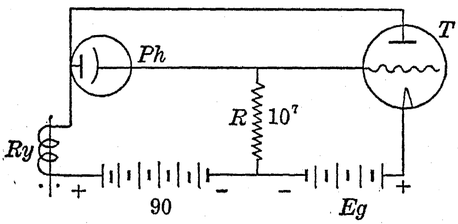


FIG. 8–27. A photoelectric cell connected to an amplifier tube may produce a signal of any desired intensity when a light beam enters or is shut out from the cell.

An Electrical " Detective."

If a person passes through a light beam which is falling on a photoelectric cell, his cutting off the light may be made to operate a counting mechanism or to give an alarm. But with no photo cell, no light beam, a circuit can be arranged to give an alarm when a person approaches a concealed apparatus. T is a screen grid thyratron (Fig. 8–28), the screen surrounds the control

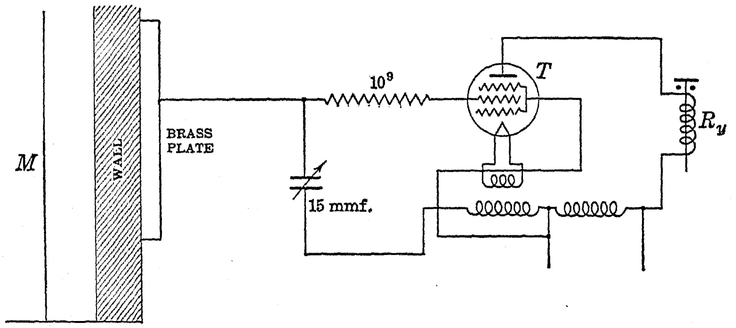


FIG. 8–28. The circuit of a screened grid thyratron may be so adjusted as to release a signal when a man in an adjoining room approaches the separating wall.

grid, thus shielding it from disturbances other than those which we desire to measure or to use. Alternating voltages are applied to the plate and control grid but so balanced that no current flows through the tube. A rather large metal plate (perhaps 2 feet square) attached to but well insulated from a wall and a very small variable condenser are connected to the control grid. When a person M preferably in another room approaches the wall opposite to the metal plate, the capacity connected to the grid is thereby changed and if the variable condenser has been properly adjusted the balance between grid and plate voltages is disturbed sufficiently to allow current to pass. The relay will then operate. A signal of any desired power can then be released.

A Millionfold Extension of Current Measurement

One illustration of the use of an electron tube in extending the bounds of human knowledge may here be given. It has been stated (page 136) that the very best galvanometer will detect a current not smaller than 10^{-12} ampere. The sensitivity of our best commercial galvanometers is about 10^{-10} ampere. An

electrometer can be used under rather special conditions to detect a current of 10^{-15} ampere. But an electron tube has been used to detect one of 10^{-18} ampere—or 6 electrons per second. Very special precautions must be used in the manufacture and use of such a tube. The control grid must be quartz-insulated; the very highest vacuum must be obtained; low voltages must be used; a space charge grid must be inserted next the filament to turn back any positive ions given out by the filament. Then with connections as given in Fig. 8–29 the current amplification may be such that a good commercial galvanometer G (sensitivity 10^{-10} amp./mm./meter) will give a deflection of 1 mm. when 10^{-17} ampere flows through the resistance R. Here the space charge grid is $+ 4$ volts, the plate $+ 6$, the control grid $- 4$ compared with the negative end of the filament. With the space charge grid 4 volts positive, no positive ions sent out by the filament can reach the control grid. This helps to isolate and insulate the control grid. The current to be measured must pass through the very high resistance ($R = 10^{12}$ ohms). Since the circuit containing the control grid and filament is shunted across this resistance, that circuit must have a high resistance compared with R. Hence the necessity of effectively insulating the control grid and of using a small plate voltage, since such a voltage would not be sufficient to ionize the residual gas in the tube.

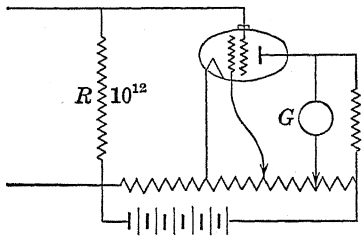


FIG. 8–29. An electron tube with a quartz-insulated grid may amplify feeble currents a million fold.

The scientific value of this millionfold extension of our limits of current measurement is seen in the fact that, using this tube and a sensitive photo cell, we are able to detect stars of the fourteenth magnitude and to get a measure of their radiant energy. This is about the one fifteen-hundredth of the minimum luminous energy which can be detected by the eye.

But for certain purposes the Geiger counter is far more sensitive than the tube above. It is described in Chapter 13 and Appendix 13.

Some applications of electron tubes in industry are given in Chapter 17.

CHAPTER 9

ELECTRICAL PHENOMENA IN GASES AND SOLIDS

Foreword.

We think of a gas as the simplest form of matter. In Chapter 1 it was shown that we could compute the mean speed of gas molecules, their distribution with regard to speeds, their average energy, their mean free path, and we could deduce or justify various laws based on the classical kinetic theory. It might be thought that electrical phenomena in gases would also be comparatively simple, that we could gather together all the phenomena under our old kinetic theory. But now we are to learn that electrical phenomena are concerned with electrons, atoms, molecules, ions. Even an atom now becomes complex. For though there may be, let us suppose, only one kind of helium atom, it may exist in hundreds of different forms, of different energy conditions, and perhaps all of these forms or conditions enter as variables in electrical phenomena. And helium is nearly our simplest atom.

In this chapter, after dealing with some most elementary demonstrative experiments, we shall discuss collisions—of electrons and photons with atoms; of excited atoms with other atoms; of exciting collisions; ionizing collisions; pilfering or leveling collisions: we call attention to the evidences of ionization left by an electron, an ion, an atom, a photon as it passes through moist vapor. It has been thought a considerable accomplishment to follow an electrified oil drop as we move it back and forth in an electrical field. But an oil drop is a vast, an astronomical, body in comparison with an electron, yet in Wilson's cloud tracks we can follow a single electron, an atom, an ion, an alpha particle by its path of destruction through gas particles. Moreover, we can identify every one of those particles and all of them as compared with a photon in its passage through a gas.

Continuing, we examine, as a spectacle, a gas column through which an electric current is passing, then present the modern experimental methods of analysis and, very incompletely, the

modern theory. Here it is seen that electrons may behave like gas molecules, that they follow Maxwell's laws of distribution among themselves, but they are also superior to such laws. We find electron temperatures of 20,000 or 200,000 degrees in an ordinary glass tube which may be at room temperature. While modern methods have solved many problems, there are still many perplexities.

Then turning our attention to a solid, we discuss the extraordinary property of electrical resistance, how it changes from one element to another, how it changes with temperature and with magnetic field. This brings us to discuss low temperatures.

An Elementary Picture of an Electric Current.

The oil drop experiment may be used to give us a very simple illustration of an electric current. We can *see* the drops as they move from one plate to the other, we know that they are electrically charged since we can change the direction of their motion by reversing the voltage on the plates, we can determine the charge on a drop and theoretically, at least, on all the drops; hence we can say that one plate has gained n_1 positive, and that the other plate has gained n_2 negative, electron units per second; therefore that the current is $(n_1 + n_2)e$ where e is the electron charge. We shall here leave out of consideration questions as to the uniformity of this "current," or as to whether one plate loses as much as the other gains. If we had placed smoke particles between the plates, there would have been many particles, as we saw when observing the Brownian motion, too many to count, and their motions would have been rapid. But aside from the difficulty of making numerical estimates of numbers and speeds we can, in this case, "visualize" a large current, large as compared to that due to the oil drop.

Smoke, Clouds Dispersed by an Electric Field.

An important application of the principle of this experiment (the Cottrell Smoke Eradicator) can be illustrated as follows: A glass tube (Fig. 9–1) about an inch in diameter and 10 or 20 inches long has a fine wire gauze cylinder or just a few copper wires in contact with its inner surface, and there is a fine insulated copper wire along the axis of the cylinder. Smoke of any kind is introduced into the cylinder and a high potential is applied between

the center wire and the wire gauze. The smoke is drawn to the electrodes very quickly—the air becomes free of smoke almost

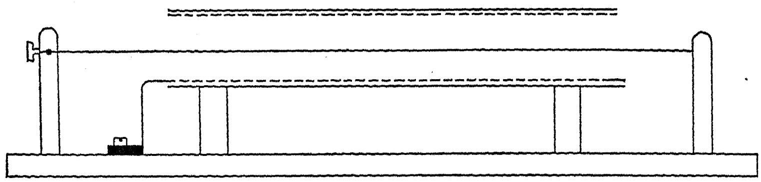


FIG. 9–1. Smoke in a tube is almost instantly dispersed when a high voltage is applied between the central wire and the gauze cylinder.

instantly. So the black smoke ordinarily issuing from the chimneys of large furnaces can be almost completely freed of its "blackness" by electric means. But it is probably better by proper combustion in the furnace to prevent the formation of black smoke. To indicate the condition of the smoke, a photoelectric cell is mounted on the outer wall of a chimney and a light beam from a light source of definite energy is sent across the rising smoke column. When the smoke becomes somewhat black, the photo cell gives a signal or an alarm which calls attention to the condition of the furnace or automatically corrects that condition.

Gases in and around a Flame Are Ionized.

Let us now consider two insulated metal plates, vertical, and facing one another, a centimeter or so apart, and connect these to the two poles of an electric machine. A gas flame, with part of it luminous, is placed between the plates and the machine is "turned up" or energized so as to charge the plates. The flame is pulled sideways chiefly towards one of the plates. It may at times be nearly broken into two parts, one part pointing towards one plate, the other to the other plate. Clearly this is the oil drop or smoke particle experiment on a large scale. The flame is *ionized;* [1] the molecules of the gases or vapors are electrified, some positively, some negatively.

Ions Recombine.

We can show that the gases around a flame are ionized. In Fig. 9–2 any one of the three insulated brass plates in the center

[1] An ion is an atom or a molecule that has lost or gained at least one electron as compared with its neutral condition.

of a vertical brass tube may be connected to an electroscope. The lower one is connected and charged. A flame is brought to the lower end of the tube. The electroscope is quickly discharged. If it had been charged positively the plate would have drawn to itself negatively charged ions, hence its charge would have been neutralized. Had the electroscope been connected to Plate 2 it still would have been discharged but more slowly; if to Plate 3, still more slowly. This shows that there are fewer ions passing the upper plate than the lower. In other words, it shows that the ions have recombined.

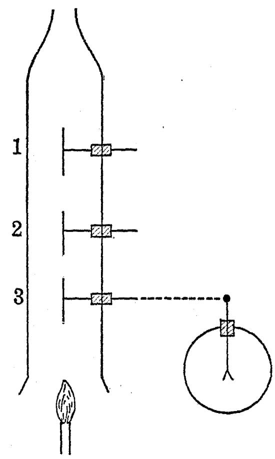


FIG. 9–2. The gases around a flame cause the charged disc, 1, to lose its charge rapidly; discs 2 and 3 slowly. Ionization and recombination.

X-Rays Produce Ionization.

A variation of this experiment is shown in Fig. 9–3. Here, instead of a flame as an ionizing agent, we use X-rays. The horizontal part of the metal tube is closed with a large rubber bulb. A damper or photographic shutter when closed partially prevents the air in the horizontal tube from diffusing into the vertical part. Lead screens shield all the apparatus, except the horizontal tube, from the X-rays. The X-ray tube is energized. There will be a slow leak of the electroscope. But if the "damper" or photographic shutter is opened and the bulb compressed, the electroscope quickly discharges. This shows that the air through which the X-rays have passed has been ionized. If we turn off the X-rays and wait a few seconds before forcing the air out of the horizontal tube, we can show that the air has lost some of its ionization.

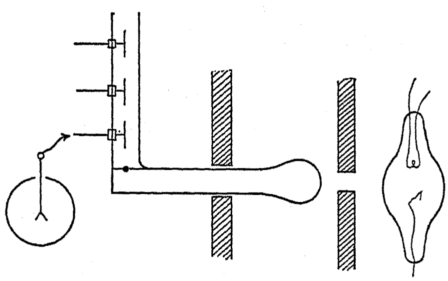


FIG. 9–3. A gas can be ionized by X-rays. Recombination can also be shown.

Ionization Due to Collisions of Molecules, Electrons, Alpha Particles, Photons.

We have given illustrations of the ionizing of gases by means of two different agencies, thermal agitation and X-rays. In the former we have molecules bombarding one another with increasing intensity as the temperature is raised; in the latter we have bombardment by high energy photons. We might then list various ionizing agents as follows: (1) collisions due to the bombardment of molecules by other molecules or by electrons or alpha particles; (2) bombardment of molecules by X-rays or light. By means of one of these agencies an electron is detached from a neutral atom or molecule, leaving a positive ion. The electron momentarily free may move under the action of an electric field towards the positive electrode. It would move fast compared with the ion. For the same potential difference the kinetic energy of the electron $\frac{1}{2}\,mv^2$ would be the same as that of the ion $\frac{1}{2}\,MV^2$; an electron would move 60 times as fast as a hydrogen ion since the mass of the hydrogen molecule is nearly 3700 times that of the electron. Thus in high electrical fields there may be cumulative ionization, an electron released from one atom may acquire sufficient energy in a short distance to ionize another atom. Thus the density of ionized particles increases in geometric proportion and the current may become very large.

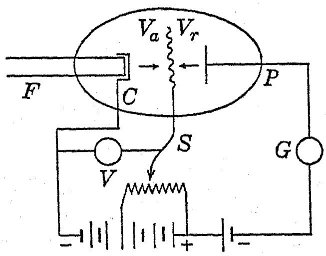


FIG. 9–4. Electrons, speeded up by the positive potential of the grid, bounce off mercury atoms without giving up energy—unless the potential is greater than 4.86 volts, then they lose all their energy. Elastic and inelastic collisions.

"Elastic" Collisions Do Not Produce Ionization; "Resonance" Collisions; Franck and Hertz Experiment.

But there is another phenomenon connected with the collisions of electrons with molecules which is of great importance. It is illustrated by the illuminating experiment which we owe to Franck and Hertz. In Fig. 9–4, C is a cathode or electron source indirectly heated by a filament F. The tube containing a few drops of mercury is heated to about 200° C. so

as to ensure a large density of mercury vapor. An accelerating potential V_a is applied between S and C, a small retarding potential V_r between S and P. The electrons leaving C are speeded up towards S. If they do not lose energy by impacts, they will have enough energy to carry them against the small retarding potential V_r and will arrive at the plate to continue their journey through the galvanometer G. As V_a increases from 2 or 3 volts up to about 5 or 6, the galvanometer current increases as shown in Fig. 9–5; then it decreases and again increases. The first peak is somewhat uncertain owing to contact potential difference between C and S. But the differences between the voltages for the successive peaks when averaged is 4.86. What is the explanation of this rise and fall? For small electron energies we regard the collisions with the molecules as perfectly elastic. The electrons lose no energy; they are able to go to the plate against the small retarding voltage. But as the grid potential rises the electron energy increases; some electrons have sufficient energy to cause an increase in the (internal) energy of a *molecule struck* by the electron; the latter loses energy and cannot go on to the plate. Hence the drop in the current in G. But as the grid potential is further increased above 5 volts, these *inelastic* collisions take place nearer the filament; the electrons having lost energy by such collisions are again accelerated towards the grid, again continue to the plate, and again the current in G rises. When the grid voltage approaches 9.7 volts (corrected for contact potential), the second maximum current in G is obtained. Then it begins to fall as the grid voltage is still further increased.

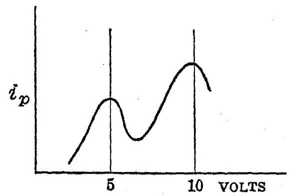


FIG. 9–5. Currents to the plate show that electrons in their collisions with mercury atoms are "tax exempt" unless their energy is greater than 4.86 volts. See Fig. 9–4.

This experiment therefore shows the difference between elastic and inelastic collisions and also illustrates the meaning of *excitation* or *radiation* potentials. (The term "resonance potential" has in the past been applied to this quantity, 4.86 volts for mercury, but as there is no resonance connected with the phenomenon, the terms above are preferable. The term excitation is the better since radiation does not always take place.)

Ionization Potentials.

If the potential V_r is applied between the plate and cathode, the arrangement serves to detect ionization potentials. Since the plate is now negative compared with the filament, electrons will not go to the plate,[1] but if the potential of S gives to the electrons leaving C an energy great enough to ionize some molecules between C and S, the positive ions may continue to the plate and give a current in G opposite to that previously obtained. For mercury vapor this ionizing potential is 10.39 volts.

It has been generally thought that all of the above points may be demonstrated by the use of commercial tubes, the G. E. thyratrons F. G. 17, the Westinghouse grid glow tube K. U. 628, the Western Electric 256 A. But in the author's laboratory it has been found that there is an effect due to temperature in the case at least of the F. G. 17 which completely blanks the resonance potential effect. It would be better to use a multigrid radio tube (the 59 of Fig. 8–5) into which some mercury or inert gas had been introduced.

Explanation on the Bohr Theory.

The old Bohr theory offers us an *explanation*, if primitive, of the above phenomena. We picture an atom with its "optical" electron in its prescribed "normal" orbit, the orbit of minimum angular momentum and of minimum energy. An electron collides with the atom, that is, comes close enough to be deflected by the electric field round the atom, but if it has not enough energy to cause an electron of the atom to go out to the next Bohr orbit, no energy is absorbed and the electron bounces off with its energy unchanged. It is an elastic collision. But if the electron has the necessary energy, the atom receives that energy, an electron goes to the next higher energy orbit, and the colliding electron loses its energy. The collision is inelastic. Later when the atom returns to its normal condition (in one step, but see later discussion), the energy previously absorbed is radiated as a quantum hf. If the energy absorbed is sufficient to remove an electron from its atom, we have the phenomenon of ionization.

It is rather evident that there may be a number of excitation or

[1] If the temperature of the cathode is high, photoelectrons may *leave* the plate; this would be equivalent to positive ions reaching the plate. This current, ordinarily very small, must be allowed for.

radiation potentials for any kind of atom. And too there is the possibility that an atom which has been thrown into an excited state by one collision may, before it has time to return to its normal condition, receive another energy contribution from another electron: this would carry it to a higher energy state or it might produce ionization. It seems to be an experimental fact that for a fluorescing vapor, iodine for example, the ionizing potential is only 6.8 volts, while that for the normal vapor is 9.4 volts. This points to the possibility of *ionization* by *cumulative effects.*

An Electron Does Not Set an Atom in Motion.

There has been left out of consideration in the above discussion the possibility that an electron might give motional energy to the entire atom and therefore lose an equivalent amount of energy. But we can see that such energy changes would be very small. For suppose a particle of mass m_0 with velocity v_0 strikes a mass m at rest "head on" (this would be the case of maximum energy change) and that the velocities after impact are v_1 and v; then, since momentum and energy are both conserved, $m_0v_0 = mv - m_0v_1$ (the electron would be thrown back) and $\frac{1}{2} m_0v_0^2 = \frac{1}{2} mv^2 + \frac{1}{2} m_0v_1$ or $E_0 = E + E_1$ where E_0 is the original energy of m_0 and E is the energy of m after impact. It is easy to show that

$$y = \frac{E}{E_0} = \frac{4\, m_0m}{(m_0 + m)^2} = \frac{4\, x}{(1 + x)^2},$$

where $x = m_0/m$ or m/m_0. For the case of an electron striking a hydrogen atom

$$\frac{m_0}{m} = \frac{1}{1850} \qquad \text{and} \qquad y = \frac{1}{460}.$$

In other words, the energy which an electron could give to a hydrogen atom would only be about one-fifth of one per cent of the electron's original energy. For the case of an electron striking a mercury atom, the loss would be only one part in 90,000. Thus it is clear that the phenomenon we have observed cannot be explained on the basis of an electron giving kinetic energy to an atom; the phenomenon is, that an electron loses no appreciable energy in passing through mercury atoms until its energy is 4.86 volts—then it loses all.

An Atom in the Normal State May Absorb an Appropriate Photon.

But if a normal mercury atom has been put into an excited state—using the Bohr picture, if an electron has been driven to an orbit higher in energy than the normal orbit—it will, on returning to its normal state, radiate a quantum of energy hf equal to the energy change. The wave length which would correspond to 4.86 electron volts is given by the relation (see page 114)

$$\lambda = \frac{12{,}336}{V} = \frac{12{,}336}{4.86} = 2537 \text{ Å}.$$

This is the very strong mercury line far out in the ultraviolet. Its spectroscope designation is $1^1 S_0 - 2^3 P_1$, indicating that the energy change has been from the $2^3 P_1$ condition to the normal $1^1 S_0$ condition.

A very important experiment demonstrates the point just discussed. Let us suppose that we have a photoelectric cell in a quartz envelope and containing a surface especially sensitive to the ultraviolet. The cell has been connected to an electron tube as indicated in Fig. 8–27 and light from a mercury arc lamp is thrown into the cell. A photoelectric current will be produced. Then let an open vessel containing mercury be placed below the light beam. The current will be quite appreciably decreased. There is sufficient mercury vapor above the vessel to absorb the 2537 line. (The vapor pressure of mercury at 20° C. is only 0.0012 mm.) It will not absorb appreciably any line longer than 2537. The mercury atoms are in their normal state. When the appropriate quantum reaches them, they absorb it and are raised to the first excited state.

This is the method used to detect the presence of mercury vapor (it is poisonous) where mercury is much used in laboratories or in a room in which the new mercury vapor turbine is operating.

Again it requires 10.39 E volts to ionize a mercury atom.[1] If there were in the mercury spectrum a line corresponding to this energy, its wave length would be (12,336/10.39) Å or 1188 Å and its wave number would be the reciprocal or 84,220 cm.$^{-1}$ This is the wave number of the head of the principal series as determined by spectroscopy.

[1] Due to a single collision; but this is rare.

The Importance of the Quantum Idea.

The nuclear picture of the atom was given to us by Rutherford in 1912, the Bohr theory in 1912–1913, Moseley's experiment in 1913. The quantum theory as applied to spectroscopy had captured the attention of all physicists. The Franck and Hertz experiment in 1914 showed that phenomena connected with the collisions of electrons with atoms could be explained on the basis of the quantum theory, that atoms could have different energy conditions, and that these energy states were apparently in accord with spectroscopic data. After the war an extensive investigation of *critical potentials* of a great number of elements showed, in so far as those measurements, relatively crude compared with those in spectroscopy, could show, that there was an extraordinary correlation between those two sets of data.

The quantum argument which we have used to account for critical potentials of electrons bombarding atoms must also apply to photons, as the exciting or ionizing agency. Light will not ionize a gas unless the energy of the photons hf is as great as the kinetic energy of the electrons which would produce the same result. Or we must have the relations $hf = \frac{1}{2} mv^2 = Ve$, as equivalents for the two agencies. Generally the photon energy required for the ionization of any ordinary gas is large; in other words, the wave length belongs in the far ultraviolet. Air is ionized by 1300 Å; mercury vapor by 1188 Å; normal iodine vapor by 1308 Å. These lines are far out in the ultraviolet. But there are a vast number of subsidiary data and qualifying conditions connected with the matter of excitation and ionization potentials.

In the pioneer Franck and Hertz experiment there was apparently only one excitation potential. But in experiments of greater refinement, many other excitation potentials were found. All of them fitted into the mercury spectrum energy levels.

The ionization potential for mercury as found by electron bombardment is given as 10.39 volts. But when mercury vapor is irradiated by the line 2536, the vapor is ionized; and 2536 corresponds to 4.86 volts. Now it has been stated that a photon of frequency f and therefore of energy hf has the excitation power of a particle of energy $\frac{1}{2} mv^2$ and this again is equivalent to that of an electron corpuscle of energy Ve. But there is a difference.

When a photon is absorbed it vanishes,[1] it ceases to exist as a photon. When an electron strikes an atom, it may part with practically all its energy—but it is still an electron. In the kind of activity we are discussing, a photon must give up all its energy, an electron need not do so. (Why must a photon give up all its energy?) So let us assume that 2536 photons have produced a number of excited atoms. These excited atoms may give energy to other atoms, as the following experiment shows.

" Pilfering " Collisions.

In Fig. 9–6, Q is a quartz tube with connecting tubes, 1 containing some mercury, and 2 containing thallium. It has been

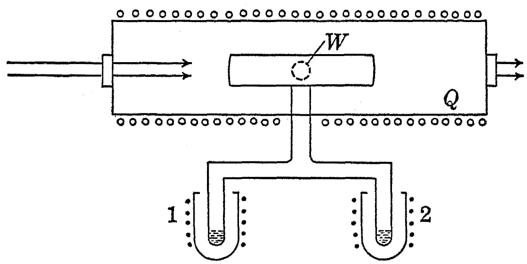


FIG. 9–6. Thallium atoms become luminous sources due to energy which they have pilfered from mercury atoms, energy which would have made the latter luminous.

exhausted, sealed off, and placed in a heating furnace as shown. Mercury 2536 Å is directed into the tube as shown by the arrows. Tube 2 is then heated, thallium vapor is driven over into Q, the vapor is viewed [2] through the window w. No light is scattered into the observing spectroscope, nor is any light absorbed, if viewed in the transmitted direction. Tube 2 is then cooled and the thallium vapor in Q condenses in the cooler tubes. Then tube 1 is heated, mercury vapor enters Q, and the scattered light shows a strong line at 2536 when viewed through W, marked absorption in the transmitted direction. But when both mercury and thallium vapors are in Q, and one observes through w, the *2536 line is weak but new lines, those of thallium, are now seen.* Now it appears that many of the mercury atoms which

[1] Unless we accept as established this result—a high energy photon may vanish in producing an electron and a positron.

[2] This must be done either by the use of a fluorescent screen in the eye-piece or a photographic plate.

were excited by the incident 2536 photons (they are the only atoms which can be excited since they are the only ones which have absorbed the photons) have given up their energy of excitation, 4.86 volts each, to a thallium atom and the latter have emitted their own characteristic photons. (The chief thallium quantum is 3.27 volts.) In some way an excited mercury atom can return to its normal state without radiating the usual quantum if it can pass on its energy to a thallium atom. This operation has been named (by its discoverer Professor Franck, then of Göttingen) *Stösse zweiter Art,* "collisions of the second kind," a rather clumsy name, not a descriptive one. The fact is that the thallium atoms, not being able themselves to absorb the energy of the incident light, take away from the mercury atoms the energy which the latter have absorbed. It might be called a pilfering or leveling collision. However, it is evident that the energy exchanges in these collisions are always from high frequency towards low. This leveling process is in accord with the Second Law of Thermodynamics. It is due to these leveling collisions that all the heat produced by a current passing through a commercial neon tube is not radiated as desired luminous photons, but a considerable fraction of that energy is lowered and leveled to heat the tube.

Later there will be discussed a phenomenon somewhat similar to the one above in which molecules of one element A, bombarded by photons from another element B, scatter out not only some of the B photons but others characteristic of the A molecules—the Raman effect. Here, however, we continue the discussion of the behavior of excited atoms and electrons.

An Electron Striking an "Excited" Atom Produces an Explosive Mixture.

Consider a mixture of electrons and Hg atoms in thermal equilibrium. In general there will be a certain fraction of atoms in an excited state and there will also be a fraction of the electrons with energies greater than 4.9 volts. We have seen that the latter can produce more excited atoms. If this performance were not reversible, all the electrons would lose most of their kinetic energy, the atoms would gain energy; thermal equilibrium would be upset: one constituent of the mixture would be reduced to nearly zero energy; perpetual motion might be possible, and the

second law of thermodynamics would be jeopardized. There is one way out of this dilemma. An excited atom must be able during a collision with an electron to give up its energy of excitation to the electron and return to its normal state without radiating a quantum. Now there is a thoroughly established law in mechanics that when two masses collide, their velocity away from one another after impact is always less than (in the ideal case of perfect elasticity equal to) the velocity of approach. But when an excited atom and slow electron collide, the latter must jump away with an enormous speed (4.9 volts means an electron speed of 1.3×10^8 cm./sec. or 800 miles/sec.). Here the "elasticity" would be many times better than perfect! Apparently a slow electron encounters an explosive mixture when it meets an excited atom. We must then think of an excited atom as a normal atom which has concealed in it for a short time—one-ten-millionth of a second—a photon. This photon may pass out as radiant energy or it may strike an approaching electron with all its energy and in doing so lose its identity in the great kinetic energy given to the electron.

But the Old Laws of Energy and Momentum Hold Fast.

Now this important fact must be noted. In all the energy exchanges here or elsewhere discussed, the laws of the conservation of momentum and of energy hold. Energy can take various forms but momentum is always motional; that is, it is always concerned with mass and velocity, and is equal to mv. When a photon is driven out from an atom or when an electron rushes away with the energy belonging to an excited atom, the momentum change is zero; the momentum of the atom with its very small velocity backward is equal to that of the photon or electron forward. Now the atom with its small velocity has a very small amount of kinetic energy. Hence in all collisions, whether of atoms, electrons, or photons, some motional energy must be involved.

Do Photons Collide with Photons?

We have been dealing with various kinds of collisions, collisions involving molecules, atoms, electrons, photons. But there is one kind of collision to which reference has not been made, that of photons with photons. Briefly we may state that there are no

such collisions. In support of this statement the author has for years past presented the following argument. Just outside the intensely hot surface of the sun there is a region in which radiant energy is crisscrossing in all directions. Photons are passing through one another without influencing one another in any way whatever. There are vast numbers of photons of definite energies. Were these to *collide,* they would give or lose energy and therefore frequency. Energies and therefore frequencies would be "distributed" as the speeds of molecules are distributed. We could not have definite spectral lines any more than we can have a single velocity for all the molecules of a gas.

The photons of X-rays have high energies. Years ago the author directed a strong beam of X-rays across a beam of monochromatic light in an attempt to find a broadening of the spectral line. No effect was found. But the sharpness of the lines in the solar spectrum shows us that we should not expect such an effect.

Black body radiation is due to the interaction of photons with matter.

The Path of Destruction, the Cloud Track of an Alpha Particle or Electron.

The demonstrations which have been given so far of ionization have been electrical. But there is an optical demonstration both enlightening and spectacular. It is known as the cloud track experiment devised by C. T. R. Wilson years ago. When dust-free moist air in a chamber is allowed to expand [1] suddenly so that the volume is increased about 25 per cent, a very light cloud might be formed. But if X-rays had been passing through the chamber immediately after the expansion, the cloud would have been dense. If an electrical field is applied quickly after the expansion to wires or plates in the chamber, the cloud is quickly pulled to the plates, as the smoke was pulled in Experiment 9–1. Obviously the cloud particles are electrified. (This was the earliest experiment for measuring the electron charge—it was the origin of the oil drop experiment.) If an alpha particle from radium or polonium (necessarily placed inside the chamber) drives through the vapor and the necessary expansion takes place, a dense fog track

[1] An expansion of 25 per cent in dust-free moist air must take place before any cloud forms on negative ions; of 31 per cent before it forms on positive ions. The expansion used by Wilson was about 38 per cent. For some other vapors the necessary expansion is small, 10 to 15 per cent.

is seen (Frontispiece and Fig. 12–1). In fact, from a minute trace of radium on the end of a needle (enclosed in the thinnest possible glass tube) a large number of dense tracks radiate. Generally they are straight to the end of their path, though towards the end they may suddenly or gradually change direction. A very strong magnetic field at right angles to the particle's path will cause it to describe a circle. When a strong electrical field is applied just after the track is formed, the single track separates into two. Ion pairs are formed all along the track. The water vapor merely enables us to see (and photograph) the ionized tracks.

The track (Fig. 9–7) of an electron [1] definitely differs from that of an alpha particle. Here it is thin—not straight—it may look

X-rays

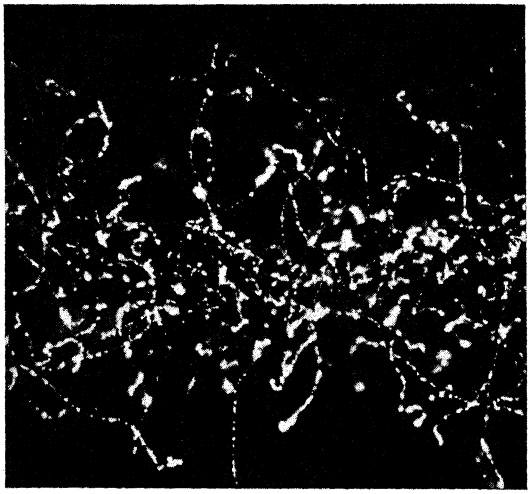


FIG. 9–7. X-rays driving through a gas flick off electrons from atoms. The electrons make thin beaded irregular tracks, as made evident by the moisture on the ions. (C. T. R. Wilson.)

like a twisted string with beads at irregular intervals. A magnetic field, even a weak one, at right angles to the motion of the electron will cause it to describe a circle. Measuring the radius of the circle and knowing the strength of the magnetic field, we can compute the velocity of the electron (Chapter 2). For equal speeds of alpha particle and electron, the former has 7300 times

[1] See also Fig. 13–6.

the momentum of the latter; for equal energies, 85 times. Hence the alpha particle is not easily turned aside.

When a narrow pencil of X-rays is admitted to an expansion chamber, the appearance of things is shown in Fig. 9–7. The X-rays themselves do not produce a track—at least it is not confined to the path of the X-rays. But it can be seen that along that path atoms have been ionized and that electrons emerge from some atoms and describe very irregular paths of perhaps a few centimeters in length, ionizing as they go. Thus it may be said that X-rays ionize some atoms and that the electrons thus set free ionize many more. It may be noted that the electrons set free move relatively large distances; the motion of the recoil atoms is so small as scarcely to be seen.

Further discussion of cloud tracks will be given in the chapters on radioactivity and on cosmic rays.

Electrical Phenomena and Gases

Let us turn now to consider the passage of an electric current through a gas. It is evident from what has gone before that, since an atom may be ionized by thermal agitation, by electrons which have been speeded up in electric fields, by photons, and by leveling collisions between excited and other atoms, the passage of a current through a gas may be and generally is an extremely complex phenomenon. In the early years of this century a text was published on this subject and for years it was the authoritative text in English. But it contained a great number of uncorrelated facts and surmises as to theoretical relations. During very recent years many of these facts have been brought together under comprehensive principles. This work has been done chiefly by Langmuir and his associates in the General Electric research laboratory and by Karl Compton and his associates in Princeton.

The Number of Ion Collisions Is Very Large Compared with That of Molecule Collisions.

Besides the necessity for dealing with the agencies which produce ionization, we must deal also with recombination. And what determines the rate of recombination? Obviously if two oppositely and equally charged particles come together, a neutral

atom or molecule will result and this will contribute to the recombination. Now the coming together of such particles will depend on their masses—the heavier they are, the slower they will move; upon the temperature—the higher it is, the faster they will move and the more recombinations will take place per second. One might think that the kinetic gas theory would tell us how many collisions (and therefore recombinations) of electrons and ions would take place per second. But that theory states that for *uncharged* atoms, n per cm.^{3}, the number of collisions per second is $n^2/10^{10}$, while for n *charged* particles, half of each sign, the number of recombinations is $4\,n^2/10^7$, a ratio of 4000 to 1. It is important therefore to know the densities of the electrons and ions.

Luminous and Dark Regions in a Discharge Tube.

Suppose a glass tube (Fig. 9–8) originally containing dry air has been exhausted to a pressure of about half a millimeter, and

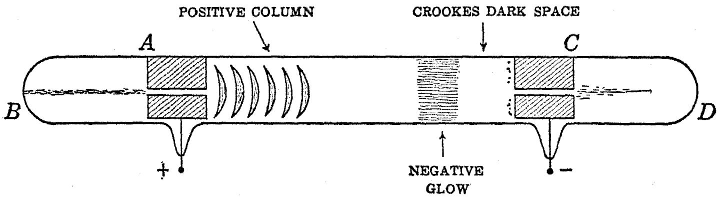


FIG. 9–8. An electric discharge through a gas; a phenomenon, complex—and beautiful.

that a potential of several hundred volts (with a high resistance in circuit) is applied to the electrodes. A rosy velvety glow will be seen on the surface of the cathode towards the anode, then there will be a dark space of several millimeters, then a luminous portion of the gas, then a dark space, then a large, probably reddish, portion up to the surface of the anode. The technical terms applied to these parts are: the cathode glow, the Crookes dark space, the negative glow, the Faraday dark space, the positive column. The first four occupy a length of a few centimeters (for a pressure of a half-millimeter), the positive column fills the rest of the tube up to the anode surface, even though this length may be several meters. Moreover, this positive column may be filled with numerous alternately dark and bright portions, striae, buttons of

light, nearly equally spaced, concave towards the anode. Why all these variations in color and intensity of light? The phenomenon may be one of luminous beauty. And the extraordinary fact is that when the phenomenon is most beautiful it is apt to be most complex.

If the pressure is decreased, the Crookes dark space and negative glow increase in length; the positive column decreases until the Crookes dark space extends to the anode, then there is seen a greenish fluorescence of the glass especially towards the anode. If both anode and cathode have holes through them (Fig. 9–8), faint luminous beams will be seen beyond the electrodes. That behind the anode can easily be shown to be due to cathode rays. That behind the cathode (if the pressure is not too low) can be shown to be due to a stream of (chiefly) positively charged *atoms* or molecules of the gas in the tube. The name *Kanalstrahlen* (channel, not canal or pipe, rays) is given to this stream. It is this arrangement and this kind of particle that is used in the modern mass spectrograph.

A few facts may here be stated. (1) When the striae are present or when there is a definite positive column if the cathode is movable and is moved towards the anode (current being kept constant by varying the voltage or outside resistance), the entire structure (glows, dark spaces) in front of the cathode moves with it. The positive column is pushed through the anode. (2) At higher pressures (several millimeters) the negative glow spreads over, or enfolds at least, a part of the cathode. If the current is increased, the fraction of the cathode covered is increased. Various applications are made of this relation. A modern radio set may display a colored column of light which rises and falls with the intensity of the signal: the extent of the negative glow shows the amount of current passing. While the current is increasing the voltage may be constant or decreasing, until the whole surface of the cathode is covered; then the voltage increases with the current.

Glow Lamps. Applications in Industry; Voltage Regulators.

The rather common glow lamps illustrate the points which have just been discussed. These may have a variety of forms and serve a variety of purposes.

We may have the *negative* glow lamp, Fig. 9–9, for which the wire spiral is the (chief) cathode and the small plate the anode. The gas usually is neon at a pressure of about one centimeter. For an AC voltage it is a partial rectifier. When we connect it to a voltage source, always with a resistance in series, we may increase the voltage up to a certain point (perhaps to 200 volts), then a discharge takes place; the potential at its terminal drops. If we decrease the series resistance or increase the applied voltage, we may have a current-voltage relation shown by Fig. 9–10. This shows a very important characteristic—*the voltage at the tube terminals remains constant as the current changes from 5 to 50 milliamperes.* Thus using a high voltage as a source we may obtain a constant lower potential even though the source may vary between rather large limits. The arrangement for this constant potential is shown in Fig. 9–11. If the so-called 220-volt source varies from 180 to 250, we may still have 90 volts between *A* and *B*. If the tube has for an anode a fine wire down the axis of a long brass cylinder, we may multiply the voltages above by 10—and we have a *constant potential source for a cosmic ray counter* (Chapter 13) or for some similar apparatus. It is necessary, of course, that the apparatus using the 90 or 900 volts shall have a resistance large compared with that of the glow lamp. The smallest neon lamps starting at 90 stabilize at 60 volts.

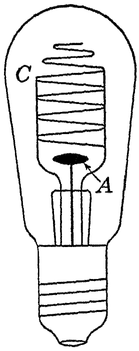


FIG. 9–9. A (neon) negative glow lamp may have a constant voltage across the terminals though the current changes greatly.

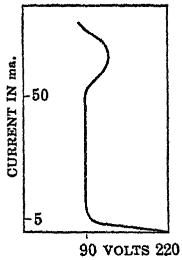


FIG. 9–10. The voltage between *A* and *C* in Fig. 9–9 remains practically constant while the current changes from 5 to 50 ma.

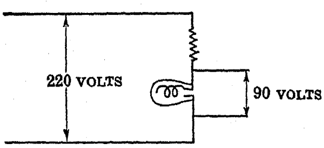


FIG. 9–11. The wiring arrangement for a constant voltage (90 volts).

The *positive* glow lamp is illustrated by the common neon sign. They are high voltage devices, up to 15,000 volts. The positive column rather than the grid glow is the source of light.

We turn from the discussion of general and qualitative phenomena to that of precision.

What Are the Temperatures of Electrons? How Great Are Their Concentrations?

Below is pictured apparatus which has been used in an attempt to answer these questions.

A long glass tube (Fig. 9–12) 150 cm. contains a small mercury pool as cathode, various anodes, and numerous probes. The

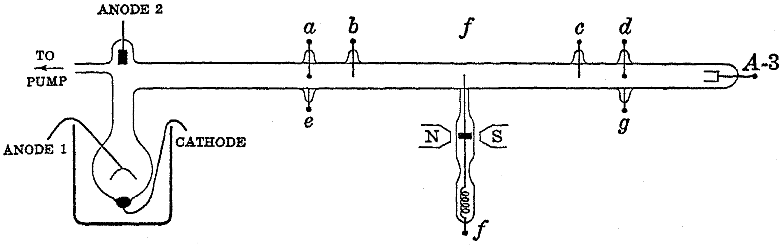


Fig. 9–12. Langmuir's scheme of numerous probes in an attempt to measure the concentration and "temperature" of electrons. Estimated temperatures of 40,000 degrees in a cold tube!

last are definite elements—tungsten, nickel; small, of definite form—cylinders, spheres; and of definite superficial area. Probe *f* can be moved across the tube by magnetic control. The tube is pumped out and all metal out-gassed. The mercury pool is kept at a definite temperature, the rest of the tube at a higher temperature. Then the pressure of the mercury vapor everywhere in the tube is the same as that in the pool. The cup-shaped anode (1) contributes to this condition by preventing the vapor blast given off by the mercury cathode from driving through the tube.

Let us suppose that there is a current of 5 amperes from anode 2 to the cathode. The positive column extends all the way to anode 3. The *random* current to a probe is measured. The data are analyzed according to the following theory.

The Temperature of Electrons May Be Extraordinarily High in a Cold Tube.

It has long been known that the potential gradient along the positive column is very small. This means that the densities of ions and electrons are nearly the same. It might be thought

that a "floating" probe in the positive column would be struck by an equal number of ions and electrons and therefore would not accumulate either. But Langmuir pointed out that the electrons would have greater velocity than the ions on account of their smaller masses, and even if the energies were the same the probes would accumulate electrons. This would mean that a positive sheath would form outside the probe, that a potential difference would be established between the probe and the *space*, and that electrons would have to break through a potential wall to arrive at the probe. There would then be a difference in the concentrations of electrons near the probe as compared with the positive column space. If V_0 and V are the potentials, n_0 and n the electron densities in the free space and near the probe, we have the relation (Appendix 1–4)

$$\frac{n}{n_0} = e^{-\frac{(V_0-V)\epsilon}{kT}}$$

where k is Boltzmann's constant and T is the (absolute) temperature of the electrons. Using Maxwell's law of distribution,[1] we easily compute the current to the probe $I = n\epsilon\bar{c}/4$ where $\bar{c}$ is the *average* speed of the electrons (Appendix 1–6). Using the relations of Chapter 1,

$$I = \left(\frac{kT}{2\,\pi m}\right)^{\frac{1}{2}} n\epsilon = \left(\frac{kT}{2\,\pi m}\right)^{\frac{1}{2}} n_0\epsilon e^{\frac{(V_0-V)\epsilon}{kT}} = AT^{\frac{1}{2}}e^{\frac{(V-V_0)\epsilon}{kT}}.$$

It is seen that this is similar to Richardson's law for thermionic currents. By measuring the current obtained when different

[1] Obviously the Maxwellian distribution does not hold in a region into which electrons are projected from the outside. Therefore it would not apply throughout a large region of intense electric field. Nor would it apply as between gas particles and electrons unless there were means of transferring energy from one set of particles to the other as contemplated by the ordinary collisions or interactions of mechanics. Having these facts in mind and further taking into account the variation of collision cross-section with velocity, Morse, Allis, and Lamar of Massachusetts Institute of Technology in an exceedingly important article in the *Physical Review* (Vol. 48, p. 412, 1935) have derived a new law of distribution of electrons in a gas. Instead of the Maxwellian distribution Ae^{-ax} they obtain $Be^{-b^2x^2}$ where x is proportional to the kinetic energy of the electron. Their theory has received rather convincing experimental confirmation. Thus we see that the Langmuir probe method, since Maxwellian distributions are there assumed, applies only to limited conditions.

We recall the fact that the Fermi-Dirac Law of distribution has replaced that of Maxwell in accounting for the emission current from a hot wire. Now it appears probable that the Morse, Allis, Lamar law will supersede other laws in realms in which electrons and gas molecules intermingle.

voltages are applied to the probe, the value of n_0 and T are determined.

Here are some of the amazing results.[1]

Temperature of mercury cathode (0° C.)	1.4	18.6	38.6
Pressure of vapor, in 0.001 mm. Hg...	0.2	1.02	5.35
Electron temperature (absolute)......	38,000	27,500	19,900
Number of electrons/cm.³ along axis....	5.17×10^{10}	9.13×10^{10}	28.6×10^{10}
Number of atoms of Hg/cm.³ (approx.)..		2.47×10^{13}	
Total *random* current across the cross-section of the tube in amperes (the current along the tube was 5 amperes)	21.5	38.5	64.0

The fact that stands out as extraordinary is that, with the mercury in the cathode near zero Centigrade, the *temperature of the electrons* in the positive column is 37,700° C.! This temperature inside a glass tube which does not melt—in fact, it may be only slightly warm! The student might explain how electrons can have such temperatures without imparting similar temperatures to the tube and why the electron temperatures decrease as the vapor temperatures increase from 1.4° to 38.6° C.

It might be thought that the electron temperatures must decrease at points near the glass wall. But the probe f can be moved across the tube and it is found that the temperature is constant along a diameter as well as along the axis in the positive column.

Some Regions in a Tube May Have Potentials Much Higher than the Anode.

Not only are the number of electrons and ions per cubic centimeter and their absolute temperatures measured in this way, but also one is able to measure the *potential* in the space through which the discharge is passing. Now it was shown (page 147) that the potential near the cathode in an electron tube might be more negative than the cathode itself. Similarly in arc and glow discharges in regions of a great concentration of ions or electrons, the potentials may be more positive than the anode or more negative than the cathode. Earlier in this chapter it was stated that the Franck and Hertz experiment was hailed as a brilliant confirmation of the quantum theory. It was shown how satisfactorily the Bohr picture fitted in with the fact established by that experi-

[1] See Killian, *Physical Review*, Vol. 35, p. 1245, 1930.

ment that nothing much could happen to an atom due to an electron collision unless the energy of the electron was at least as great as the lowest excitation potential of the atom. We came to the view that atoms in a discharge tube could not be ionized unless the potential was greater than that definite minimum. But then experimentalists (rude fellows!) began to announce that self-sustaining arcs could be maintained in argon, for example, with only 4 volts between cathode and anode. Now the resonance or excitation potential of argon is 11.5 volts, of ionization 15.7. Cumulative collisions or second-kind collisions (leveling) could not be invoked to explain this disturbing fact. But the probe method showed that in this argon arc there was a region with a potential of 15 volts relative to the cathode, 11 volts higher than the anode itself! The quantum theory was saved and all the quivering philosophy that had been used to account for this fact evaporated.

Potentials Measured by the Deflection of Electron Streams.

The probe method with modern interpretation of its data has given us a great deal of information regarding the behavior of electrons and ions in electrical discharges. But it ought to be checked by other methods. One of these, used by J. J. Thomson and later by Aston, stands out in importance. Figure 9–13 shows a double discharge tube. A_1C_1 are the anode and cathode of one tube. They are connected by glass rods so that either by gravitation or magnetic control they can be moved together along the tube T_1. A_2C_2 are the electrodes for tube T_2. A_2 is pierced by a fine hole so that an electron stream passing through will impinge on a fluorescent screen S. When no discharge is passing through T_1 this electron stream will strike S at some definite point O. When the discharge is passing through T_2, the electron stream will go through the ionized region in T_1

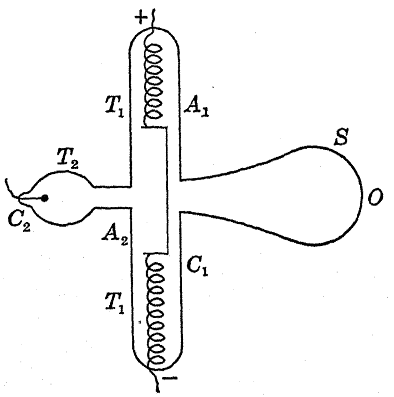


FIG. 9–13. The electrical field at various points in the discharge along A_1C_1 can be measured by shooting across it an electron stream.

and, being subject to the electrical field there, will be deviated to another point on S. From the displacement and the known geometrical magnitudes and the potential in T_2 we can measure the electrical field (see Appendix 2–3). As A_1C_1 can be moved along T_1, a new part of the discharge can be tested.

It is obvious that the vacuum in T_2 must be high in comparison with that in T_1 and therefore two separate pumping systems must be used. Also it is evident that the stream of electrons passing through T_1 may modify the discharge in the region through which it passes. By using a hot cathode in T_2 so that the density of the electron stream can be altered by known amounts, the extent of this modification may be studied and allowed for.

An interesting variation of this method is to use only one discharge tube with a cathode in the form of a tungsten ribbon on which there has been placed an oxide spot. When the ribbon is heated and the positive potential applied to the anode, the electron stream from the oxide spot can be seen, Fig. 9–14, as a straight line down the center of the tube towards the positive column. When a magnetic field is applied at right angles to this direction, the electron stream is curved. Presumably we can measure the curvature at every point of the stream. Then knowing the magnetic field we are to compute the velocity and the electrical field at every point in the main discharge. This method applies of course to fields near the cathode.

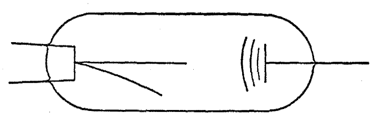


FIG. 9–14. The magnetic deflection of an electron stream issuing from a cathode gives a measure of the electrical field near it.

One rather startling conclusion came from the method above. The electrons appear to come from the oxide spot with appreciable energy. (If they came out with zero velocity, the radius of curvature would have been zero.) Now the ordinary view is that electrons arriving at the anode cannot have a greater energy than Ve where V is the total potential; that would imply that they start from the cathode or its near vicinity with very small kinetic energy. Yet the electrons from the hot spot seem to come out with considerable energy and are still to be accelerated by the potential in the tube. However, that there is a *high electrical field near the cathode* is indicated by this experiment, as it is by the probe method and by the following experiment.

In the methods of examining the electric forces operating in a discharge tube so far discussed, it has been necessary to insert a metallic probe in, or to direct a stream of electrons through, the discharge. But there is another method where nothing is inserted that would disturb the discharge. The light from the discharge is analyzed in a spectroscope.

Spectroscopic Method of Measuring Potentials; The Stark Effect.

In most spectroscopic work the light source is a luminous flame or the luminous part of an electric discharge; for example, the positive column in an arc or glow. In these sources the electrical field is very small. But when we examine light coming from a luminous gas which is subject to a strong electrical field, the bright lines originally sharp and single [1] are broken up into components shifted nearly symmetrically to the right and left. This is known as the Stark effect. The shifts are nearly proportional to the electrical field. When the slit of the spectroscope is close to the cathode and parallel to the axis of the tube, a mercury line, instead of being a single straight line, has the appearance shown in Fig. 9–15. The shifts of the components are small near the negative glow, increasing in the Crookes' dark space, large near the cathode. The way in which the electrical field changes as we go from the cathode towards the negative glow can be determined from the shift of a component of the line from its zero position. In Fig. 9–15 the position of the cathode is indicated at the bottom. The displacement and therefore the field is large near the cathode, increasing slightly for a short distance, then decreasing, becoming nearly zero at the top. This is in the vicinity of the negative glow.

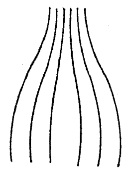


FIG. 9–15. The high electrical field near the cathode is shown by the breaking up of a spectral line.

In a uniform conductor like a metal wire, the electric intensity or field, being the potential difference per centimeter, is constant. But within a glass tube of uniform cross-section through which a discharge is passing, the electric intensity may be very large in one region, very small in another. How do we account for this variation? The explanation lies in the concentration of positive

[1] These are qualitative terms. Relatively few lines are sharp and single.

ions or electrons in certain regions and the influence of these clouds upon the fields. It has been found by the modern probe method that the density of the electrons in the negative glow is one hundred times that in the positive column. Clearly a gas through which a discharge is passing is highly complex.

Luminous Intensity of One Gas Greatly Increased or Decreased by Admitting Another Gas.

One very important commercial result is being looked for in these studies of electrical phenomena in gases. It is, as the man in the street would phrase it, the production of light without heat. In the ordinary incandescent lamp, only a small part of the energy is in the luminous form. (The temperature of the filament is about 2400 K: therefore the maximum intensity is at about 1.2 μ or 12,000 Å, whereas the visual sensitivity is a maximum at 5700 Å.) But by combining gases in discharge tubes we can alter greatly the luminous efficiency. For example, if one admits hydrogen up to a pressure of 1.2 mm. in a tube containing mercury vapor at room temperature (pressure about 0.003 mm.), the intensity of the mercury lines in the visual region is increased over a hundred times,[1] the spectrum of hydrogen is scarcely visible. Similarly in the so-called neon (blue) lights used for advertising, the neon is present up to a pressure of perhaps one centimeter, yet the intense blue light (almost all the tube length is a positive column) is due to mercury vapor, the pressure of which would be only one-three-thousandth of that of the neon. It seems reasonable to believe that a combination will be found which will give the chief part of the energy in the visible spectrum. That is what is meant by the unscientific expression "light without heat."

Electrical Phenomena in Solids

If we have come to the conclusion that electrical phenomena in gases are complex, what are we likely to think of the corresponding phenomena in solids? There is only one answer—yet let us indicate some theoretical ideas and gather together some outstanding experimental facts.

[1] There is another, an entirely different phenomenon, called "the quenching of fluorescence" in which the intensity of the mercury line 2536 is reduced to about 10 per cent of its initial value by the admission of hydrogen. The fluorescence of vapors will be briefly discussed in the next chapter.

We have seen that a procession of a vast number of electrons constitutes an ordinary electric current. One ampere means six million million million electrons passing a point per second. In conductors therefore there must be an enormous number of electrons more or less free to move when urged by the slightest electromotive force. But there is one rule or regulation which they must observe—they must obey Ohm's law. (Just now we leave out of consideration superconductivity.) Ohm's law (1825) states that in a conductor the potential difference is proportional to the current. The ratio is called the resistance. The law has been tested through an enormous range of current density and is found to be substantially correct throughout that range.

Now electrical resistance is found to be a very remarkable property of matter. No other property executes such amazing jumps as we pass from one material to another. For example, the resistance of a cube of sulphur compared with that of an equal cube of silver is a quantity which is easily written down, about 10^{22}. We may possibly get some appreciation of this number by putting into the picture an astronomical quantiy. The distance to the nearest fixed star is about 25×10^{12} miles, or 4×10^{18} cm. If then we were to take a sheet of sulphur 1/2500 cm. thick, its resistance to current flow would be equal to that of a rod of silver of the same cross-section extending out to the constellation of Centaur!

We Try to Account for Electrical Resistance.

Why are electrons so difficult to move in the case of sulphur? Or so easy in silver? What happens to the electrons in silver when we apply a voltage difference at the ends of a wire?

Let us picture a metal rod 1 square centimeter in cross-section as having n electrons per cm.^{3}, and let us apply an electric field E (potential drop per cm.). Every electron of charge e would experience a force Ee and would have an acceleration $a = eE/m$. If an electron moves freely for a time t, it would acquire a speed at and would have an average velocity of $\frac{1}{2}\,at$ or $\frac{1}{2}\,(eE/m)t$ during that time. Since current density $i = nev$, we would have an average current of $i = (ne^2E/2\,m)t$. The conductivity (the reciprocal of resistance) is then $ne^2t/2\,m$. This procedure pictures an electron being put into motion for a certain time, then colliding with and giving up its energy to a molecule, then starting over again. Thus the work done in putting the electron into motion

finally goes to energy of agitation of the molecules, that is, to heat in the body.

The relation above for conductivity $ne^2t/2m$ is satisfactory in that it does not contain E. In other words, it looks as though Ohm's law is satisfied. But we have said nothing about n and t.

Suppose we change the temperature T. May we (neglecting certain constants) put $\frac{1}{2}mv^2 = kT/2$ as we did[1] in Chapter 1? If so, $t \propto 1/v \propto 1/\sqrt{T}$. and we would have resistance $2m/ne^2t$ vary as the square root of the (absolute) temperature. But the experimental fact is that (roughly) R varies as T; certainly we do not have $R \propto \sqrt{T}$. In other words, the considerations above which looked so hopeful do not lead to the right result in the matter of temperature and do not enlighten us regarding the difference as between sulphur and silver.

But in Chapter 8 we saw that electrons in metals do not follow the Maxwell distribution law. We must use the Fermi-Dirac law which at the present time is the law used for such matters. This works better. But we are still a long way from a complete theoretical explanation of many facts regarding resistance.

We Come to a Startling Phenomenon—Superconductivity.

There is one fact which stands out. As we lower the temperature of an ordinary conductor (copper, silver, . . .), the resistance decreases, rather uniformly; certainly there are no sudden jumps. But as we approach absolute zero of temperature, a strange phenomenon is witnessed; the resistance of certain materials (but not of copper, silver, . . .) suddenly vanishes. At least, for one-tenth of a degree drop from a certain critical temperature (called the transition point) the resistance may drop to less than one-millionth of its previous value. We may state this in another way. If we lower the temperature of a ring of lead to 7° absolute (7° K.) and start a current of a few hundred amperes in it (as we can do by magnetic induction), the current once started continues to flow for hours or for days if the ring can be kept at a temperature below the transition point. In general, the transition points are not as sharp as suggested above. Indeed, there are a vast number of facts which refuse to be gathered together into one brief, precise, statement.

[1] The reader will object to this on the ground that temperature change is due to random motion, while we have been dealing with unidirectional velocity.

The transition (absolute) temperatures for some materials may be here given: lead 7.2, tantalum 4.4, mercury 4.22, thorium 1.5, aluminum 1.14, zinc 0.79, cadmium 0.6. On the other hand, copper, silver, and tungsten have not been found to be superconducting even at temperatures as low as 0.75.

We might illustrate the consequence of the above facts in another way. Copper is about 17 times as good a conductor as lead at room temperature, but at 7° K. lead is many million times as good a conductor as copper. Note further that if a current of 200 amperes had been generated in the above-mentioned ring of lead at room temperature, it would have been melted almost instantly, but below 7.2° K. there is no appreciable heating effect. This phenomenon of superconductivity was discovered by Onnes in 1911.

A Superconducting Galvanometer.

How do we know that a current continues to flow in the lead ring? By its effect upon the needle of a magnetometer. This must necessarily be placed at some distance from the lead coil which must be kept in liquid helium. (The temperature of liquid helium at atmospheric pressure is 4.2 K.) Consequently it would be better to use a galvanometer especially constructed for this kind of work. This has been done in the physics laboratory of the University of Toronto by H. Grayson Smith and F. G. A. Tarr. Their instrument is shown in Fig. 9–16. The moving coil consists of 100 turns of lead wire of 0.28 mm. diameter. This is connected by fine lead ribbons 0.01 mm. in thickness to a fixed coil of the material to be tested. The whole dips into liquid helium. A current is induced in this lower coil by the changing current in the inducing field coils. The performance of this galvanometer as indicated by theory and as tested by experiment for known conditions have been found to agree. Hence it is used quantitatively in a study of unknown materials. We return now to discuss further the topic of superconductivity.

Extraordinary Interrelation between Magnetic Field, Temperature, and Superconductivity.

Let us suppose that we have a ring of *white* tin (tetragonal) in liquid helium at a temperature just above 3.71° K., and we impose on it at right angles to the plane of the ring a weak magnetic

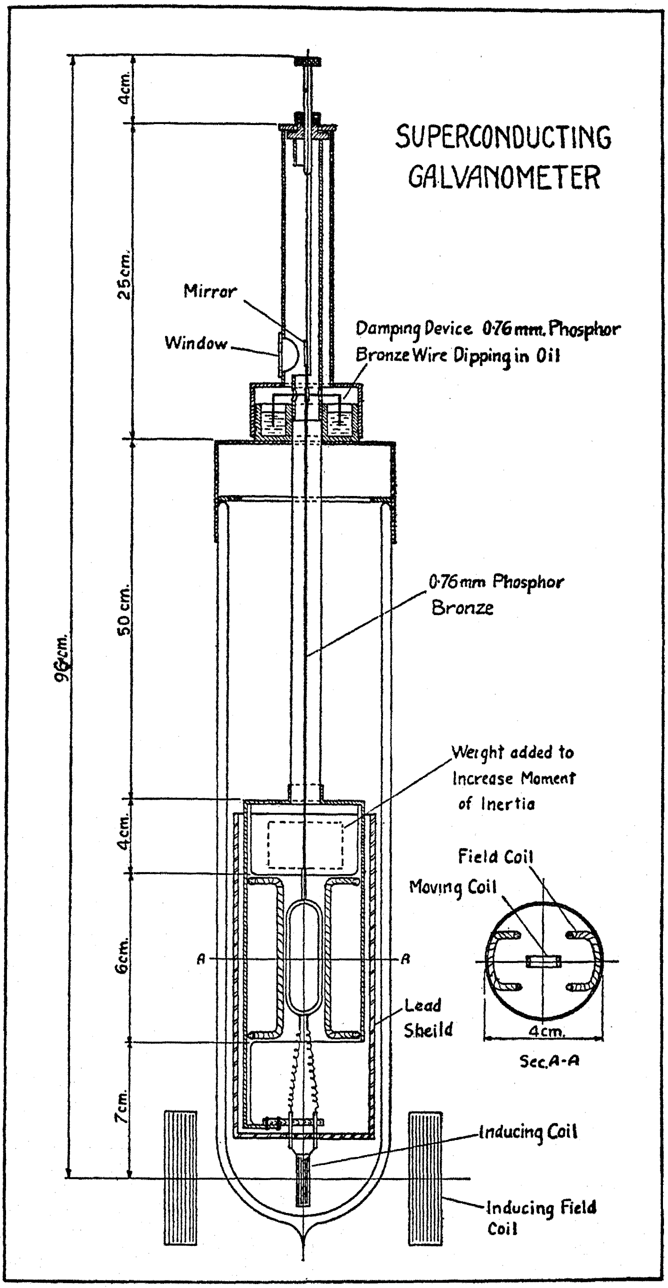


FIG. 9–16. A galvanometer whose coils dip into liquid helium.

field. If we lower the temperature of the ring below the transition point, 3.71, the ring becomes superconducting. If the magnetic field be annulled, an induced current will be set up in the ring, perhaps 100 amperes, which will continue to flow for days without heating the ring. But if the magnetic field had been strong (200 gauss), the temperature would have had to be lowered to 2° K. before superconductivity appeared. Had the temperature been lowered to 2.5° K., then held there while the field was decreased to 100 gauss, superconductivity would have appeared.

In general we may arrive at the super state by decreasing the temperature, keeping the field constant or vice versa. *A strong magnetic field operates against the appearance of superconductivity.* This is a fact of very great importance. It shows that there is a critical interrelation between temperature, magnetic field, and conductivity in the region of the absolute zero. There are other effects due to a magnetic field to which we now call attention.

It is necessary here to discuss three rather technical terms, diamagnetic, paramagnetic, ferromagnetic substances. A ferromagnetic rod (of iron or nickel) will set itself very strongly along a magnetic field; it is strongly attracted towards the intense part of the field. Lines of force run through the iron or nickel rather than through air. We say that it is permeable to the field; its permeability μ may be 2000 times that of air. A paramagnetic rod will set itself along the field; μ for it is greater than 1. A diamagnetic rod (bismuth) sets itself across the field; it is forced away from the intense part of the field; μ for it is less than 1. Now it is found that *a superconductor is a perfect diamagnetic body.* For it $\mu = 0$. In other words, when we push a magnet up towards a superconductor, the latter is repelled by the magnet; the lines of force appear to run around the body. Can we account for this?

Let us think of atoms in the superconductor as having electrons rotating in orbits about the nuclei. Now an electron rotating in an orbit produces a magnetic field; it is equivalent to a small magnet at right angles to the plane of the orbit. In a diamagnetic body these fields are in all possible directions; there is no resultant magnetic field. When we push a magnet towards the body, these orbits orient themselves so as to produce a resultant magnetic field opposing that which is being thrust upon them. (This is Lenz's law, the law of the electric motor.) Hence the

imposed magnetic field does not seem to enter the body. This qualitatively accounts for the diamagnetism of a superconductor.

We cannot follow this argument out for the other two classes, but we note here that up to the present *no ferromagnetic body has been made superconducting.*

Later we shall see that the very lowest temperatures are produced by decreasing (suddenly) a magnetic field in a body. Altogether it must be evident that *there is in this region of very low temperature a most important field for exploration.*

There are a vast number of facts connected with this phenomenon of superconductivity.[1] Transition points for high frequency fields are lower than those for steady fields; for thin films they are lower than for thick films. White tin of ordinary thickness has a transition point of 3.71° K., but a film of tin less than 0.004 mm. thick could not be made superconducting at 2° K. The transition point when the metal carries a large current is lower than that for a small current. But this may be due to the magnetic field of the large current. Specific heat, thermal conductivity, thermo-electric power, all have abrupt variations in the region of the transition point, but the photoelectric effect is unchanged. This indicates a striking difference between photoelectrons and conduction electrons.

Any theory that gathers together all of the above phenomena and facts must have several humps in it.

Low Temperatures: Obtained by Boiling Liquid Helium, Measured by Pressure of Helium Vapor.

But perhaps the reader would like to know how temperatures like 0.75° K. can be obtained; and measured!

Presumably all our readers are acquainted with "dry ice," solid carbon dioxide. It is obtained by allowing liquid CO_2 under high pressure (60 or 70 atmospheres, depending on the temperature) to expand and vaporize. The solid CO_2 results. Its temperature is − 79° C. But when dry ether is mixed with it, a temperature of about − 87° C. may be obtained in the open air; this is due to the evaporation of the ether, accelerated in part by the bubbling through it of CO_2. Under reduced pressure the evaporation is

[1] A very complete account will be found in *The Phenomenon of Superconductivity* (The University of Toronto Press) by E. F. Burton of Toronto, one of the foremost investigators in this field. See also *Reviews of Modern Physics*, H. Grayson Smith and J. O. Wilhelm, Oct., 1935.

increased and − 110° C. or − 166° F. may be obtained. This was done by Faraday one hundred years ago. We might continue this story through the stages of liquid oxygen (1883), and liquid hydrogen (1892). Then came one great triumph, the liquefaction of helium by Kamerlingh Onnes in Leiden in 1908. (That story reads like an account of an expedition to the North Pole.) The temperature of 4.2° K. was reached and the last known gas had been liquefied. By making liquid helium boil rapidly (by reducing the pressure above it), Professor Keesom of Leiden has reached (1933) a temperature of 0.71° K. In the meantime (1926) helium had been solidified. (At 800 atmospheres its melting point is 12° K.)

The Lowest Temperatures: Obtained by the Magnetic Method.

There is one other way of lowering the temperature besides making a liquid boil. A *paramagnetic* salt is placed in a small glass tube and this inside a small Dewar flask (thermos bottle) containing liquid helium at very low temperature. This flask is then placed between the poles of a powerful electromagnet. After it is clear that the salt has reached the low temperature of the helium, the magnetic field is greatly reduced (from 30,000 to 1000 gauss). The temperature of the salt is considerably lowered. In the case of manganese ammonium sulphate, the temperature of 0.1° K. was reached (1934); with potassium chrome alum, 0.0044° K., the lowest on record!

How are such temperatures measured? If not too low, by the vapor pressure of helium. The table in Appendix 9–3 indicates how very low temperatures may be computed from the reasonably well-known relations at higher temperatures. But it is clear that for temperatures below 1° K. we cannot compute temperatures from vapor pressures. We then must resort to computations based upon energies due to magnetic fields.

We pause to note that low temperature has been pushed down to 0.0044° K. It might seem that practically we have reached the absolute lowest limit of temperature. But it must be remembered that in this region differences must be measured by geometric ratios; that the interval from 3° K. to 0.3° K. is the same as from 0.3° to 0.03° or 0.03° to 0.003° K. At least there is still room for very extraordinary events to take place below 0.0044° K. They belong in the future.

We do not have space for the discussion of other topics which ought to be included under the general heading of electrical phenomena in solids. They are numerous—and important. We have chosen the topic of resistance as outstanding.

In the Realm of Great Pressures.

Here we can only note that a new realm in physics has been opened up by P. W. Bridgman of Harvard by means of the great pressures to which he has been subjecting matter. Pressures of the order of 50,000 atmospheres have been used. Under such extraordinary pressures matter might be expected to display extraordinary properties—as has been found to be the case. Physics, chemistry, geophysics—are greatly concerned in the results of those experiments.

CHAPTER 10

THE ZEEMAN AND RAMAN EFFECTS

In the preceding chapter we have discussed the interaction (collision) of electrons with atoms and of photons with atoms and have seen that the picture we have formed of the atom based on the quantum idea fits in with the phenomena resulting from those collisions. In this chapter we desire to consider other interactions —the action (a) of a magnetic field, (b) of a strong electrical field, upon atoms emitting light, (c) of photons on atoms, resulting in photon emission. The three outstanding phenomena connected with these actions are known as (a) the Zeeman, (b) the Stark, (c) the Raman effect. Very especially we want to see to what extent these effects may be explained by the classical wave theory and to what extent by the quantum theory.

THE ACTION OF A MAGNETIC FIELD UPON LIGHT AND LIGHT SOURCE

Faraday as early as 1822 tried to see if a magnet affected in any way a beam of light. He found no effect at that time, but in 1845 he discovered the phenomenon known as the Faraday effect, which may be described thus: when a beam of plane polarized light passes through a piece of glass in the direction of a magnetic field, the plane of polarization is rotated.[1] This was the first time that any interaction was shown between light and magnetism. Faraday continued his interest in this kind of interaction and in 1862, among his last experiments, he attempted to find if the light emitted from a source was altered or modified when the source was placed in a strong magnetic field. Using a sodium flame as a source and examining it by means of a spectroscope, he was unable to see any change in the yellow lines when he turned on or off the field. We now know that he failed on account of the limitations of his apparatus. Thirty-four years after his attempt, Zeeman (1896) discovered the effect.

[1] A special case is this—a change is produced in polarized light when it is reflected from the polished pole of a magnet, as discovered by Dr. Kerr (1875).

The Zeeman Effect.

Using as source the blue-green line of cadmium placed between the poles of a powerful electromagnet (Fig. 10–1), Zeeman observed that the line was broadened. Fortunately he was later guided in his analysis by the theoretical work of Lorentz of Leiden and Larmor of Cambridge, and he critically examined the line as he viewed it across and along the field. (For the latter purpose one of the pole pieces was bored.) He found that the spectral line, originally single, was a triple viewed across the field, double along the field. Moreover, the latter two components were circularly polarized, the former three plane polarized, as the theorists had predicted.

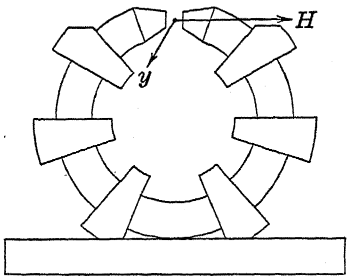


FIG. 10–1. A spectral line from a source placed between the poles of a powerful magnet may be broken up into several lines when the magnetic field is turned on. The Zeeman effect.

An Elementary Explanation Using "Old" Ideas.

The theoretical operations were based upon Maxwell's electromagnetic wave theory. A very simple and qualitative treatment is as follows. Light was supposed to be due to electric charges rotating in orbits or vibrating back and forth. (The cause of such motions need not here be enquired into.) The frequency of the light sent out would be the frequency of the electric oscillation or of the electric charge in its orbit. Let us then suppose that an electric charge is rotating in a circle and that some central force F holds it on that orbit. Then $F = mv^2/r$. Let the electric charge be e. (The electron had not been discovered in 1896.) If the magnetic field is H at right angles to the circle, then there is a force towards the center $= Hev$ either increasing or decreasing F. Let us take only the former case. Then the particle must speed up in order to stay in the same circle. (Suppose that for some reason the radius cannot change. This foreshadows Bohr's innermost orbit. We assume that the kinetic energy of the electron automatically changes so that it describes the original orbit in equilibrium with the new force.) Then we must have the relation

$$F + He(v + v') = \frac{m(v + v')^2}{r}.$$

We suppose v' to be very small compared with v, and therefore ignore the square of v'. Then $He = 2\,mv'/r$. But the original frequency f is $v/2\,\pi r$ and the new frequency $f + f' = (v + v')/2\,\pi r$ or $f' = v'/2\,\pi r$ or $f' = (e/4\,\pi m)H$. In other words, the *change* of *frequency* should be *proportional* to e/m and to the *magnetic field.*

To explain the polarization effects on the basis of the old theory, we note that the vibration at any instant may be resolvep (Fig. 10–2) along and at right angles to the magnetic field. The former is not influenced by the field; the latter may again be broken up into two circular components, clockwise and counterclockwise, as viewed along the field. The former is quickened, the latter slowed down by the frequency difference above. The vibration along the field cannot be propagated in that direction; the other two components emerge in that direction as circularly polarized light. When we view along y across the field all three components emerge, the central component emerges as plane-polarized light with frequency unchanged, but the circles are viewed edge on and only the linear components emerge to give plane-polarized light with frequencies greater and less than the original by $(e/4\,\pi m)H$ per/sec. All this can be illustrated graphically. In Fig. 10–2 OA is the component of the vibration along H, OB is at right angles. The two circular components at right angles to H into which OB can be resolved are shown in perspective. Light due to OA is sent out along Oy plane polarized with vibrations horizontal and with unchanged frequency. Light due to the circles is sent out along H circularly polarized in opposite directions, and along Oy plane polarized with the vibration vertical. Circle No. 1 is hurried by the magnetic field;

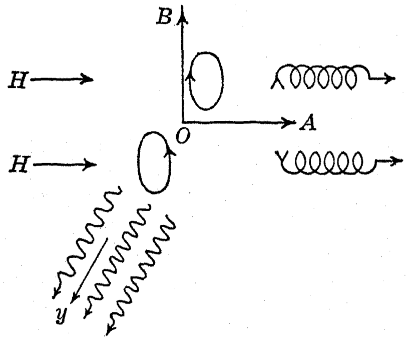


FIG. 10–2. Looking along the magnetic field, one sees a single line broken up into two circularly polarized lines; across the field three plane polarized. The old explanation of the Zeeman effect.

the frequency is therefore greater than that of OA or OB; the opposite for No. 2.

After the discovery of the electron and the accurate measurement of e/m, it was known that, since $e/m = 1.76 \times 10^7$ e.m.u., the frequency change computed above was $1.40 \times 10^6 \times H$. Of greatest practical importance is the wave number change which is $1.40 \times 10^6\ H/2.998 \times 10^{10}$ or $4.674 \times 10^{-5}\ H$ per cm. It results that the wave length change for a line about 6000 A is, in angstroms, $1.7 \times 10^{-5}\ H$. A well-designed magnet is required to give a field strength of 10,000 over an area large enough for this experiment. This would produce a shift in wave length (right or left of center) of 0.17 Å. To measure this to an accuracy of one per cent requires considerable care even with good optical instruments. Still for several years after the discovery of this effect its experimental data were used for the computation of e/m. The values so computed were in accord with those obtained by the methods of Chapter 2. But will the reader note the great difference in these methods of measuring e/m? In one case we find the deflection of a stream of electrons due to a magnetic field, in the other the separation of the fine lines in the Zeeman effect.

The Zeeman Effect Was Hailed as Another Confirmation of the Electromagnetic Wave Theory of Light.

For it followed from that theory that whenever an electric charge oscillated, an electric wave (light) would be radiated from that oscillating charge as a source. A magnetic field should modify the oscillation in such a way that, viewed along the field, a single spectral line should be a doublet, viewed across the field it should be a triplet; that the lines should be now polarized, whereas the original line was unpolarized. Moreover, after the discovery of the electron and the measurement of the ratio of its charge to mass, the displacements of the components of the spectral line were quantitatively confirmed. And will the reader note how complete and satisfactory this theory appeared to be? Note again in Fig. 10–2 with what assurance we draw the vibrations, linear, circular, how we draw wave motions resulting from those vibrations, how clearly we can picture linear and circularly polarized light, how satisfactorily we explain the doubling and tripling of the spectral lines. Every part of it lends itself as a model.

Then Disturbing Facts Enter.

But it was soon found that the effect was far more complicated than the first results suggested. For example, had the most ordinary spectrum been examined—the yellow (D) lines of sodium—it would have been found that, instead of there being three components of each of the D lines, D_1 would have had 4 and D_2 6 components. There may be 3, 4, 6, 8, 9, 12, 15, $\cdots$ components of some lines! Heroic attempts were made to explain these complex results. Then the Bohr theory came and gave us the view that the frequency of light radiated was not the frequency of an electron in its orbit; that the frequency of radiation was merely an energy change divided by h. The idea of *ether vibration*, plane or circular, did not enter. The picture which had been carefully drawn faded away, and with it the explanation.

The photon, whose frequency was merely its energy divided by h, now dominated the scene.

The Quantum Idea Must Be Introduced. The Bohr Theory Helps Only a Little.

But in the Bohr theory we still had the rotation of electrons in orbits. Moreover, the laws of electromagnetism held and an electron describing an orbit when a magnetic field was imposed upon it might have an energy which it did not have before the creation of the field. Let us see how this follows.

We know that a wire in the form of a circle carrying a current will set itself in a position of stable equilibrium at right angles to a magnetic field. It acts like a magnet of definite magnetic moment. If turned away from its equilibrium position, it will try to return. Therefore in any position away from this zero or equilibrium position it possesses potential energy. Now an electron in a circular orbit is equivalent to a current if we distribute its charge uniformly around the circle and allow it all to rotate with the angular speed of the electron. Consequently an electron, either revolving in an orbit or spinning on its axis, has an energy due to the existence of a magnetic field.

Even Space Must Be Quantized!

It has been stated above that the laws of electromagnetism hold. But do they? Or are they the same for an electron in an orbit as for a circular wire carrying a current? In the latter case,

though the wire would want to set itself with its plane at right angles to the magnetic field, it might have practically any direction relative to that field. But theorists proposed, and experiments seem to confirm, another behavior for electrons. Just as the Bohr theory indicated that electrons could occupy only certain nonradiating orbits, so here it is proposed that the orbits cannot have any direction whatever relative to a magnetic field; only those directions would be allowed for which *the projections on the zero plane* would have *integral* (or half-integral) *quantum numbers*. To use the technical term, *space is now quantized.*

A very elementary picture might be given of this space quantization. Suppose we have a top spinning about a vertical axis. We may think of the axis of that top as departing from the vertical and of having any position whatever between the vertical and horizontal. But if the idea of space quantization prevails, the axis could have only a limited number of positions, depending upon the total original angular momentum; the greater the latter, the greater the number of allowed positions. If now the top is an atom, the angular momentum of which is associated with an electron's motion, and if the maximum angular momentum in Bohr units, $h/2\pi$, is 3 (that is, if $2\pi mv = 3h$, see Chapter 6), then the allowed directions are such that the components of the angular momentum about the *vertical* axis (now the direction of the magnetic field) are $3, 2, 1, 0, -1, -2, -3$, in Bohr units.

Silver Atoms Show Space Quantization. The Gerlach and Stern Experiment.

An experimental result may illustrate the above statement. A silver atom in the normal state is believed to have one outermost electron. According to the quantum theory, this electron may spin so that the normal to its plane of motion—its spin vector—is parallel to the field or is in the opposite direction, so that it is antiparallel. The electron may spin clockwise or counter-clockwise as one views it along the field. This is *space quantization* in this simple case. Now a small compass needle merely turns to set itself along a *uniform* magnetic field. There is no total force pulling the whole magnet north or south. But if the magnetic field is not uniform, if a bar magnet is brought towards a compass

needle, the latter not only turns towards one pole of the magnet, it may jump into the magnet. Since the field is not uniform near one end of the magnet, the attractive force on one pole of the compass needle is greater than the repulsive force on the other pole. Hence there is a resultant attractive force on the compass needle.

Now suppose that silver atoms from a very small furnace are shot out between the poles of a magnet, Fig. 10–3. The pole

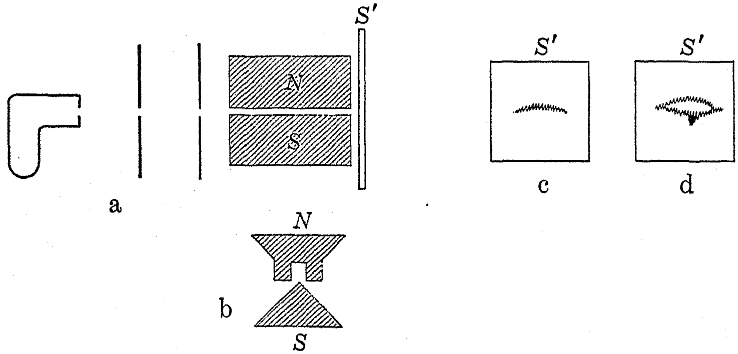


FIG. 10–3. Silver atoms shooting out from a furnace are condensed on a plate S'; c, when there is no field; d, with a magnetic field between N and S. b is a section of NS at right angles to a.

pieces are so constructed that the intensity of the field is very large near the sharp edge of S and small near the groove in N. The space quantization idea requires that for normal silver atoms there are only two directions for electron rotation; the north polarity of the electron spin would be up, or down. Hence some atoms would be pulled up, some down, and there ought to be two lines (one on either side of the original undeviated line) on the polished metal (or glass) plate S' upon which the atoms condense. The experimental result is seen in b. The silver stream has divided into two parts, confirming the idea of space quantization. This is known as the Stern and Gerlach experiment.

For a more complex electron condition we may have 3, 4, 5, $\cdots$ directions [1] for the electron orbit. Hence we may have 3, 4, 5, $\cdots$ energy values of an orbit due to a magnet field.

[1] For an atom of resultant orbital momentum J there are $2J + 1$ directions such that the components perpendicular to the field are J, $J - 1$, $J - 2$, $\cdots$, $(J - 1)$, $-J$. Here J may be integral or half-integral.

The Zeeman Effect Explained by the Use of the Quantum Idea.

In Appendix 10–1 is given an elementary and quite unorthodox method of finding the energy increase in an orbit due to a magnetic field. Strangely enough, this increase for a normal triplet comes out to be the same as was computed by the electromagnetic theory. But in the Bohr theory or in the quantum theory based on the Bohr model, the frequency belonging to a photon is due to a transition from one energy state to another. If all energy states had their energies altered in the same way and by the same amount due to the imposing of a magnetic field, there would be no Zeeman effect according to this theory. But (see page 96) though an electron may change from a large energy condition to a small one, its angular momentum change is limited to a maximum of one Bohr unit, $h/2\,\pi$, and the magnetic moment

$$\frac{h}{2\,\pi} \times \frac{e}{2\,m}$$

is proportional to the mechanical moment. Hence since the energy change hf' due to the magnetic field is $(eh/4\,\pi m)H$, then the maximum frequency change is $\pm\,(e/4\,\pi m)H$. This is the experimental value for the normal triplet. It is the value found by the old electromagnetic method (page 206).

The Explanation May Not Appear Simple—but Neither Is the Phenomenon.

Attention has been called above to the fact that orbits are *space quantized.* So also is electron spin. And the resultant condition of the atom is dependent upon the combination of the two components. It results that the increase in energy for a mechanical moment increase M is not $(Me/2\,m)H$ but this quantity multiplied by a constant g. Now there is no clear, complete method of deriving the value of g on the old quantum theory, but there is a *rule* for finding its value. Hence if we know the energy values of the various *space quantized states* of the higher and lower levels, we can compute the Zeeman shifts.

For example, consider the D_1 and D_2 lines of sodium, viewed at right angles to the magnetic field. Spectroscopists know these as the $2\,P_{1/2} - S_{1/2}$ and $2\,P_{3/2} - S_{1/2}$ transitions. (See also Fig. 6–5.)

The space quantizing requires half-integral quantum numbers. The different energies may be represented by points in Fig. 10–4.

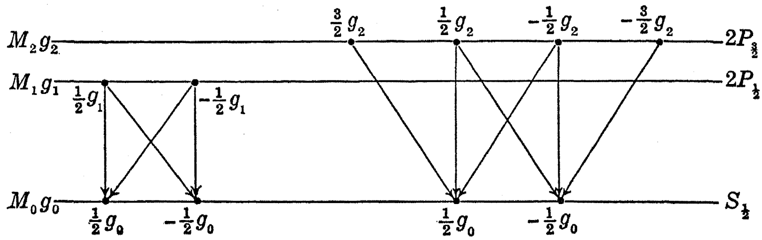


FIG. 10–4. A magnetic field causes a single energy state $S_{1/2}$ or $P_{1/2}$ of sodium to be broken up into two; the $P_{3/2}$ into four. Hence the D_1 line is broken up into four and the D_2 into six components.

The two space conditions of $S_{1/2}$, $\frac{1}{2} g_0$, and $-\frac{1}{2} g_0$ are shown twice, since $S_{1/2}$ is the common lower level. Here the spectroscopic evidence shows that there are two space conditions both for the $S_{1/2}$ and $P_{1/2}$ levels and four for the $P_{3/2}$. (Constants have been omitted in the figure.) Hence the mechanical moments must be in half-integral quantum numbers. The arrows show the allowed transitions. The g's, as computed by the rule in Appendix 10–6 are $g_0 = 2$, $g_1 = \frac{2}{3}$, $g_2 = \frac{4}{3}$. Then the Zeeman shifts for the D_1 line are $\frac{1}{2} g_1 - \frac{1}{2} g_0 = -\frac{2}{3}$; $\frac{1}{2} g_1 + \frac{1}{2} g_0 = \frac{4}{3}$, etc., or 4/3, 2/3, − 2/3, − 4/3; or, in wave numbers, $4/3 \times 4.674 \times 10^{-5} H$, etc. For the D_2 line it is seen that there are 6 components with displacements of 5/3, 1, 1/3, − 1/3, − 1, − 5/3 units. For all lines it is found that the vertical arrows represent lines polarized with their electric vectors parallel to the magnetic field; the diagonal arrows, lines with vectors at right angles.

We give another illustration. We consider the neon line 6678 Å. Spectroscopic evidence shows that there are 5 space conditions for the upper level involved in the transition, three in the lower. The mechanical moments are in integral quantum numbers. The g rule gives $g_1 = 1$, $g_2 = \frac{5}{4}$. There are nine arrows, Fig. 10–5, showing 9 Zeeman components. The shifts (in units of $4.674 \times 10^{-5} H$) are 6/4, 5/4, 1, 1/4, 0, − 1/4, − 1, − 5/4, − 6/4. These results are in accord with experimental fact.

Had the g's come out equal to 1, we would have had the normal triplet; if zero, there would have been no Zeeman lines.

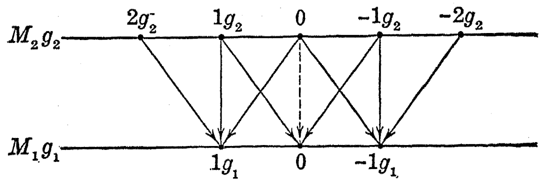


FIG. 10–5. A single neon line is broken up into nine components.

REVIEWING THE EVIDENCE, WE SEE THAT THE CLASSICAL THEORY COULD NOT ACCOUNT FOR THE ABNORMAL ZEEMAN EFFECT. THE OLD [1] QUANTUM THEORY CAN—PROVIDED WE ALLOW THE USE OF AN EMPIRICAL RULE FOR DETERMINING THE VALUE OF THE G CONSTANTS.

Will the student note the extraordinary contrast in the two methods of explaining the Zeeman effect. In the classical theory every element of the explanation is complete and is represented by a model. The logic is rigorous. In the old quantum theory a vast amount of experimental evidence is used to construct the energy levels which are necessary to account for the Zeeman components. Then there are the g values. Polarization is not accounted for as it was in the classical theory. Logical deduction has given way to "rules" of the game.

A Digression regarding Spectra.

One important point regarding the Zeeman effect is this—those lines in the spectrum of an element which have the same Zeeman pattern belong to the same series. Hence we are able to pick out a number of lines of one series and, taking wave number differences, we have differences in energy levels. Plotting a number of these, we may compute a formula—Balmer, Rydberg, Ritz—to include them and by making n equal to infinity in the relation

$$\nu = RZ^2\left(\frac{1}{(n_1 + a)^2} - \frac{1}{(n + b)^2}\right)$$

we may find the convergence level of the series and the energy levels of the different states. Various other phenomena or characteristics may assist in picking out lines of a series, singlets, doublets, triplets. And the Zeeman patterns may determine the

[1] The "old" quantum theory is based on the Bohr idea. It is so named in contrast to the newer wave mechanics theory. In this newer theory the "explanation" of the Zeeman effect is complete—but algebraical.

space-quantized states. Spectroscopy is an inductive science. We build up an atom picture by piecing together jig-saw experimental facts.

THE STARK EFFECT

After the discovery of the Zeeman effect, the influence of a magnetic field upon spectral lines, physicists discussed the possibility of an *electrical* analogue. Larmor, using the only theory then current, the classical theory showed that if luminous particles were set into rapid motion by an electrical field, the frequency of the radiating electron would be increased—but the change, involving the square of the ratio of the velocity of the particle to that of light, was small. On the whole, classical theory, contrary to its prediction in the case of the magnetic influence, did not predict an effect due to an electrical field. In 1905–06 the author tested the matter experimentally. Using as a light source the luminosity just in front of the cathode, in which region the electrical field is large (see Appendix 9–15) he compared the hydrogen lines of such a source with those originating in a weak field and found that the former lines were broadened.[1] Apparently the optical apparatus used was unsuited to the resolving of the lines or the electrical field was too weak; otherwise the breaking up of the line into components would then have been discovered (in 1906). It was found by Foster in an identical set-up. Stark, however, in 1913 with a very intense field discovered the effect.

Returning to the author's work, it is interesting to recall the argument he used in justifying the effect he found—a broadening of the lines. He pictured an electron in an orbit about a *nucleus* which was the rest of the atom. (Rutherford's "nuclear" atom was discovered in 1912.) The frequency was that for an elliptical orbit,

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu}{a^3}}$$

where μ is a constant and a is the semi-major axis of the ellipse. (See discussion of the Bohr theory and Appendix, Chapter 6.) Now the region near the cathode is one of intense ionization; atoms are frequently subject to collisions in that intense field.

[1] "An Investigation of the Influence of Electrical Fields on Spectral Lines," G. F. Hull, *Astrophys. Jour.*, XXV, 1, 1907.

Hence it was thought that the "rest of the atom" might be struck by an electron, in which case there would be a change in the relative velocity of the revolving electron; this would produce a change in a, the major-axis of the ellipse, and consequently a change in the frequency. Since it was everywhere supposed at that time that the frequency of light was due to the frequency of a vibrating electron, it followed that the line would be broadened. Moreover, the *methods of the calculus were used, which implied that changes in the diameter of the orbit could be continuous* (Appendix 10–2). The quantum idea of Einstein which had just been stated had yet to be tested. Bohr's theory of definite energy levels was set forth seven years later. The discussion in the paper cited used ideas which are in sharp contrast with those which came into physics after the quantum ideas were established. But the computations showed that a perturbation of the atom's speed of 100 centimeters per second would account for the broadening which had been found.

The Stark Effect.

Stark's method of producing a strong field is shown in Fig. 10–6. With the proper pressure in the tube, luminous particles stream through the perforations of the cathode C. A high potential (perhaps 10,000 volts) is applied between B and C. Since the distance between B and C is small, the intensity of the field is large. The luminosity between B and C is viewed across the field. It was found that a spectral line was broken up into numerous components generally distributed symmetrically on either side of the original position and that these components were polarized in the direction of the field or at right angles. The appearance of the components is shown by Foster's photograph (Fig. 9–12).

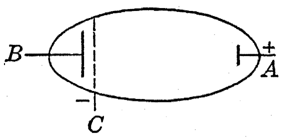


FIG. 10–6. There may be a very strong electrical field between B and C. Spectral lines from luminous sources in that region may be broken up into several components. The Stark effect.

The breaking up of a line due to an electrical field cannot be explained on the classical theory. Nor in all detail by the old quantum theory. The general idea is as follows. Imagine an electron revolving in an elliptical orbit and an electrical field to be imposed along the major axis. *The average potential energy*

would be increased (*or decreased*). (If the orbit were circular or at right angles to the field, there would be no change in energy.) Since a spectral line is due to a transition between two energy levels, there would in general be a change in the energy of transition and therefore in the frequency. But the theoretical operations and experimental details for the Stark effect are very complex. A very brief discussion is given in the Appendix for this chapter.

The Raman Effect

The two effects which have just been discussed demonstrate the superiority of the quantum, as compared with the older classical, theory, in the domain of atomic spectra. Another effect, discovered by Professor Raman of the University of Calcutta in 1928, adds testimony to this superiority and shows that all motions of *molecules* are quantized.

May Scattered Light Differ in Color from Incident Light?

When light traverses a liquid, even a very clear liquid, some light is scattered. Classical theory would say that the scattered light should have the same wave length as the incident (except in the case of fluorescence). But Raman using monochromatic light as source found that though some of the scattered light had the same frequency as the incident, there were in addition various scattered frequencies which depended on the liquid particles.

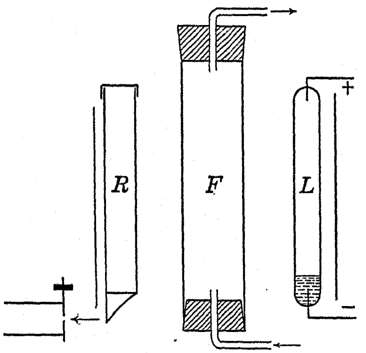


FIG. 10–7. A liquid in R receiving light of a definite frequency from L may scatter out several other frequencies characteristic of the energy states of the R molecules. The Raman effect.

The experimental arrangement is shown in Fig. 10–7. A very intense source as nearly monochromatic as possible is necessary. This may be provided by a quartz mercury arc lamp L. All the mercury lines except one can be partly, perhaps nearly, filtered out by a liquid F. Since the filtering liquid absorbs most of the light sent out by the arc, it may boil. Con-

sequently it may be necessary to have the liquid flowing continuously. The liquid whose "Raman spectrum" is to be obtained is placed in the tube R. The filter not only serves to remove (nearly) all frequencies except one but also serves to focus the light on R. Light scattered from R is thrown into the spectroscope S by a right-angled prism as shown. The greatest care must be taken to exclude all stray light and all scattered light except that which comes from R.

Suppose we desire to find the Raman spectrum of carbon tetrachloride and to use as our exciting line the mercury blue 4358 Å. The filtering solution which allows this line to pass through and weakens greatly the other mercury lines is a solution of sodium nitrite. It is advisable to use photographic plates specially sensitive to the region near 4358. A very satisfactory comparison spectrum is the copper arc. A fifteen-minute, or longer, exposure is necessary for the Raman spectrum, one second for the copper arc. Then when the plate is developed the results shown in Fig. 10–8 are obtained. The exciting line L of wave

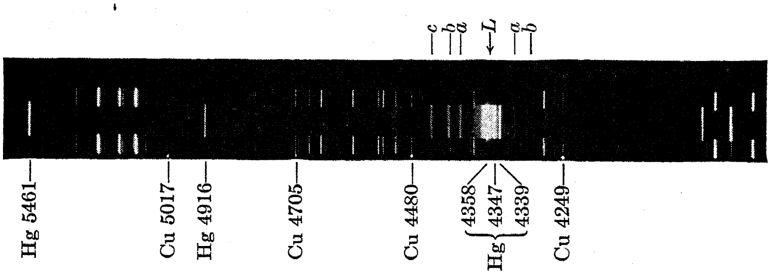


FIG. 10–8. The Raman spectrum for carbon tetrachloride is shown in the center. The exciting line L is the 4357 Å of mercury. The Raman lines are to the right and left of this line. Lines of the copper arc, above and below, give the comparison spectrum.

length 4358 Å has a wave number of 22,944.5 cm.$^{-1}$; that is, there are that number of wave lengths in one centimeter. (We may recall that the frequency for that line would be 22,944.5 multiplied by the velocity of light, 2.99776×10^{10}.) On the plate that line is strong. But on either side of L there are fainter lines obviously occurring in pairs, the two a's are equally spaced from L, so the b's, c's, $\cdots$. But there are apt to be more and stronger lines on the side of longer wave lengths, smaller frequencies, than on the other side. When the wave

numbers of these lines are measured, it is found that the *difference* in *wave number* between that of the exciting line and that of $a, b, c, \cdots$ is 218, 314, 460, 760, 793, 1533.[1]

Had any other radiation been used as the exciting agent, the same *differences* in *wave number* would have been found. Consequently the wave numbers 218, 314, etc., must be characteristic of carbon tetrachloride.

How do we account for these lines? The answer is that the old classical theory cannot account for them; the quantum theory, with certain adjustments, gives a clear explanation.

Explanation of the Raman Effect.

We picture a photon of energy hf striking a *molecule*. We have shown previously (see p. 125; also see Appendix 7–2) that a photon of the luminous variety can give only a very small motional energy to an atom or molecule. But it may definitely alter the *energy state* of the molecule. A photon of energy hf may raise the energy state of a molecule from E_0 to E_1, E_2, E_3, etc. The photon would lose a corresponding amount of energy and would have its frequency lowered. Algebraically

$$hf + E_0 = hf_1 + E_1, \qquad \text{or} \qquad f - f_1 = \frac{E_1 - E_0}{h}.$$

This would account for the decreased frequencies of the scattered photons. But if some of the molecules were in excited states, E_1, E_2, $\cdots$, and were about to return to the normal state or if the arrival of a photon precipitated the return, the energy set free might attach itself to the striking photon and would increase its energy, therefore its frequency. Moreover, since the number of molecules in excited states would be small compared with those in the normal state, photons would be apt to lose rather than to gain energy. In other words, the more intense Raman lines would be on the side of longer rather than shorter wave lengths.

The explanation above implies that a molecule in energy condition E_0 can absorb energy from light by amounts $E_1 - E_0$, $E_2 - E_0$, etc., where the E's are definite energies of the molecule. Hence the liquid should absorb light of frequency $(E_1 - E_0)/h$ or in wave numbers of 218, 314, $\cdots$. But light of wave number 218

[1] Data and experimental details taken from a thesis by G. F. Hull, Jr., 1934, in the Dartmouth College Library.

has a wave length of 0.0458 mm. That is, it is more than one hundred times as long as the exciting line; it is very far out in the infra red. We must then examine the absorption spectrum of carbon tetrachloride in the far infra red.

Briefly we may state that some Raman lines and some absorption lines agree but that there are some lines in the absorption not found in the Raman spectrum and vice versa. The explanation of this result is deferred until after a discussion of *band spectra*, or spectra due to molecules.

Band Spectra

Let us consider a molecule of hydrochloric acid. We picture the two atoms, hydrogen and chlorine, at a definite distance apart. We now inquire as to the various ways in which this molecule may acquire or exhibit energy. There may be a vibration of the two molecules along their connecting band; there may be rotation about an axis at right angles to the band and through the center of mass; there may be electron energies in either or both atoms. Now we have presented abundant evidence to show that electron energies can be of only definite amounts, as suggested by the Bohr theory. Similarly, spectroscopic evidence suggests that certain limitations hold with regard to the other two forms.

Let us accept the spectroscopic evidence and state that the vibrational energy may have any value given by $E_v = (n + \frac{1}{2})\, hf_v$ where n can take any of the values 0, 1, 2, 3, $\cdots$, h is the usual Planck's constant, and f_v is the vibration frequency; that the rotational energy $E_r = R(r + \frac{1}{2})^2$ where R is a rotational characteristic of the molecule (Appendix 10–3), and r may also have values 0, 1, 2, 3, $\cdots$. Then the total energy of the molecule at any time is given by $E = E_e + E_v + E_r$ where E_e is the energy due to an electron arrangement. Now the energy of the molecule may change due to the change of any one or any two or all three of these forms. We have reason for believing that the E_r's are small compared with the E_v's and these small compared to the E_e's.

If only E_r changes, we have a series of energy levels due to rotation. These are relatively close together. Similarly for the other E's. Hence we may give a graphical representation of

certain energy levels, as in Fig. 10–9. At the bottom of the figure we show low energy states for E_e and E_v with a number of possible rotational energies. Above this the same E_e but a higher E_v, to which may be added any one of the E_r's. Then there is a long jump to the top group representing an electron change, consequently a change in E_e. We desire to show how this diagram may be used to explain the following phenomena or processes: (1) absorption lines due to rotational changes far out in the long wave lengths; (2) absorption lines still in the infra red but of shorter wave lengths and more complex than (1); (3) the Raman spectrum (we shall use the diagram to give the "reason" for the fact that the Raman spectrum has some lines in common with the infra red absorption spectrum, some lines that the latter does not have, and that some lines are lacking in it that are in the absorption spectrum); (4) absorption or emission band spectra in the visible or near-visible region.

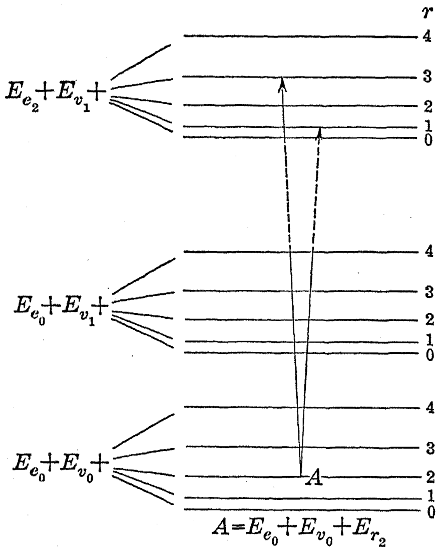


FIG. 10–9. A molecule may possess energy due to rotation E_r, vibration E_v, or electronic arrangement, E_e.

1. Infra Red Absorption.

As in the spectral rules for atoms,[1] we legislate that the E_r's can change by only one step, that is, r can change from 0 to 1, 1 to 2, $\cdots$, 7 to 6, or 6 to 7. If only low energy photons are available, the rotational states of the molecules may be changed by absorption from $r = 0$ to $r = 1$, or from $r = 1$ to $r = 2$, etc. Since the energies of these states are represented by $R(1/2)^2$, $R(3/2)^2$,

[1] It may be recalled that in the Bohr picture it was found that an electron could change by any energy amount (keeping to the prescribed orbits), but that the angular momentum could change by only one Bohr unit $h/2\pi$.

$R(5/2)^2$, etc., the energy *differences* between successive levels are $2\,R, 4\,R, 6\,R, \cdots$. We would then have absorption lines far out in the infra red of frequencies proportional to $1, 2, 3, \cdots$. Measuring these absorption frequencies, we are able to determine R or $h^2/8\,\pi^2 I$ (Appendix 10–3), *and consequently the moment of inertia of the molecule about its center of mass.* This for a polar molecule like HCl. For a more complex molecule the matter is more complex. We return to this later.

2. Vibration-Rotation Absorption Lines.

In Fig. 10–10, two vibrational levels are shown, each with five rotational levels. If a molecule is in the energy condition A_0, its energy may be expressed by $E_{e0} + E_{v0} + E_{r0}$. If it absorbs sufficient energy it may go to the higher vibrational group and r may change from $r = 0$ to $r = 1$. If the initial state is A_1 for which $r = 1$, it may absorb energy to carry it to either $r = 0$ or $r = 2$ in the upper group. Similarly for the initial condition A_2. Hence the energies absorbed are (from left to right in the figure) $hf_v + 2\,R$, $hf_v - 2\,R$, $hf_v + 4\,R$, $hf_v - 4\,R$, $hf_v + 6\,R, \cdots$. Here f_v is the vibration frequency of the molecule and $R = h^2/8\,\pi^2 I$ where I is the moment of inertia of the molecule. Consequently the *frequencies* of the absorption lines are

$$f_v \pm \frac{h}{4\,\pi^2 I}, \qquad f_v \pm \frac{2\,h}{4\,\pi^2 I},$$

$$f_v \pm \frac{3\,h}{4\,\pi^2 I}, \qquad \text{etc.}$$

These frequencies will be those of an *absorption band;* the

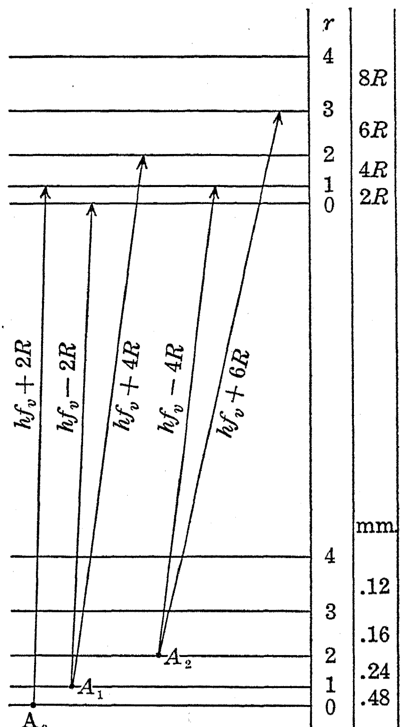


FIG. 10–10. The absorption spectrum of HCl can be explained if the rotation energies differ by amounts proportional to 1, 2, 3, 4.

difference in frequency between the successive maxima will be $h/4\,\pi^2 I$, the same as for the difference in frequency between the successive *rotation lines.*

If the energy absorbed had been sufficient to change the condition by two vibration levels, the frequencies of the absorption lines would have been

$$2\,f_v \pm \frac{h}{4\,\pi^2 I}, \qquad 2\,f_v \pm \frac{2\,h}{4\,\pi^2 I}, \qquad \text{etc.}$$

This would have given an absorption band of nearly twice the frequency of the first band.

It is to be noted that the lines belonging to the frequencies f_v, $2\,f_v$, $\cdots$, are absent. The values of f_v, $2\,f_v$, $\cdots$, can be determined, however, by measuring to the center of symmetry of the band.

A drawing approximately representing an absorption band of HCl is shown in Fig. 10–11.

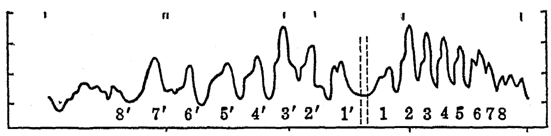


FIG. 10–11. A drawing showing the intensity of an absorption band of HCl. The small humps are due to Cl 37, the high peaks to Cl 35. Isotopes have been discovered from band spectra.

The missing absorption lines are indicated by the dotted lines. There is a clear evidence that the lines are slightly doubled. The small hump is interpreted as follows. The chlorine atom may be either of atomic weight 35 or 37. The vibration frequencies [1] of the two atoms would be different. Very simple, elementary, classical theory shows (see Appendix 10–4) that the *relative* vibration frequencies should be given by

$$\frac{f_2 - f_1}{f_1} = \frac{1.03}{2}\left(\frac{1}{35} - \frac{1}{37}\right).$$

Since the wave length of the line pictured above is (about) 0.00176 mm., the distance between the small and large humps is

$$\frac{1.76}{1270} \times 10^{-4}\ \text{cm.} = 13.9\ \text{Å}.$$

[1] There would be differences between the rotational frequencies of the two molecules, but these would be small compared with the vibrational differences.

Spectroscopic results are in accord with this interpretation. Moreover, the intensities of the two components are in the ratio of the abundance of the two molecules, 3.35 to 1, as determined by the atomic weight 35.46.

Thus we see how *band spectra* may be used for the identification of *isotopes* of elements.

3. The Raman Spectrum.

We now assume that the incident or colliding photons have larger frequencies than those considered in (1) and (2), that they belong in the visible or ultraviolet range. If these are absorbed, electronic as well as rotational and vibrational transitions in the HCl molecule may be necessary. We picture (Fig. 10–12) the initial condition of the molecule as in A_2. Absorption of the incident energy would change the state of the molecule represented by A_2 (for which $r = 2$) in the bottom group to either of the tops of the up-pointing arrows ($r = 1$ or $r = 3$) in the group. The molecule might then change back to the points of the downpointing arrows in either of the lower groups. In doing so it would radiate (scatter) energy of a frequency belonging to one of these transitions. If it came back to $E_{e0} + E_{v0} + E_{r2}$, it would scatter a frequency equal to that of the incident. If it came back to one of the other levels, the frequency scattered would differ from that of the incident by the frequency

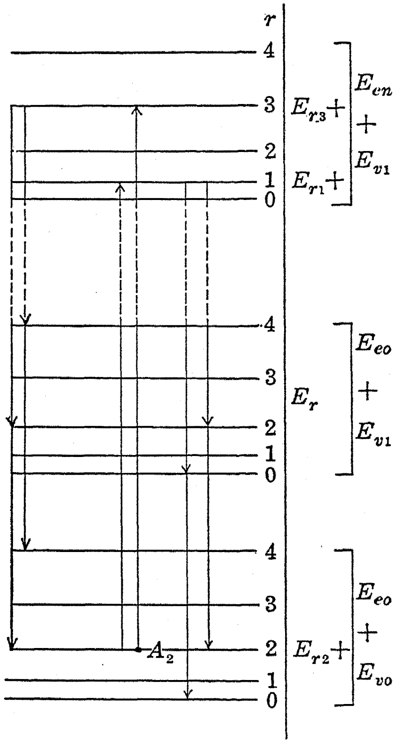


FIG. 10–12. If the initial condition of the molecule is A_2, the final condition after absorption and emission may be A_0, A_2, or A_4 in either of the lower groups.

represented by the transition $r = 0$ to $r = 2$ or $r = 2$ to $r = 4$, or by the vibrational frequency f_v. *But in the absorption spectra these transitions are "forbidden."* However, since the rotational levels differ by energies proportional to 1, 2, 3, $\cdots$, it is seen that the transition from $r = 2$ to $r = 0$ represents the same change as that from $r = 2$ to $r = 3$.

It is now seen that, though we legislated that rotational levels in absorption or emission could change by only one rotational unit, in the Raman spectrum for this type of molecule, as it has been here explained, these levels can change, if at all, only by two units. Hence it follows that we have the relations between the Raman and rotational frequencies as indicated by Fig. 10–13.

For HCl gas the Raman spectrum consists of one line differing in wave number from that of the exciting line by 2886 cm.$^{-1}$ (for the liquid 2800 cm.$^{-1}$), together with lines very close to the exciting line. The 2886 wave number is the "missing" line in the absorption spectrum at 3.47 μ (or 0.00347 mm.). The other lines are due to the differences in rotation levels as shown in Fig. 10–13.

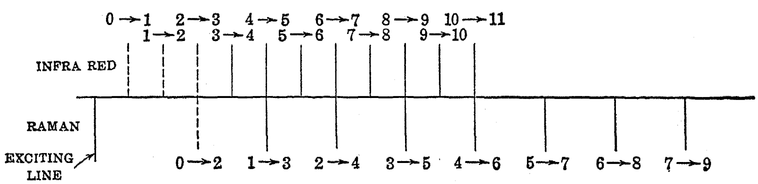


FIG. 10–13. The preceding shows why some of the Raman frequencies should agree with some of the infra red frequencies, and why some should disagree.

Absorption and Raman Spectra for Symmetrical Molecules.

In the case of HCl we are dealing with a *polar* molecule, hydrogen positive, chlorine negative. But for oxygen, nitrogen, etc., the molecules are symmetrical. The quantizing rules change. Moreover, for these molecules it seems to be necessary to emphasize odd states ($r = 1, 3, 5, \cdots$) or even states. One of these classes, odd or even, may be abundant, the other rare. If, for example, the odd states are rare, then the transitions $1 \rightarrow 2$, $3 \rightarrow 4$ are rare and the corresponding absorption lines feeble. Similarly the Raman lines corresponding to $1 \rightarrow 3$, $3 \rightarrow 5$ are also

feeble. This is illustrated by the Raman spectra for nitrogen as photographed by Rasetti (Fig. 10–14 a).

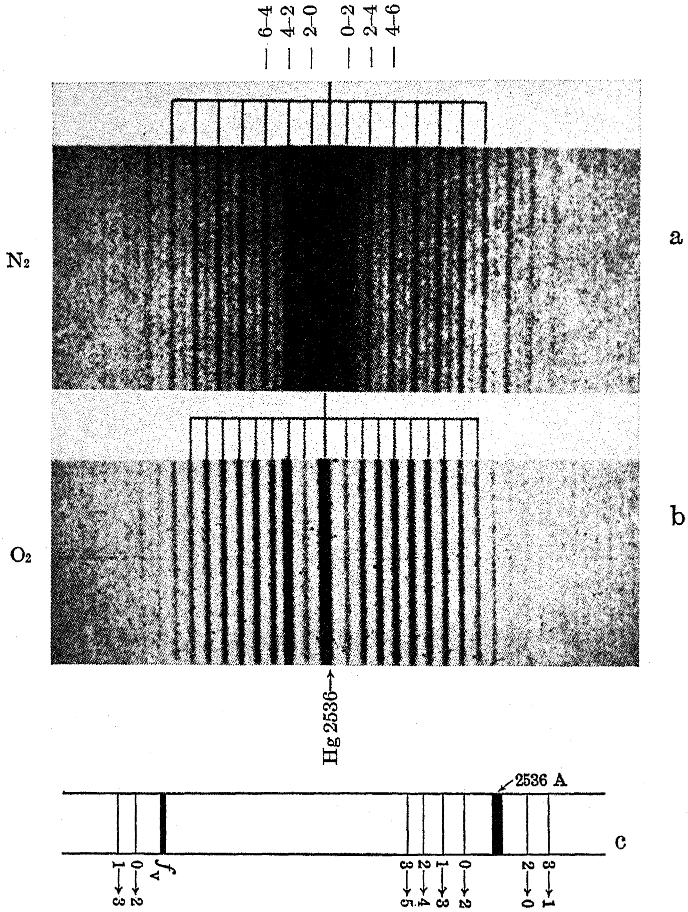


FIG. 10–14. The Raman spectra for (a) nitrogen, note the strong even and the faint odd transitions; (b) oxygen, only the even transitions appear; (c) hydrogen (drawing); a vibration frequency difference, f_v, appears; the rotational energy differences are large.

The lines are alternately strong and weak. The average distance between the strong lines in wave numbers is 16. From this it follows that $h/\pi^2 I = 16 \times 3 \times 10^{10}$; or I, the moment of inertia of the nitrogen molecule, $= 13.8 \times 10^{-40}$. As for the oxygen molecule (Fig. 10–14 b), its Raman spectrum shows that

the odd states do not exist. The strong lines show only even transitions $2 \rightarrow 4$, $4 \rightarrow 6$, $\cdots$. The moment of inertia of the molecule is 20.2×10^{-40} gm./cm.^{2}.

The Raman lines due to vibrational transitions are in wave numbers: for oxygen 1552 and for nitrogen 2331; or in wave lengths 0.0064 mm. and 0.0043 mm. Each of these is the "missing" line in the corresponding absorption spectrum. For the hydrogen molecule the average distance between the rotational lines is 355 cm.$^{-1}$. This makes $h/8\,\pi^2 I = 59.1 \times 3 \times 10^{10}$. Hence I for hydrogen is 4.6×10^{-41} gm./cm.^{2}. Note that the smaller the moment of inertia, the greater is the rotational energy. The vibrational wave number is 4162 cm.$^{-1}$.

The Raman spectrum of H_2 is shown in Fig. 10–14 c. The exciting line Hg 2536 Å is the heavy line on the right. There are two rotational lines of hydrogen on the side of higher frequencies (anti-Stokes) and four on the side of lower frequencies (Stokes). The displacements in wave numbers from the exciting line are 354.6, 587.3, 814.4, and 1033.9. These represent changes in r $0 \rightarrow 2$ or $2 \rightarrow 0$, $1 \rightarrow 3$ or $3 \rightarrow 1$, $2 \rightarrow 4$, $3 \rightarrow 5$. According to the theory above, they should be in the ratios of $3 : 5 : 7 : 9$. It is seen that this is not the case, for dividing by 3, 5, 7, 9, we have the numbers 118.2, 117.4, 116.3, 114.9.

The "pure" vibrational line is really a quadruplet with wave numbers from 4127 to 4162. There are two vibrational-rotational lines of approximate displacements from the vibrational line of 345 and 568 instead of 354.6 and 587.3.

These departures from the results to be expected from our simple theory show that the energy of a state must be a more complex function of n and r than we have discussed above.

Effect of Temperature on Raman Spectra.

In Fig. 10–8 showing the Raman spectrum for carbon tetrachloride, there appear lines of frequencies higher than the exciting frequency. We say that an excited molecule, about to return to a lower state, added its energy to that of the exciting photon to give the higher frequency. Now the number of excited molecules would increase with increase of temperature, consequently the intensity, and perhaps the number, of these high frequency lines should increase also with the temperature. Experimental results support this point of view.

Concerning Rotational and Vibrational Levels.

In deriving the values of the rotational energies we assumed, as was done by Bohr for the electron, that the angular momentum of the *molecule* could take only definite values, multiples of $h/2\pi$. From these definite values we can derive values of the energies in these possible states. However, some adjustments are necessary. Instead of the energy of the n^{th} state being $n^2h^2/8\pi^2I$ with n a whole number, we change it to $(r + \frac{1}{2})^2 h^2/8\pi^2I$ where r can be 0, 1, 2, $\cdots$. In wave mechanics we derive the value $r(r + 1)\, h^2/8\pi^2I$. The difference between corresponding terms is a constant, and since the frequency of either an absorbed or emitted radiation depends on differences of energies in the levels concerned, the two expressions $(r + \frac{1}{2})^2$ and $r(r + 1)$ are equivalent.

It is pointed out in the Appendix that the *average* rotational frequency when the molecule changes from the $r + \frac{1}{2}$ to the $r - \frac{1}{2}$ state is the same as the frequency radiated for the same change of state. This would appear to make a frequency of light depend upon a mechanical frequency of a molecule. Or, if the components of the molecule are electrified, that we are getting back to the point of view of the electromagnetic theory—that when an electric charge rotates, radiation results, its frequency being that of the rotation.[1] But the significant point is that these frequencies are small (compared with luminous frequencies) and for such small frequencies the classical and quantum theories of radiation come together. As has been pointed out, the frequency of rotation of the HCl molecule in the first state is about 6×10^{11}, of the order of the frequency of the (hydrogen) electron in the 25^{th} orbit.

This is one of the earliest conclusions at which Bohr arrived—that for a transition between two electron orbits of very large radius the frequency of the photon was (nearly) equal to that of the electron in its orbit.[2] It is known as Bohr's correspondence principle. Its justification was seen in the fact that for very

[1] There is of course an outstanding difference. From the quantum point of view no energy is radiated unless the energy of rotation changes ($r + \frac{1}{2}$ to $r - \frac{1}{2}$). The electromagnetic view was that energy should be radiated during the rotation of electrical charges, the frequency of the radiation being equal to that of rotation.

[2] The way in which the *average* of the rotational frequencies of the electron in the Bohr orbits approaches the frequency of the photon radiated due to transitions between these orbits is seen in the table on p. 228. The general relation giving the orbital frequency is $f = 2\,RcZ^2/n^3$ (see Chapter 6). (*Continued next page.*)

long waves, small frequencies, the black body radiation curve could be derived by the methods of classical physics, as was done in deriving the Rayleigh Jeans law (Appendix 4–2).

We have assumed that the energy due to *vibration* is $(n + \frac{1}{2}) f_v h$ where f_v is the frequency of vibration. If the frequency is fixed, how can the energy change? If we are dealing with a vibrating elastic body, the energy is proportional to the *square* both of the *amplitude* and of the frequency. It is necessary therefore to assume that the amplitude changes abruptly, taking values proportional to the square roots of 1, 3, 5, 7, $\cdots$; and that the frequency remains constant.

We can pursue this mechanical problem further. If we picture two masses m_1, m_2, held in an equilibrium position by a spring, then displaced toward or away from one another, they would oscillate with a frequency

$$f_v = \frac{1}{2\,\pi}\sqrt{\frac{K}{M}}$$

(see Appendix 10–4) where K is the elastic constant of the spring.

Now from either the Raman or the band spectrum we find the vibration frequency of the HCl molecule $= 8.6 \times 10^{13}$, the wave length being 0.00347 mm. Substituting in the relation for f_v, we find the elastic constant K along the line of the chemical bond.[1]

For the innermost orbit $n = 1$.

n	f	n	f	Average Orbital f	Photon Frequency
2	0.82×10^{15}	1	6.58×10^{15}	3.70×10^{15}	2.47×10^{15}
6	3.04×10^{13}	5	5.26×10^{13}	4.15×10^{13}	4.02×10^{13}
10	6.58×10^{12}	9	7.71×10^{12}	7.80×10^{12}	7.71×10^{12}
25	4.21×10^{11}	24	4.76×10^{11}	4.48×10^{11}	4.48×10^{11}
50	5.264×10^{10}	49	5.593×10^{10}	5.43×10^{10}	5.43×10^{10}

[1] We may find the force along the bond necessary to hold the hydrogen atom during its rotation. Take the frequency of rotation $= 6 \times 10^{11}$/sec. (the same as that of the difference between the rotation lines), and r, the radius of the hydrogen orbit, as 1.3×10^{-8} cm. Then $F = mr\omega^2 = mr(2\,\pi f)^2 = 1.65 \times 10^{-24} \times 1.3 \times 10^{-8} \times 4\,\pi^2 \times 36 \times 10^{22}$ dynes $= 3.1 \times 10^{-8}$ dynes. If we assume that each atom has a charge of one electron, then the attractive force between them would be $(e/r)^2 = 3.7 \times 10^{-3}$ dyne—quite sufficient to hold the molecule together even at a rotational frequency of 10^{13} per sec. Compare the so-called centrifugal force at such a speed with that for rotational speeds which we can produce, 10^4 per sec.

The average energy of vibration would be $2\,\pi^2A^2f_v{}^2m$, where A is the amplitude. If this is put equal to $(n + \frac{1}{2})\,f_vh$, we find

$$A = \frac{1}{\pi}\sqrt{\frac{(n + \frac{1}{2})\,h}{2\,mf_v}}.$$

From this formula, making n zero, we find that $A = 0.15 \times 10^{-8}$ cm. or 0.15 angstrom. The distance between the hydrogen and chlorine atoms is taken as 1.3 angstroms. Hence it appears from this computation that the amplitude of vibration of the hydrogen atom in the HCl molecule in its first vibrational state is about one-ninth of the distance between the atoms.

More Complex Atoms.

We gave ourselves easy problems when we decided to consider the vibration and rotation frequencies of the HCl atom. If we tried to work the corresponding problems for a slightly more complex molecule, sulphur dioxide SO_2, we would at once encounter difficulties. How are the three atoms arranged? Along a straight line O—S—O or O—O—S; or on the vertices of a triangle $_{\mathrm{O}}\mathrm{S}_{\mathrm{O}}$? If in the first mode, how many kinds of vibrations are possible? There might be opposite motions of the oxygen atoms towards and away from the sulphur; or the two might move both right, then left; or the sulphur atom might vibrate at right angles to the line of symmetry. We might work out approximate solutions for the various vibration, but in view of the uncertainty of our assumptions the best that we can hope for is to show what mode or modes of motion are in accord with the Raman spectrum. Fortunately we have other tests as to the structure of the molecule, one being the behavior of the gas in an electrical field. The evidence from the electrical source is that the molecule is polar and that the atoms are on the vertices of a triangle, as shown in the third case above.

If we were to consider the case of carbon tetrachloride, CCl_4, we would naturally assume that the four chlorine atoms were on the corners of a regular tetrahedron with the carbon atom at the center. There are obvious modes of vibration, a symmetrical motion of the chlorine atoms towards and away from the carbon, converging motions of three of the chlorines on the surface of a sphere of which the carbon is the center, other distortional motions. Again we can find approximate solutions of the vibration

frequencies. These operations have been only partially successful.

Very recently there have been ingenious attempts to build models of molecules so that vibration frequencies could be studied. Steel balls of masses proportional to the masses of the atoms separated by steel springs were set in vibration in definite directions by a motor of variable speed. The natural frequencies for various modes of motion were found by the condition of resonance. The results were very suggestive but not altogether satisfactory. Below are the "spectra" as worked out from the models for carbon tetrachloride, CCl_4, and chloroform, $CHCl_3$. These are compared with the known Raman spectra (Fig. 10–15).

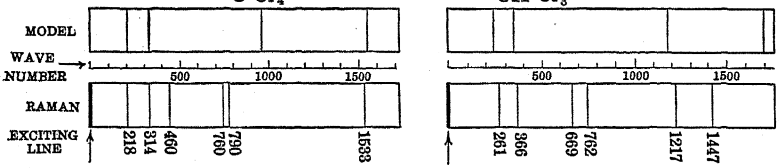


FIG. 10–15. Models of molecules composed of steel balls and springs can be made to possess frequencies similar to the Raman frequencies.

Perhaps it should be pointed out that some of the Raman lines are combination lines. For example, in the case of carbon tetrachloride the mean of 760 and 790 is equal to 314 + 460; and 1535 is (nearly) twice 775. So the Raman spectrum is likely to have more lines than are obtained from the model.

Raman Spectra for Similar Molecules.

If we compare the Raman spectra for various tetrachlorides, we find variations which are suggestive of the Moseley spectra of the X-rays of the elements. We arrange the spectra in order of atomic weight of the element in combination with the four chlorines as in Fig. 10–16.

	MASSES	RAMAN LINES			
$C\,Cl_4$	12+140	218	314	460	760
$Si\,Cl_4$	28+140	151	221	425	607
$Ti\,Cl_4$	48+140	120	144	386	497
$Sn\,Cl_4$	119+140	105	130	368	403

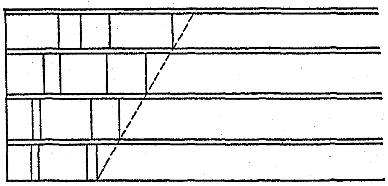


FIG. 10–16. Raman lines for similar molecules; the frequencies decrease as the atomic masses increase.

While it is clear that no simple relation like that arrived at by Moseley, $\nu = K(Z - k)^2$ (see Fig. 7–6), holds for these numbers, still it is evident that there is a dependence of these upon the atomic weight of the central atom; the numbers decrease as the atomic weight increases.

Detective Work.

Perhaps the student has noticed that by means of the Raman effect molecules are tricked into giving up information regarding their rotation and vibration frequencies, even though we might be unable to determine these by absorption or emission spectra. For the exciting line may be in the visible spectrum and the Raman lines quite near it. The spectral lines due to rotational frequencies ordinarily would be far out in the infra-red region, a region in which it is difficult to work since glass absorbs all such radiations. But in the Raman effect the small frequencies of the molecular spectrum become the differences of *frequencies* in the visible spectrum—or in that part in which we choose the exciting radiation. *Hence variations in molecular energies which would give emission or absorption lines in very long wave lengths, make themselves evident as differences in lines in the visible or ultra-violet region.*

4. Band Spectra Involving Electronic Changes.

As has been stated above, we accept the point of view that the energy of a molecule E is made up of E_r, E_v, and E_e, of rotational, vibrational, and electronic levels. E_r depends upon the mass and distribution of mass (I) of the atoms. As long as the electronic levels are unchanged, the moment of inertia, I, of a molecule remains constant, but not otherwise. Hence if the molecule absorbs a photon of large energy, we picture a change of energy thus: the initial $E_{e0} + E_{v0} + E_{r0}$ becomes $E_{en} + E_{v0} + E_{r1}$. Thus the frequency absorbed is due to changes in the three forms of energy. We now recall that the rotational energy is of the form $R(r + \frac{1}{2})^2$ or

$$\frac{(r + \frac{1}{2})^2 h^2}{8\, \pi^2 I}.$$

But I is now not constant. Hence when we take differences we have this result, the frequency absorbed $f = f_e + f_v + B \pm 2\, Br + Cr^2$, where f_e is due to electron change, f_v due to

vibrational change (it may be zero), and B and C are constants formed by making r change by $+1$ or -1. For some one electron change we have f = const. $\pm 2Br + Cr^2$ where $r = 1, 2, 3, \cdots$. Obviously if this relation be plotted, we have a parabola of two branches, a positive and a negative branch.

Figure 10–17 shows the graph of this relation as r changes from 0 to 17; the corresponding values of f are shown on the horizontal

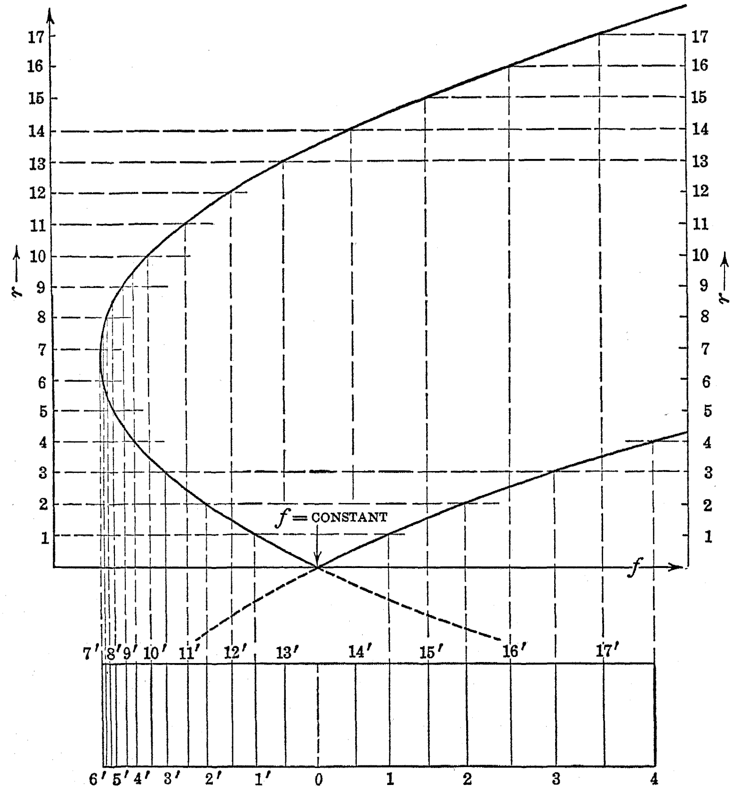


FIG. 10–17. The arrangement of the lines of a single band appears extraordinarily complex until they are pictured as points on two intersecting parabolas.

line. For $r = 0$ we have f = const.; this corresponds to the missing line. As r increases, f becomes less on the negative branch, rounds the vertex of the parabola at $r = 5^1, 6^1, 7^1, 8^1$, then increases; the lines in the band spectrum, at first strong and rather far apart, now become faint since the rotational levels are

now quite removed from the normal. On the positive branch we have strong lines for $r = 1, 2, 3, \cdots$, rather far apart with the fainter lines of the negative branch, $r = 14^1, 15^1, 16^1, 17^1$, in between.

Concerning the Specific Heat of a Gas

In Chapter 1 we derived the relation for the pressure of a gas in terms of the speed of the molecules. From that relation $p = (1/3)nmC^2$ and from the experimental relation between pressure and temperature, $pv = RT$, we arrive at a very clear picture concerning temperature and the motion of molecules. We finally state one of our conclusions thus: when the temperature of a gas changes 1° C., the *translational energy* (KE) of a molecule changes by 2.058×10^{-16} erg; or to leave it in symbol form, the KE per 1° C. = $(3/2)k$ (where k, 1.37×10^{-16} erg, is Boltzmann's constant and $= R/N$, R being the gas constant, 8.315×10^7 ergs, and N Avogadro's constant, 6.06×10^{23}).

Now a gas molecule may move in any direction and we may resolve its motion along three axes mutually at right angles. It has, as we say, three degrees of freedom. For one degree of freedom the energy above is $kT/2$. Finally we give this energy to every degree of freedom that the molecule may possess at a given temperature. A monatomic gas can have only translational motion. But a dumb-bell molecule, like H_2 or O_2, may also rotate about either of two axes at right angles to the bond. Such a molecule has 5 degrees of freedom—if it is rotating.

We may translate the above into other terms and state that the energy required to change N molecules of helium 1° C. is $(3/2)R$ and (since 1 calorie $= 4.18 \times 10^7$ ergs) this is equivalent very nearly to 3 calories. In other words, the specific heat per gram atom of a monatomic gas is 3, of a diatomic gas is 5.

But the phenomena we have been discussing—absorption, emission, and Raman spectra—show that the rotational energy of a molecule may have a value

$$\frac{(r + \frac{1}{2})^2 h^2}{8\,\pi^2 I}.$$

For a change from the zero to the first rotational state the energy change is $h^2/4\,\pi^2 I$. From the Raman spectrum we have the distance between the hydrogen rotational bands = 355 cm.$^{-1}$

and deduce $h/8\,\pi^2 Ic = 59.1$ cm.$^{-1}$ or $h^2/4\,\pi^2 I = 2.4 \times 10^{-14}$ ergs. In other words, the energy required to change the rotational condition of hydrogen from the zero to the first state is more than 100 times that required to change the translational energy 1° C. We should expect therefore that hydrogen at low temperatures would behave like a monatomic gas, that its specific heat per mole would be 3, and that as it rose in temperature it would acquire the properties of a diatomic gas. From 200° K. to 800° K. its specific heat per mole rises from 4 to 5. When the temperature gets very much higher, the molecular energy should be sufficient to excite the vibrational state. Then the number of degrees of freedom would be increased from 5 to 7, since in a vibration there is both kinetic and potential energy along the bond. At still higher temperatures there would be dissociation of the molecules into atoms. Every atom would have 3 degrees of freedom, and as the number of atoms is double the number of molecules the specific heat per mole is now 6. At still higher temperatures we would have electron rearrangements. If now we attempt to "heat" the gas, most of the energy would go to increase the potential energy of the electrons in the atom, but little of it in random translational energy of the atoms; that is, in temperature change. Figure 10–18 gives all of this in graphical form.

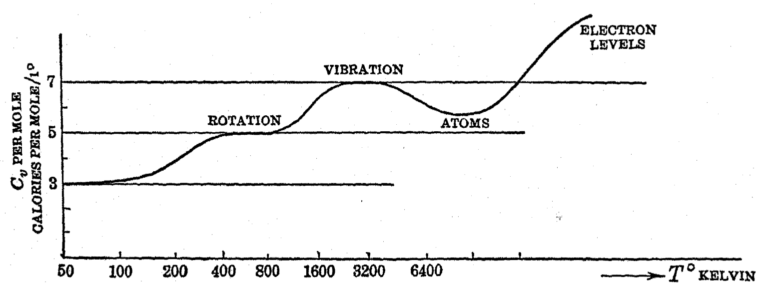


FIG. 10–18. According to the quantum theory, a hydrogen molecule cannot be given rotational energy until the temperature is about 200° K.; vibrational about 1600° K. Hence the specific heat changes as shown.

It is significant that in the above discussion we have not considered the heat required to change the kinetic energy of the electrons. But does the kinetic energy of electrons change with temperature? The old theory of thermionic emission assumed that it did. But facts connected with the specific heat of metals

lead to the opposite conclusion. Moreover, we saw in Chapter 8 that the Fermi-Dirac ideas were satisfactory; that the thermionic laws could be derived without requiring electrons to be appreciably disturbed at ordinary temperatures. Then what about the Bohr picture? There the electrons have definite speeds in their various orbits. These speeds do not change with temperature.

Summary—Molecular Spectra and Raman Effect

1. The rotation of molecules is quantized by making the angular momentum proportional to $h/2\pi$, as was done in the case of the electron in the Bohr orbits. The energy in the successive states $= R(r + \frac{1}{2})^2/8\pi^2 I$ or $Rr(r + 1)/8\pi^2 I$. But a special rule is necessary for symmetrical molecules; r changes from odd to odd or even to even. Note that the energy required for a quantum change, being inversely as I, becomes large as I becomes small. Hence we would expect that the energy required to set into rotation the hydrogen molecule or a simple molecule of which hydrogen is a constituent would be large and would be evident in phenomena other than spectra. The data for the specific heat of hydrogen support this view.

2. The vibration of molecules is quantized merely in terms of the frequency of vibration, E varies as hf. There may be vibrations due to extension, shearing, tension, etc.

Problem. The center of the lowest vibration-rotation band for HCl is at 0.00356 mm. Show that it takes 0.35 e.v. to excite this band.

CHAPTER 11

RADIOACTIVITY: SPONTANEOUS TRANSMUTATION OF ELEMENTS

The Story of Radium.

We go back to the wonder years 1895 to 1898. X-rays had just been discovered. They produced phosphorescence in glass and other substances. Is phosphorescence in various materials due to a radiation like X-rays having its source in the materials? Henri Becquerel of France, early in 1896, asked that question. He enclosed a photographic plate in a light-tight box and placed on it a phosphorescent salt, uranium potassium sulphate. After exposure for several hours, the plate was slightly darkened. Further experiment showed that uranium was the source of the (Becquerel) rays. Phosphorescence was not necessarily associated with the phenomenon. Madame and Pierre Curie took up the search. They treated a ton of pitchblende, constantly testing the various residues by their *ionizing power* and obtained a substance, which Madame Curie named polonium, which was many times as active as uranium. Continued treatment produced (in 1898) a compound of bromine, which they called radium bromide, which was about two million times as active as an equal weight of uranium.[1] Thus began the story of radium.[2]

And what a story! Perhaps a million pages have so far been written in telling the details. It has profoundly influenced all branches of science, all philosophy. And it continues to provide the most exciting news of the physics of today. The latest

[1] The amount of radium obtained by the Curies was about one ten-millionth of the ore treated. Now radium ores which will yield by commercial treatment one part of radium in two hundred million of ore are considered satisfactory. Recently (1934–35) physicists have become interested in new forms of hydrogen, H^2 and H^3. After boiling down 75 tons of water, a half-gram of water has been obtained *rich* in H^3, that is, containing one part of H^3 in ten thousand of all the hydrogen present. This would make the concentration of H^3 in ordinary water one part in ten thousand million, or 1 in 10^{10}.

[2] The most important text in English is *Radiations from Radioactive Substances*, by Rutherford, Chadwick, and Ellis. A substantial report containing much data is the Bulletin of the National Research Council No. 51 (1929), *Radioactivity*, by Kovarik and McKeehan.

chapter is called Artificial Radioactivity. It is a special case of Transmutation of the Elements.

Three Kinds of Radiations.

Let us suppose that we have a very small amount of radium bromide in a narrow well in a lead block; above and around it a photographic film; a magnetic field variable at right angles to the paper and directly away from us. It will be seen that for a very strong field we have a stream of particles deflected towards the left; a weak field is sufficient to deflect a stream, or streams, to the right; another "stream" is undeflected. We have the alpha (α), beta (β), and gamma (γ) rays. It will later be shown that from radium there come alphas of several velocities, betas of many velocities, and many gammas, but all alphas are helium nuclei. Occasionally an alpha particle picks up an electron and becomes a helium atom with a single positive charge. All betas are electrons, all gammas are very short waves like X-rays.

The Radiations from Radon, the Gas from Radium, Are Analyzed.

Radium gives off a gas, an emanation, now called radon. The "standard" or unit in radioactivity is the "curie." It is measured by the gamma rays which come from the amount of radon in equilibrium with one gram of radium. Obviously the millicurie (1/1000 curie) is the ordinary unit. Let some of this gas be pumped off and confined in a slender *very thin-walled* glass tube S. This is placed as shown (Fig. 11–1) in a glass tube T

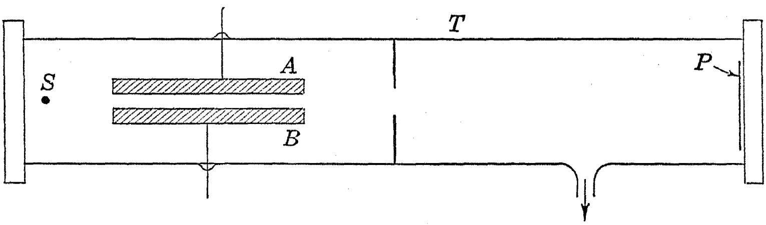


FIG. 11–1. The positively charged particles (α rays) from a radioactive source S, passing between the electrified plates A and B, are dispersed and registered on a photographic plate P.

in which are two metal plates, A and B, and a photographic plate P wrapped in aluminum leaf. The tube T is pumped out and a potential of a few thousand volts is applied to A and B. After

an exposure of five hours, the potential on A and B is reversed and another exposure made. The lines obtained on the plate are shown in Fig. 11–2 a. The heavy central line is the undeviated line, the first line on either side is due to the swiftest (as determined by this experiment) α particles, coming from a radio-

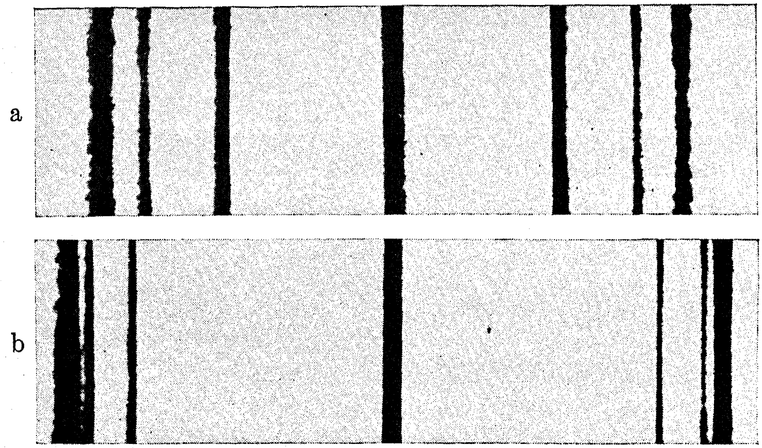


FIG. 11–2. The central line is due to undeflected particles. In a, an electrical field causes radon alphas of three different velocities to form three separate lines; in b, the separation is due to a magnetic field.

active constituent, radium C; the next line, α rays from radium A; and the outer line the α rays from radon itself. All gamma rays are undeflected, all β rays are bent out of the picture. The application of an intense magnetic field gives the picture Fig. 11–2 b.[1] From these two pictures we are able to measure the velocities of the various α rays and the ratio e/m of the charge to the mass of the carrier. The value of e/m comes out the same for the three rays, viz., 4820. Now it was shown (page 22) that when a charge of one unit is attached to a hydrogen atom (when a hydrogen atom loses its electron), the ratio of e/m is 9570, if e is measured in e.m.u. This would make a ratio of 4820 correspond (nearly) to one electron carried by a hydrogen molecule or of two units of charge carried by a helium atom. In the very early days, there was some doubt as to which of these two views was correct,

[1] The displacements of the three lines in Fig. 11–2 b differ from those in a because in b they are inversely proportional to the velocity, while in a they are inversely proportional to the velocity squared.

but Rutherford quickly devised experiments which decided the question. It has been completely established that the α particle is the helium atom with two positive units of charge. This is also the accepted picture of the helium nucleus.

Very Swift Particles Pass through Thin Glass. The Alpha Particles Are Identified.

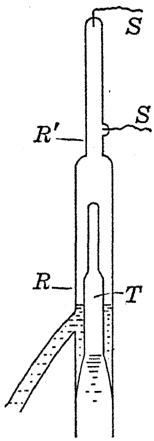


FIG. 11–3. Alpha particles shot out by radon gas in T have such great speeds that they penetrate the thin walls of T and are identified as ionized helium atoms by the spectrum in SS.

While there are several experiments which make practically certain the exact nature of the α particle, there is one which stands out as convincing. It is known as the Rutherford and Royds experiment (1909). Radium emanation, radon, is introduced into a glass tube T and forced up by means of mercury, as shown in Fig. 11–3. The top part of T is drawn out into a very thin-walled tube. This is inside another glass tube originally exhausted. After a couple of days the mercury in the outer tube R is raised to R^1 and an electric discharge is passed between S, S. The spectrum of helium is seen. Before radon was admitted to T, no spark would pass, the vacuum was too high. It appears that particles having the great speed of 1.92×10^9 cm./sec. (about 12,000 miles per second) are able to pass through the thin glass walls of T. If helium gas instead of radon is introduced into T, no spectrum of helium appears. The slow-moving helium atoms cannot penetrate even thin glass walls.[1] Recall that swift hydrogen atoms can pass through thin platinum into an X-ray tube (Fig. 7–1 b).

[1] It is a rather astonishing fact that helium atoms will pass slowly through thick (1 mm.) plates of quartz either fused or crystalline.

There is another illustration of the penetrating power of high-speed particles which might be here recorded. A mechanic working with oil in a tank under high pressure found that a very fine jet of oil was escaping. This jet struck him on the bare arm. He brushed off the oil and thought nothing about it. Later, experiencing great pain in his arm, he was examined by a surgeon who probed into his arm and found an accumulation of oil near the bone.

A fine high-speed jet of water striking the back of a person's hand may produce an accumulation of water under the skin of the palm. This penetrating power of high-speed jets of liquid may possibly be used as a mode of injecting serums.

The authority for the above statements is Dr. H. B. Williams of Columbia University.

The Radiations Discharge an Electroscope. Thus Radium Was Discovered.

It has been shown that we can separate the various radiations given out by radium by means of electric and magnetic fields. And we can detect or measure them by photographic and ionization methods. The photographic method is relatively slow. The ionization methods are quick and may be made extremely sensitive. They are of various forms. The simplest form is shown in Fig. 11–4. The brass rod of a gold or aluminum leaf electroscope is well insulated from the case by an amber or sulphur plug E. A radium or thorium salt is placed in a shallow dish R. The electroscope is charged by a battery of small cells or by an electrified rod by means of the contact C. The deflection of the leaf is read by a low-power microscope and the readings are plotted against time. If the electroscope has been calibrated, the rate of change of these readings gives the (relative) ionization current at any time.

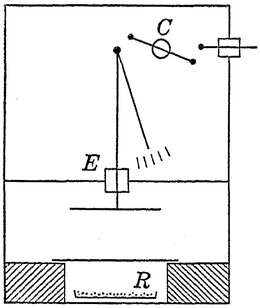


FIG. 11–4. A simple electroscope arranged for the study of radiations from R.

The Most Sensitive Detector—The Geiger-Müller Counter.

But there is a variation of the ionization method which has come into large use in recent years. Originally due to Rutherford and Geiger, it has been extended by Geiger and his associates so that it is now known as the Geiger-Müller counter method of detecting a single α, β, or γ "particle" as it enters or passes through a chamber.

Figure 11–5 shows one form of the G-M counter (see also Appendix 13–1 and 13–2). Inside a small brass cylinder C is a rather small well-insulated steel needle with its fine point ending near a hole in the cylinder. It generally is best to cover the hole with the thinnest aluminum foil so as to keep stray ions out of the cylinder. A potential of about 2000 volts is applied between the cylinder and needle through the large resistances shown in the figure. This potential ought to be nearly large enough to produce

a spark discharge between point and cylinder. Critical factors are: (1) the steel point—it must not be too sharp, it must be clean; (2) the high resistance (about 10^{10} ohms) connecting the

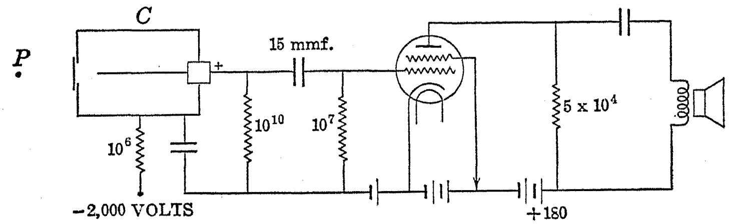


FIG. 11–5. Alpha particles from polonium at P shooting through a thin window into C produce a momentary rush of current. The electron tube amplifies the effect and the loud speaker barks. Thus we count the number of α particles (or β or γ) entering the cylinder.

needle to the earth; and (3) the small condenser 10 to 30 micro microfarads. When an alpha particle enters the chamber through the thin window W, the air is ionized along its path as was shown in the cloud tracks. There is a rush of electrons and ions towards the needle, of ions towards the cylinder. The intense field in the neighborhood of the sharp point greatly assists this action as it helps to produce ionization by impact or collision. The needle experiences a sudden drop in potential and this is partly passed on through the small consenser to the grid of the first amplifying tube, and is finally made audible by a loud speaker or is registered by a mechanical counter. The drop in potential of the needle prevents the continuation of the current. It is the function of the high resistance to help maintain the low potential in the needle for a very short time, perhaps 10^{-5} seconds, then after the rush of ions is over, to restore the potential so that the chamber is again in condition to be disturbed by the entrance of another alpha particle.

The most convenient source of α particles is polonium, Radium F. This is a commercial article, ordinarily in the form of a thin deposit on nickel, copper, or silver, and we can show that practically no β or γ rays come from it. If it be placed at P (Fig. 11–5), two or three centimeters in front of the window, a hail of taps is heard in the loud speaker or in the mechanical counter; if it be placed more than 4 centimeters away, no taps are heard. The taps start at about 3.9 cm. for normal barometric pressure.

Thus we find the *range* of the α particles from polonium. If we place a sheet of the thinnest cellophane (0.001 inch) in front of the window, we must bring the polonium within a centimeter of it before taps are heard. Two such sheets of cellophane stop the α particles. Thus we can measure the stopping power of gases or of materials which we can obtain in thin strips.

The Number of α Particles from One Gm. of Radium per Sec. $= 3.7 \times 10^{10}$; Determined by Counting a Few.

By means of an experiment which was the forerunner of the one just described, Rutherford and Geiger (1908) counted the number of α particles which entered a chamber per second from a known amount of radium. Measuring the total charge, the charge per particle was computed to be 9.3×10^{-10} e.s.u. Assuming that this was twice the electron unit, the latter came out to be 4.65×10^{-10} e.s.u. This was approximately the value obtained by other methods and was the earliest accurate method of measuring the charge on the electron. Moreover, this measurement helped to establish the view that an α particle carries two units of charge and therefore that it is a helium nucleus—not a hydrogen molecule with one unit. By means of this experiment we are able to compute the number N of α particles coming from a gram atom [214 gms.] of radium C^1 per second. As there are 6.06×10^{23} atoms in this gram atom, we know the *fraction* of the atoms that are breaking up per second. The *fraction* is a constant, but the *number* is constantly decreasing unless the source is renewed. $N = 3.7 \times 10^{10}$.

We "Weigh" a Minute Amount of Radium Scattered through a Ton of Ore—by the γ Rays.

We have just used the phrase, α particles from a known amount of *radium*. But we have just shown that such particles [1] cannot pass through even a tenth of a millimeter of aluminum. Obviously they cannot emerge from the deeper layers of radium in bulk. Metal surfaces covered with the thinnest possible layers of radioactive materials must be used for α ray sources. How then can we measure the amount of radium in such a thin film and in such a small amount? The answer is, by the γ radiation.

[1] The most energetic (ordinary) α particles originating in the radium family come from RaC^1, but even these are stopped by 0.06 mm. of aluminum.

This is not appreciably absorbed by the thin walls of a container. Consequently we are able by this electrical method to *weigh* a small amount of, or to determine the mass of, radium even if it is distributed through a rather large mass of inactive material.

An Alpha Particle Is Most Destructive Near the End of Its Range.

The counting method shows that the α particles from a certain source have the same range. (This is true only to a first approximation. See Tables I and II.) Otherwise there would not be an abrupt change from *counts* to *no* counts. The cloud tracks demonstrate the same fact. Moreover, these tracks show that near the end, except for the last two or three millimeters, a track thickens, the ionization is intense. More definitely the way in which the ionization changes along the path of the particle can be shown by the following experiment. A source of α rays is at S (Fig. 11–6). An aperture in a screen definitely limits the rays. The ionization current is measured between two pieces of wire gauze, A and B. The distance from S to A is altered and again the current is measured. Or all of these may be inside a long glass tube and the density of the air or gas changed. The equivalent distance in standard air is then computed. (It equals $xp/760$ where p is the pressure in mm.) The result obtained is shown in Fig. 11–7. The ionization is seen to increase towards the end of the path, reach a maximum, then fall quickly to zero. Reference will be made to these curves when we discuss the nature of cosmic rays.

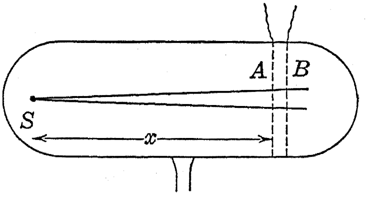


FIG. 11–6. By changing the pressure of the gas in the tube we can find the ionizing power of an alpha from point to point along its path.

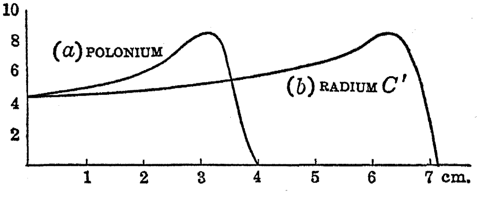


FIG. 11–7. An alpha is most effective near the end of its path, then rather suddenly ceases to ionize. The longer range is due to alphas of 7.68 million electron volts (M.E.V.), the shorter to 5.25 M.E.V.

Perhaps the student has detected an error in the logic in the preceding paragraph. It was there stated that the way in which the ionization along the path of a particle changes can be shown by the experiment which leads to the curves of Fig. 11–7 (*a*) and (*b*). But this experiment shows the ionization for a *great number* of *paths*. But suppose the paths are not all equal in length. We would then have a falling off at the end of the curve due to statistical straggling. By means of the cloud tracks we can find the variation in the ranges—then we can correct the curves and obtain the ionization along a single path. The correction does not greatly alter the curve.

One other exceedingly important fact may be established by the experiment above. By measuring the current between A and B, we measure the number of ion pairs formed by all the α particles which pass through the screens. Given a known source S, the total number of particles going through the aperture can be computed. Hence the number of ion pairs formed by an α particle all along its path can be determined. Since the velocity of ejection of a particle is known, also its mass, its total kinetic energy is known: *therefore the energy required to produce one ion pair.*

We Know Several Facts about an α Particle.

Now we may gather together some facts which have been established by the experiments so far discussed. The number of α particles from 1 gram of radium C^1 is about 3.7×10^{10} per sec. The velocity of that particle is 1.92×10^9 cm./sec.; its initial energy is 12.1×10^{-6} erg $= 7.68 \times 10^6$ electron volts; its range in air is 6.9 cm.; the number of ion pairs produced by it is 2.2×10^5; energy required to produce one ion pair in air is 35 volts. Attention is called to the fact that the energy required for ionization by an α particle is about twice that for a very slow electron; that is, it is about twice the ionization potential. Why?

The Law of Decay, of Diminishing Returns. The Exponential Law.

But what about the activity of radioactive material? How does it change with time? Polonium gives a clear answer to these questions. Let us observe its activity, either by the ionization or counting methods, at intervals of a month and plot the

results. We have the curve of Fig. 11–8. The activity decreases in geometrical fashion. Whatever its activity is at any time, it is reduced by one-half in 136.3 days. In 1363 days hence it will be $(\frac{1}{2})^{10}$ or 1/1024 of what it is now. In 2726 days it would be $(\frac{1}{2})^{20}$ or less than one one-millionth of its present activity. The curve[1] is given by $I = I_0e^{-\lambda t}$ where I_0 is the activity when we take our first observation, when $t = 0$. The time required to reduce the activity one-half is called the half-life[2] or the period. It obviously means that of a large number of molecules of polonium, one-half will in 136 days each have ejected an α particle and changed over to another material, in this case radium G or *lead.* Or we might state the matter in another way—the probability is one-half that any atom of polonium will eject an α particle in the next 136 days. As far as any atom is concerned, there is nothing gradual about this process. *It may experience sudden death in the next second* or *it may live forever. It is a matter of chance.* But when we deal with a great number of atoms the law of chance becomes a law of precision; one-half of them will change into lead in 136 days. In Chapter 14 we discuss the discovery, in 1934, of artificial radioactivity. This exponential law of decay helped to make the phenomenon evident.

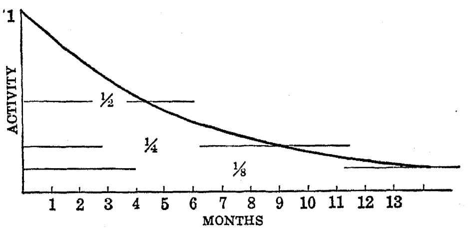


FIG. 11–8. The law of radioactive decay. (See Appendix 1–1 and 11–1.) Whatever the activity is now, it will be reduced by one-half in a certain time; 136 days for polonium.

Similarly the number of α particles entering a chamber from a source during a short time will be a matter of chance. If the number is small, there will be relatively large fluctuations from second to second. But when the number is large it becomes a fairly definite quantity. Attention will be called to these statistical variations when we discuss cosmic rays.

[1] A similar law holds (Appendix 1–1) for the absorption of light $I = I_0e^{-\mu x}$ where x is the thickness of the absorbing material passed through; also for the density of a gas with height above the earth $d = d_0e^{-kh}$.

[2] The *average life* of a great number of radioactive atoms is equal to $1/\lambda$; the period or time required to reduce the activity one-half equals $(\log_e 2)/\lambda$ or $0.693/\lambda$ (Appendix 11–1).

The Law of Growth.

Polonium, RaF, changes into lead of the non-radioactive sort. Had its product been radioactive or had it been composed of different radioactive products, the curve would not have been so simple. For example, radium gives off the exceedingly active gas radon. (It is 100,000 times as active as an equal weight of radium.) If a bare wire be inserted in this gas and charged negatively for several hours, then removed, it is found that the wire has become radioactive and that the *decay* curve is not a simple exponential curve but a complex one. The wire must be examined immediately upon removal, because one of the products, called radium A, giving off α particles of moderate energy, has a half-life of 3.05 minutes; another, radium B with a half-life of 26.8 minutes, gives off β particles; and a third, radium C with a half-life of 19.7 minutes, gives off very energetic α particles. One would think that this wire would have lost all its activity in a few hours, but a new phenomenon appears. Though the old α and β radiations have dropped nearly to zero, new α's and β's are slowly increasing. In Fig. 11–9, curve I shows the manner of decay of RaA, B, C, and curve II the slow rise of RaD, E, F.

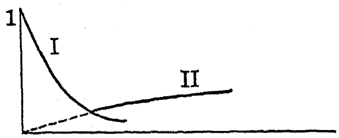


FIG. 11–9. The decay of one element causes the increase of its product.

Thus is the story of radium written, with experimental fact piled on fact, with critical analysis at every step, and with scientific imagination constantly a guide. Though the story is still unfolding, the data so far obtained may be partly summarized in the following radioactive tables.

Radioactive Genealogies.

In the fifth column of Tables I and II is given the period or half-life of the element. The extremely short periods are computed (almost guessed at) from a formula connecting range and half-life for those cases for which both can be experimentally determined. The greater the range, the smaller the period; thus

$$\text{Log}\,\lambda = a + b\log R \text{ and } T = \frac{0.692}{\lambda}.$$

In the sixth column is given the energy of the alphas in million electron volts (M.E.V.)[1] or the ranges of energies of the betas in the same unit. The energy of disintegration must take into account the energy of recoil of the nucleus, as well as that of the alpha. To get this we must add about two per cent to the energy of the α particles.

In the seventh column is indicated, for a few cases, the number of different kinds of alphas coming from the element. The nucleus must have at least that number of energy levels.

Some feebly radioactive elements—tellurium, potassium—are placed at the bottom of the uranium series.

TABLE I

Uranium Series	Atomic Number	Atomic Weight	Disintegration Particle	Range in Air, cm.	Period or Half-Life	Energy in M.E.V.	Number of Alphas
Uranium I	92	238	α	2.70	4.5×10^9 yrs.	4.05	
Uranium X_1*	90	234	β		24.5 days	.12	
Uranium X_2	91	234	β		1.14 min.	1.31	
Uranium II	92	234	α		10.6 years	4.63	
Ionium	90	230	α	3.28	7.6×10^4 yrs.	4.55	
Radium	88	226	α	3.19	1600 years	4.74	2
Radon	86	222	α	3.39	3.82 days	5.44	
Radium A	84	218	α	4.12	3.05 min.	5.97	
Radium B	82	214	β	4.72	26.8 min.	0.037 0.39	
Radium C	83	214	β		16.7 min.	0.042 0.31	2
Radium C′	84	214	α	6.97	10^{-6} sec.?	7.68	12
Radium D	82	210	β		25 years?	0.019 0.071	
Radium E	83	210	β		5 days	0.08 4.58	
Radium F	84	210	α	3.92	136 days	5.25	
Lead	82	206	Stable				
Potassium	19	41*	β				
Rubidium	37	87*	β				
Tellurium	81	207* 208* 210*	β				
Neodymium	60	144					
Samarium	62	150					

* Elements of the same atomic number are isotopes.

[1] In the physics journals usage now tends toward MEV or even MV. But the non-technical readers of this text may prefer the symbol above since it suggests three separate words.

TABLE II

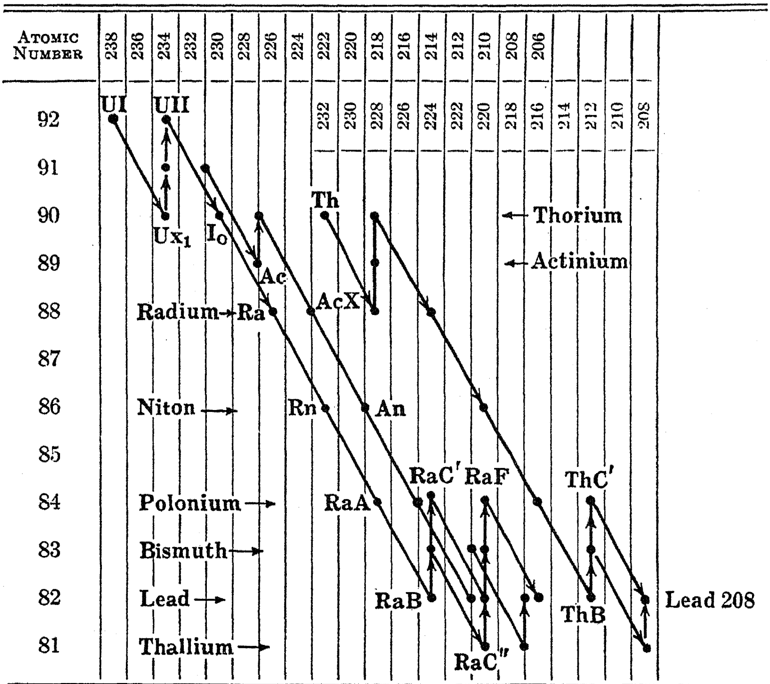
	Atomic Number	Atomic Weight	Disintegration Particle	Range in Air, cm.	Period or Half-Life	Energy in M.E.V.	Number of Alphas
Protactinium	91	231	α	3.67	1.25×10^4 yrs.	5.01	
Actinium	89	227	β		13.4 years		
Radio act.	90	227	α	4.67	18.9 days	5.92	11
Actinium X	88	223	α	4.37	11.2 days	5.66	3
Actinon	86	219	α	5.79	3.92 sec.	6.82	3
Actinium A	84	215	α	6.58	0.202 sec.	7.40	
Actinium B	82	211	β		36.0 min.		
Actinium C	83	211	$\alpha\beta$	5.51	0.005 sec.	6.61	2
Actinium C′	84	211	α	6.5	0.001 sec.?		
Actinium C″	81	207	β				
Actinium D / Lead	82	207	Stable				
Thorium	90	232	α	2.90	2×10^{10} yrs.?	4.23	
Mesothorium I	88	228	β				
Mesothorium II	89	228	β		6.13 hours	0.038 / 7.54	
Radiothorium	90	228	α	4.02	1.9 years	5.35	2
Thorium X	88	224	α	4.35	3.64 days	5.65	
Thoron	86	220	α	5.06	55 sec.	6.24	
Thorium A	84	216	α	5.68	0.15 sec.	6.74	
Thorium B	82	212	β		10.6 hours	0.1 / 0.28	
Thorium C	83	212	$\alpha\beta$	4.79	1.0 hour	6.12	6
Thorium C′	84	212	α	8.62	10^{-11} sec.?	8.76	
Thorium C″	81	208	β		3.2 min.	0.02 / 1.21	
Thorium D / Lead	82	207.77	Stable				

We start with uranium, the element of highest atomic number [1] 92, of greatest atomic weight 238. Occasionally an atom fires out an α particle; the chance is even that it will do so in the next *five billion years.* Having lost a helium nucleus, its atomic number becomes 90 and its atomic weight 234. And so we continue from element to element. When an α particle is ejected, the changes are as above; when a β particle, the atomic weight is not appreciably changed but the atomic number *increases* by one. Ruther-

[1] It is true that Fermi of Rome announced (1934) the discovery of element No. 93; but there was some doubt about his interpretation of an experimental result. Later it appeared probable that he was right.

TABLE III

See also Figs. 14–10 and 14–11.



Oblique arrows indicate α particles, vertical lines β particles. The Thorium series is stepped over to avoid confusion.

ford and Soddy made an extraordinary leap forward when they announced the *law of radioactive change*, developed chiefly by Soddy, that a radioactive atom in expelling an α or β particle changed to an atom of another element. After the idea of the nuclear atom was evolved, it was seen that those particles must come from the nucleus. One word of caution should here be given. With our present picture of an atom, an α particle must, and an electron may, come from the nucleus[1]; but an electron may also be one of the extra nuclear kind. We have many lines of evidence to support the view that when the expulsion of an elec-

[1] It has been stated (see footnote, Chapter 6, page 103) that there are no electrons in the nucleus. Then how do we justify the above statement? We say that a neutron may be transformed into a proton and an electron. See discussion on page 326.

tron causes a change in the element, the electron which is driven out comes from the nucleus. *It is only when the nuclear charge changes that we have transmutation or a change in the element.* Later we shall show that it is frequently true of radioactive bodies that electrons of several different energies emerge, some of which come from the nucleus, some from the outer energy levels, and that α particles of several different velocities may emerge from one atom—all from the nucleus.

The Idea of " Isotopes " Came from These Phenomena.

An inspection of the data in Table I shows that we may have several different atoms of the same atomic number but of different atomic weights. In other words, we may have a number of isotopes. Pick out, for example, all the products of atomic number 82; the atomic weights are 206, 207, 208, 209, 211, 212, 214; these are all *lead*, some forms changing rapidly, some slowly like RaD, some stable, of atomic weights 206, 207, 208. (The ordinary chemical value is 207.22.) It is of importance here to note that the discovery of isotopes came as a result of study of these radioactive products.

Concerning Beta and Gamma Rays.

Beta rays are electron projectiles. When these projectiles strike matter, X-rays are given off. Gamma rays are like X-rays. When a radioactive atom fires out a β ray, we are likely to have X-rays (γ rays) as companion radiation. But again, X-rays absorbed by matter may cause electrons to be ejected with (nearly) the same energy as the X-rays. The Einstein quantum law holds. That is, change in energy of the atom equals hf where f is the frequency of the X-ray.

Beta and Gamma Ray Spectra or Spectra Produced by These Rays Striking Matter.

Let us recall the Bohr picture of the atom, a nucleus with electrons outside in definite energy states. If one of these electrons, say one in the least energy state, is removed from the atom, there may be a series of readjustments, resulting in the emission of quanta of various frequencies. And vice versa, if γ rays are absorbed by atoms, electrons of different energies may be ejected. It may be expected then that both β and γ radiations may be

complex. Let us suppose that a γ ray, coming from the nucleus, ejects an electron from the minimum energy state, the K shell in the old picture. Then the energy, E, of the emerging electron or β particle is given by $E_1 = hf - K$ where f is the frequency of the γ photon and K is the energy of an electron in the minimum state. Similarly for other electrons. Then we may have β rays of energies $hf - K$, $hf - L$, etc. The *differences* between these energies would be $L - K$, $N - K$, etc., and these are the energies of the $K\alpha$, $K\beta$, X-rays of the element.

To test this experimentally we place at S, Fig. 11–10, a fine wire, the surface of which is covered with a radioactive film. A photographic plate at P, lead block L. The air is pumped out and a magnetic field is set up at right angles to the paper. Electrons emerging from S will be bent in circular arcs of radius r given by $mv^2/r = Hev$, or $Hr = mv/e$. Electrons of one velocity will be (nearly) focussed at a point in P; of another velocity, at another point. The photograph in general will show a number of lines, a "beta ray spectrum" proving that electrons of different energies emerged from S. It is found that the *differences* of these energies are those of the X-rays of the element. What element? Suppose radium B is the source. The X-rays are those of radium C. And this proves that the disintegration β particle was ejected from radium B, then the γ ray, then the electrons from the K and L shell of radium C. It also proves that electrons giving us line spectra come from outside the nucleus.

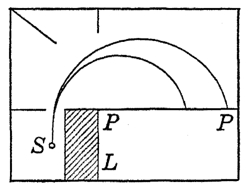


FIG. 11–10. The β rays from a source S are bent by a magnetic field so as to give a "spectrum" on the photographic plate PP.

(Had the radioactive film around S been surrounded by a very thin cylinder of platinum, some electrons ejected would have been from the K and L . . . shells of platinum.)

In addition to these groups of electrons of definite energies, there are high energy disintegration electrons forming a (limited) continuous spectrum.

Once more we desire to point out that a physicist is apt to become a detective of the first order. Think of identifying the origin of electrons when various elements are present in the source, of fixing the order of events, disintegration, γ-ray emission, β-ray ejection, from certain levels.

Concerning Complex α Radiations.

For many years it has been supposed that the following general law had been established. When disintegration takes place in an atom due to the expulsion of an α particle, only one kind of particle is ejected; that is, except for statistical straggling, all the alphas from one element have the same energy. Very recently we have learned that that law is not precise. There may be a whole "spectrum" of α particles emerging from one atom.

Suppose we have a magnet of very large pole pieces (2 feet diameter) and can obtain between the poles very large field strengths (10,000 gauss). Between the poles we place a radium source, screens to limit the rays, and a photographic plate. We arrange it so that the air can be pumped out between the pole pieces. An α-ray spectrum is obtained. Instead of having pole pieces terminating in flat circular discs facing one another, we may dispense with the central part of the discs and have narrow rings (Fig. 11–11), 5 cm. wide, 40 cm. diameter, facing one another. An α-ray source is placed near one end of a diameter, an ionization chamber at the other end. The air is pumped out and the magnetic field is increased until the ionization chamber *detects* rays which have been bent around the semicircle. Only alpha rays of a very definite velocity can enter the ionization chamber for a particular value of H, the magnetic intensity. As H is changed, other groups are detected.

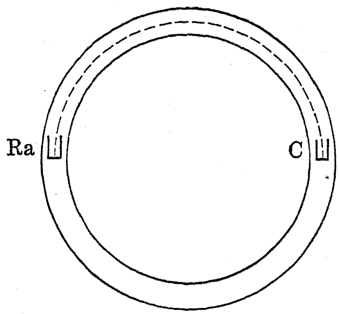


FIG. 11–11. By using as pole pieces very large rings, the Cambridge workers have found that RaC^1 gives out alphas of twelve different energies.

This analysis shows that RaC^1, for example, which according to our table gives out one kind of α ray of 6.97 cm. range and of 7.68 M.E.V. energy, now is known to send out *twelve different alphas* with energies up to 10.6 M.E.V.! But the *intensities* of these radiations are very small compared with that of the usual 6.97-range particle. In other words, the *number* of α particles coming from RaC^1 of energy 7.68 M.E.V. per second may be one

hundred thousand or a few million times the number of the other newly discovered alphas. Radium itself gives out two groups, which differ in energy by[1] 0.185 M.E.V. Now it is known that a gamma radiation of 0.185 M.E.V. comes from radium. We are led then to this picture—the radium atom differs from its daughter, radon, by a definite energy W; an alpha particle may escape from the radium atom with all this energy or an alpha particle and a gamma ray of 0.185 M.E.V. energy combined may carry it away.

The Various α Radiations Make Known the Various Energy Levels of the Nucleus.

And now we come to a new view regarding the nucleus of an atom of a radioactive element. It has structure; it has energy levels. These are made evident by the different alphas which it sends out and by the different γ rays which accompany them. Moreover the energies of the γ photons are equal to the differences between the various energy levels of the *product* nucleus. This is illustrated in Fig. 11–12. The energy levels as determined by the alphas from ThC′ are shown in M.E.V. The *differences* should represent the energies of the γ photons which accompany the alphas. Except for a few lines which are missing, the γ rays from ThC″ agree with these differences to a rather close accuracy.

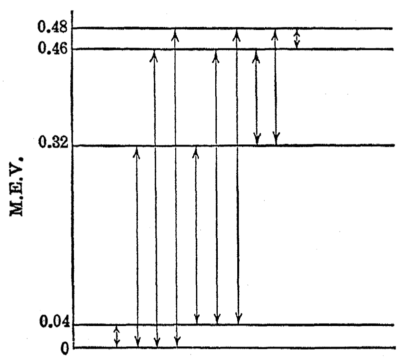


FIG. 11–12. The nucleus of an atom (ThC′) has definite energy levels; the energy differences are measured in M.E.V.

Summarizing, we see that α's have definite energy levels, γ's have energies equal to the differences in the α's. But—*the spectra of the nuclear β rays are continuous.* The curves of Fig. 11–13 show the limits of energies of the β rays from various sources. It is seen that there is an upper limit of energy for each element. A beta particle from RaB, for example, may have an energy less than 600,000 volts. The nuclei of RaC therefore

[1] Nearly, leaving out the energy of the recoiling nucleus.

may differ in energy by any amount less than this. But the α and γ rays from RaC are all definite. There is no evidence of a continuous variation in their energies. These two sets of facts appear clearly to contradict each other—and we have no explanation. Perhaps we might find comfort in this—if our experimentalists are right, the beta disintegration particles refuse to be members of a stratified society, they are individualists!

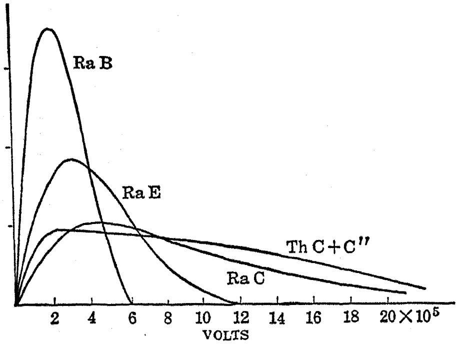


FIG. 11–13. Here is a conundrum. Unlike the α rays, the electrons coming from the nucleus give a continuous spectrum!

Is the Nucleus, Which Has a Diameter of About 10^{-13} Cm., as Complex as a Grand Piano?

Let us return to Fig. 11–12. The energy levels are slightly suggestive of electron levels for atoms. But the latter may differ by only a few volts, the nuclear levels must differ by a thousand or million electron volts. The atom diameter is of the order of 10^{-8} centimeter; the nucleus is of the order of 10^{-13} centimeter; a ratio of 100,000 to 1. Yet we have energy levels in the nucleus indicating complexity.

In dealing with spectra, we frequently called attention to the curious restriction of freedom of an atom when energy is radiated; the angular momentum changes only by one Bohr unit ($h/2\pi$). The photon must acquire that unit. A gamma ray is a photon, it must have a Bohr unit of angular momentum. Hence the nucleus must have lost or gained one unit when the gamma ray was ejected. It must have spin. It must have excited states. But discovering these excited states is part of the physics of today (1934). No Balmer has arrived to give a formula for these states. There is no Rydberg constant for a nucleus. Wave mechanics is setting forth a number of conclusions, some of which seem to be correct. But a vast number of facts must be disclosed before we can generalize about the structure of a nucleus. In the next chapter we present and discuss more facts.

CHAPTER 12

THE BEGINNING OF ARTIFICIAL TRANSMUTATION

Artificial disintegration of the elements.——Alpha particles were the first projectiles to be used in smashing atoms.——Then atomic nuclei hurled by man-made devices.——The discovery of the Neutron, of Heavy Hydrogen (Deuterium).——New mammoth generators of high potentials or of high atomic velocities.

We Propose to Fire Atomic Projectiles at Atomic Targets.

We have seen that the alpha particles from certain radioactive elements possess enormous energies. Those from RaC′ for example have a speed of about 12,000 miles per second. The greatest speed we can give to any ordinary kind of projectile is considerably less than one mile per second. Since energy is proportional to the square of the velocity (up to this value), we see that an alpha particle may have an energy a few hundred million times that of an equal mass projected by our most powerful gun. The temptation is strong therefore to use alpha particles as projectiles and to direct them at other atoms.

How Many Hits Should We Make?

From one of the experiments described in the last chapter we learned that an alpha particle would "ionize" about 220,000 air atoms. How many should it ionize? What is the diameter of an alpha particle? Of an air molecule? The student may refer back to earlier chapters in attempts to answer these questions. Just now we continue. How are gases other than air affected by these particles? There is a large amount of experimental data on this point. We present in Table I a few facts regarding ordinary gases.

For slow electrons the ionization potential is given in Column 3. But an alpha particle loses almost twice as much energy in ionizing an atom as a slow electron is required to lose—except for helium. The energy spent by an alpha particle in ionizing argon is considerably less than is spent in air, hydrogen, nitrogen, etc. Consequently argon under a pressure of from 20 to 50

atmospheres is frequently used in ionization chambers in the study of cosmic rays.

TABLE I

Gas	In Electron Volts	
	Energy Spent by an α Particle per Ion Pair	Ionization Potential by a "Slow" Particle
Hydrogen	33.0	16.5
Helium	27.8	24.6
Nitrogen	35.0	17.0
Oxygen	32.3	15.5
Neon	27.4	21.5
Argon	25.4	15.3
Krypton	22.8	
Xenon	20.8	

But ionizing an atom merely means flicking off an electron. That does not disturb an alpha very much, as is shown below. But a full-speed alpha may make a near collision with the nucleus of an atom. If the latter is not free to move, we will have scattering of the alphas, as was shown in Chapter 3. If the atom nucleus is free to move, as in the case of a gas, we may have a collision similar to that of one billiard ball upon another. We come now to consider the collision of detached, isolated particles.

Elastic Collisions. Atomic Billiard Balls.

We assume that the particles are perfectly elastic. Then kinetic energy is conserved. Momentum is always conserved.[1] We picture an alpha, mass M, velocity V, "striking" a nucleus of mass m at rest. We can show (see Appendix 12–2) that for a "head-on" collision the velocity v given to m is $2\,MV/(M+m)$ and that u, the velocity of M after collision, is

$$\frac{M-m}{M+m}V.$$

It follows from this that a hydrogen atom or nucleus struck by an alpha of speed V would be given a speed of 1.6 V; a helium atom or nucleus, V; a carbon atom, $V/2$; an oxygen atom, 0.4 V.

[1] Auger and Perrin have obtained some data which seem to show that in some cases momentum is conserved and energy not; some where energy is conserved and momentum not; some where neither is conserved. Kinetic energy, of course, need not be conserved, it may take another form; we shall give many illustrations of that case. But it seems probably that *momentum is always conserved.*

If the collision is not "head on," the alpha would be deflected. By an electron it could not be deflected more than 30 seconds: by a hydrogen atom, not more than 14° 30′. If then by means of cloud tracks we find that an alpha has been deflected on account of a collision by more than 14° 30′, we know that the other particle was not a hydrogen atom or a proton. If the two paths after collision are at right angles, the two masses are equal. In every case if we can measure the angles between the paths and the original direction, we can find the mass of the particle which has been struck.

Collisions Made Visible by the Cloud Chamber.

We may use the cloud track chamber for this kind of target practice. We arrange matters so that photographs can be taken very soon after expansion. And we must have two simultaneous [1] photographs taken in directions (nearly) 90 degrees apart. For it is necessary to know accurately the directions of the motions before and after collision. Had only one photograph been taken at right angles to the plane of the chamber, we would have had only the projection of the paths on that plane.

We must take many photographs in order to find a collision. Having found one, we proceed as follows. The velocity of the alpha just before the collision is known. (We can measure the distance to the source if necessary and allow for the loss in energy up to that point.) The range of the alpha after collision is known, therefore also its velocity just after collision. The directions of motion of both particles being known from the photographs, the mass and velocity of the atom can be computed. As an indication of the vast amount of experimental work which has been done along this line, some figures should be given. Blackett and Lees [2] examined 1,300,000 alpha tracks in various gases. As a result of their work they drew up tables giving the ranges corresponding to the energies of hydrogen, nitrogen, oxygen, argon atoms—in air or in other gases. Thus, starting with the known ranges of alpha particles of known energies, they end with known ranges for other atoms.

[1] Sometimes this is done by two cameras, sometimes by one camera taking a direct picture and another picture reflected from a mirror. However, only one picture by a short-focus camera need be taken; then it can be projected so that "depth" may be measured.

[2] *Proc. Roy. Soc.*, Vol. 134, p. 659, 1931.

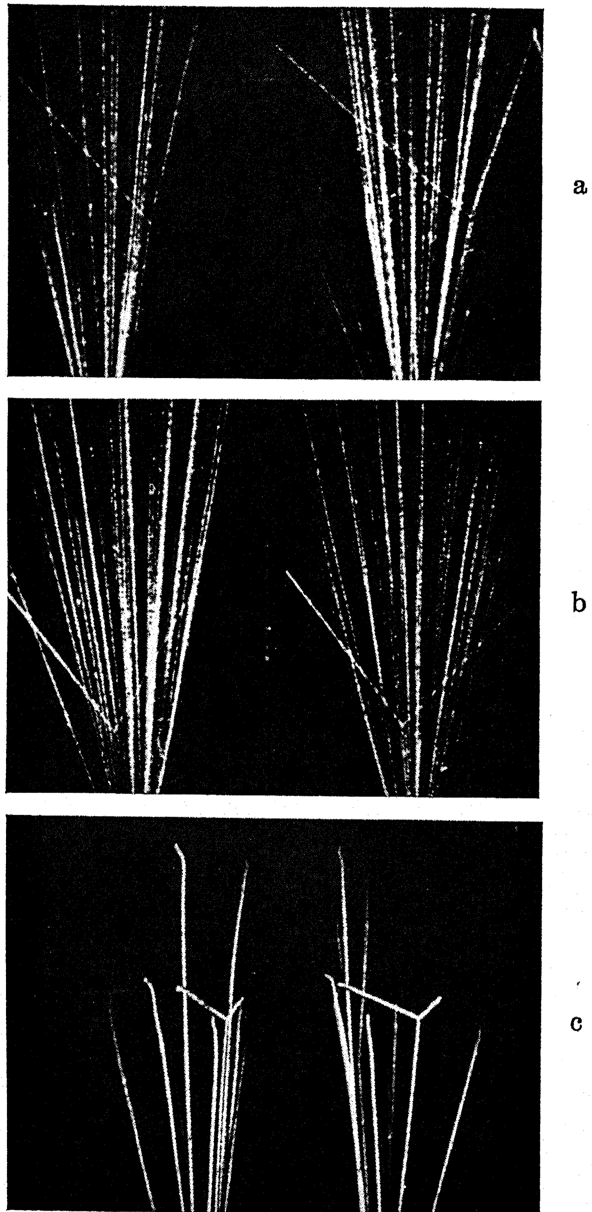


FIG. 12–1. Elastic collisions of alpha particles with (a) hydrogen, (b) helium, (c) oxygen. (Rutherford, Chadwick, and Ellis.)

The student should pause to consider this matter: 1,300,000 alpha tracks, photographed in duplicate. Think of the skill required to obtain photographs of these fleeting phenomena, of the labor involved in the measurements of ranges and angles. Foolish drudgery?

We Identify the Atoms in Collision.

Figure 12–1 a shows a collision of an alpha with a hydrogen atom. It can be seen that the alpha is only slightly deviated from its original direction, that the hydrogen nucleus, the proton, has been given a large velocity. In Fig. 1 b for helium, the two paths which are nearly equally inclined to the original alpha direction are nearly equal in length. In Fig. 1 c for oxygen, the alpha is deflected about 70 degrees; the oxygen atom about 45. The accuracy in the identification of these particles may be seen from these data: the known ratios of the masses of hydrogen, helium, and oxygen atoms to alpha mass are 0.252, 1.00, 4.00; the ratios computed from the photographs were 0.253, 0.98, 4.18.

Rutherford's Earliest Experiments. A Proton Is Ejected from a Nitrogen Atom.

It would be wrong to give the impression that the data in this work were obtained only from cloud tracks. As a matter of fact much of it was obtained by the use of apparatus of Fig. 12–2.

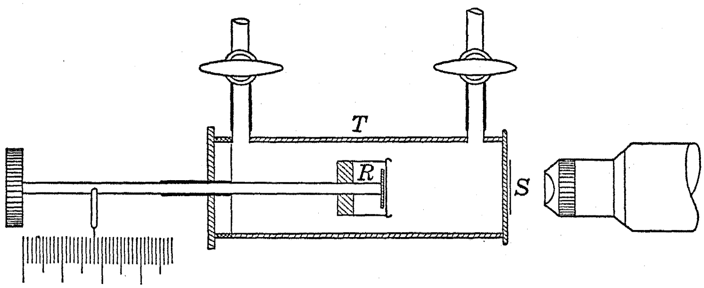


FIG. 12–2. Rutherford's historic apparatus. By observing scintillations on S due to radium at R, he noted that there were new long range particles. Thus was the disintegration of the nitrogen atom discovered. (Rutherford, Chadwick, and Ellis.)

Inside a long glass tube T there is a thin layer, R, of Radium C or F. The opening in S is covered by very thin metal foil of definite stopping power for alphas, equivalent to a few centi-

meters of air. Other thin strips of foil may be added outside, if necessary, then a fluorescent screen. The distance of R from S can be adjusted and a gas can be introduced into the tube to any desired pressure. Alphas coming from R passing through the thin window cause the screen to fluoresce in spots, to scintillate. The range of the alphas in the gas was known. But if the gas were *hydrogen*, it was found that *new particles produced scintillations*, particles of a maximum range of 28 centimeters of air. By means of magnetic and electric deflections e/m was measured and it was proved that these particles were *protons*. An alpha colliding with a hydrogen nucleus had given the latter a large velocity, a 28-centimeter range. When nitrogen was introduced into the tube, nitrogen atoms were occasionally struck and knocked forward. *But upon rare occasions a new phenomenon was observed, a particle of 40-centimeters range* caused a scintillation. These particles seemed to drive out from the point of collision in all directions, but with a preference for the forward direction. They also were shown to be protons. (No such long range particles were obtained when hydrogen alone was present.) The energy of one of these protons might be greater than that of the impinging alpha. These results could best be explained by the hypothesis that a proton had been driven out of, or exploded out of, the nitrogen nucleus.

A cloud track photograph by Blackett and Lees (see frontispiece) *illustrates the process.* An alpha particle shooting into a nitrogen nucleus is captured, a proton is ejected (to the left and backward), and the new atom makes a short track forward. We represent the transformation thus:

$$_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$$

which being interpreted states that normal nitrogen (atomic number 7, mass number 14) added to helium (2, 4) produces an oxygen isotope (8, 17) and a proton (1, 1). Thus, *Rutherford's experiment was the first in history in which we had evidence of one element being transmuted into another.* The dream of the alchemist had been realized. This work begun by Rutherford in 1919 was continued by him and by Chadwick, Blackett, and other workers in the Cavendish laboratory. It was taken up by Kirsch and Petterson in Vienna. Now it is carried on in many laboratories throughout the world. At first only the energetic alphas

of radioactive atoms were used as projectiles; now (1935) we have various projectiles hurled at desired speeds by man-made devices.

A New Phenomenon May Appear—Mass May Decrease and Energy Increase.

The above relation is incomplete; energy has not been considered. We now state the general principles which hold. In all cases of *elastic* collision we must have the following relations: (1) the momentum before impact equals the momentum after impact, (2) the kinetic energy before impact must equal the kinetic energy after impact. But in the case of *capture* of the bombarding particle and therefore in the case of transmutation or disintegration, a new item enters—there may be, there practically always is, *a change in mass.* This mass difference must be transformed into energy by the relation $E = mc^2$ where c is the velocity of light. This is apt to be the chief part of the kinetic energy of the products of disintegration. Or it may take the form of the energy of a gamma ray.

The Discovery of the Neutron

A Celebrated Experiment Which Finally Led to a Great Discovery.

We now describe an experiment which at first was supposed to illustrate the gamma ray idea just stated. In 1932 Bothe and Becker of Giessen shot alphas into beryllium and found that a very penetrating radiation was given off. They supposed it was entirely due to a gamma ray, a photon of high frequency. (It is true that γ rays are given off in this reaction, but the chief action is that described later.) They represented the action thus:

$$_4Be^9 + {_2He^4} = {_6C^{13}} + \text{quantum } (hf).$$

Mme. Curie-Joliot and M. Joliot[1] showed that when this radiation passed through paraffin the ionization, instead of being decreased by absorption, was greatly *increased,* apparently due to the ejection of protons of a range of 26 cm. The experimental

[1] Irene Curie, daughter of the most famous of women physicists, became the wife of the physicist M. Joliot. With justified pride in her descent, she and the physicists of the world hyphenate the name. Now that all the world knows her, we still honor her by speaking of the two as the Joliots.

arrangement is shown in Fig. 12–3. Alphas from polonium shoot into beryllium. A penetrating radiation enters a special form of

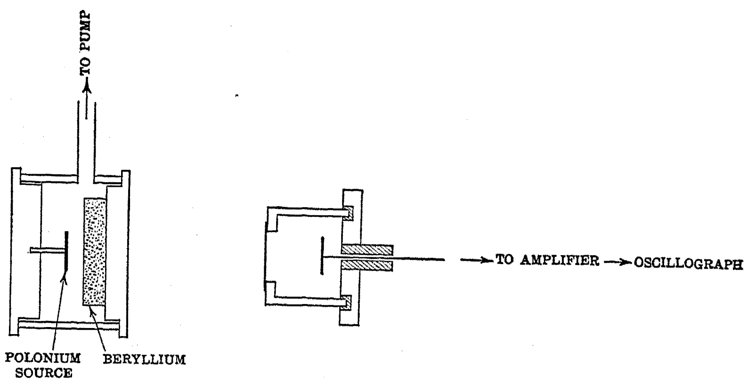


FIG. 12–3. Alphas from polonium smashing into beryllium let loose a new radiation, very penetrating as shown by sheets of lead in front of the detector. Thus was the neutron discovered.

Geiger counter and is "counted." A block of paraffin (which is rich in hydrogen) is then placed as shown. The count is thereby greatly increased. The range and nature of the particles can then be determined. It was found that the particles which came from the paraffin were protons of energies 5.7×10^6 e.v. These experimenters assumed, as did Bothe and Becker, that the radiation which came from the beryllium was of the gamma-ray type. Then it followed that the protons which came from the paraffin were driven out by Compton encounters, by the impinging of photons on hydrogen nuclei. But then entered Chadwick of the Cavendish laboratory to carry out an unusually clever piece of detective work and to make *the discovery of the neutron.*

Chadwick remeasured the energy of the protons which came from the paraffin and also the energy of nitrogen atoms set in motion by this strange radiation. He computed the energy of the (assumed) bombarding photon in each case. To give the proton the energy which he found, the photon would necessarily have an energy of 55×10^6 volts; to give the nitrogen atom its measured energy the required photon energy would have to be 90×10^6 volts (Appendix 12–2). These values were far from agreement; moreover they seemed unnecessarily large. He tried another hypothesis. Suppose that an alpha striking the beryl-

lium nucleus knocked out or caused to be ejected a particle having a mass of the order of that of the hydrogen atom, and that this particle going into the paraffin knocked forward a proton. Since the energy of the proton, as determined by its range, was known to be $5.7 \times 10^6\,e$ volts, that of the unknown particle, if of the same mass as the proton, would be, for a head-on collision, 5.7×10^6 volts. Such a particle would give to a nitrogen atom a maximum energy of $1.4 \times 10^6\,e$ volts. The measured value was 1.2×10^6. As the difficulties in the way of accurate measurement were great, this was considered a satisfactory agreement. Chadwick's hypothesis seemed to be correct.[1]

But what kind of a particle could this be, having the mass of the hydrogen atom? It could not be a proton, its range was too great. It did not make an ionizing track as did a proton. Its great penetration could be accounted for only if it were electrically neutral. Such a particle had been visioned at various times. Finally its discovery was assured. It was named the *neutron*. Since its discovery in 1932, methods of producing and using neutrons have been so improved that now the number of disintegrations that may be obtained from a source using neutrons as bombarding particles may be a million times as great as the number[2] available to Chadwick and the Joliots.

The Neutron Is the Only One of Its Kind.

The neutron may be pictured as a neutral nucleus. Its atomic number is zero, its mass number 1. Its symbol is ${}_0n^1$. Its mass in terms of O^{16} is (nearly) 1.0085. (Chadwick computed it to be 1.0067, but that appears to be too low.) Having no electron atmosphere and no charge, it can go through matter with ease.[3] Whereas an alpha particle would produce about 30,000 ion pairs per centimeter of path in air, a fast neutron would make one in several meters. But it seems to be the business of the neutron to bump into nuclei, not to flick off electrons from atoms. It has a ghostly existence after it leaves an atom. It may pass through (?) millions of molecules without announcing its existence,

[1] For this discovery Professor Chadwick has just been awarded the Nobel Prize in physics, 1935.

[2] The number of disintegrations due to neutrons depends not only on the number of neutrons but upon their energy. It will be seen later that 1 mm. of cadmium will almost completely absorb "slow" neutrons. See Chapter 14.

[3] This is true for swift neutrons. For slow neutrons, the story is very different. See Chapter 14.

then it bumps into a nucleus. The collision may be elastic or inelastic. In the former case the struck atom bounces off with part of the energy of the neutron; the laws of mechanical momentum and energy hold. In the latter case the neutron is captured, a new atom is formed, or a complete disintegration may take place.

The Neutron Enters an Equation for the First Time.

Chadwick revised the Bothe and Becker relation

$$_4Be^9 + {}_2He^4 \rightarrow {}_6C^{13} + \gamma$$

to read

$$_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1 + E.$$

The masses of the first three atoms are known. The energy of the alpha is known. Hence the sum of the rest mass of the unknown particle and its energy is known. To distribute this sum was Chadwick's problem.

As a Ghost It Enters Photographs.

Just after Chadwick made his important discovery, Feather also in the Cavendish laboratory photographed a number of neutron collisions with atoms. A few of these are shown in Fig. 12–4. Feather examined 6900 cloud chamber photographs. From the tracks of the recoiling nuclei he deduced the speeds and the speed distribution of the neutrons. Yet the paths of the neutrons could not be seen; they left no tracks.

THE DISCOVERY OF HEAVY HYDROGEN

Before taking up the general problem of the artificial transmutation of the elements, we desire to record another important discovery. In 1896–1898 there were discovered X-rays, the electron, radium. In 1932 there were discovered the neutron, heavy hydrogen, and the positron. Just now we deal with heavy hydrogen.

It had been suggested by Birge and Menzel, among others, that an isotope of hydrogen of atomic mass 2, in concentration of 1 in 4000 of ordinary hydrogen, would account for the difference between the chemical (1.008) and the mass-spectrograph ratio of hydrogen (1.0078) to oxygen (16). (In view of the new value, 1.008, just found by Aston, this argument loses its weight. It is a rather extraordinary fact that the error which Aston made

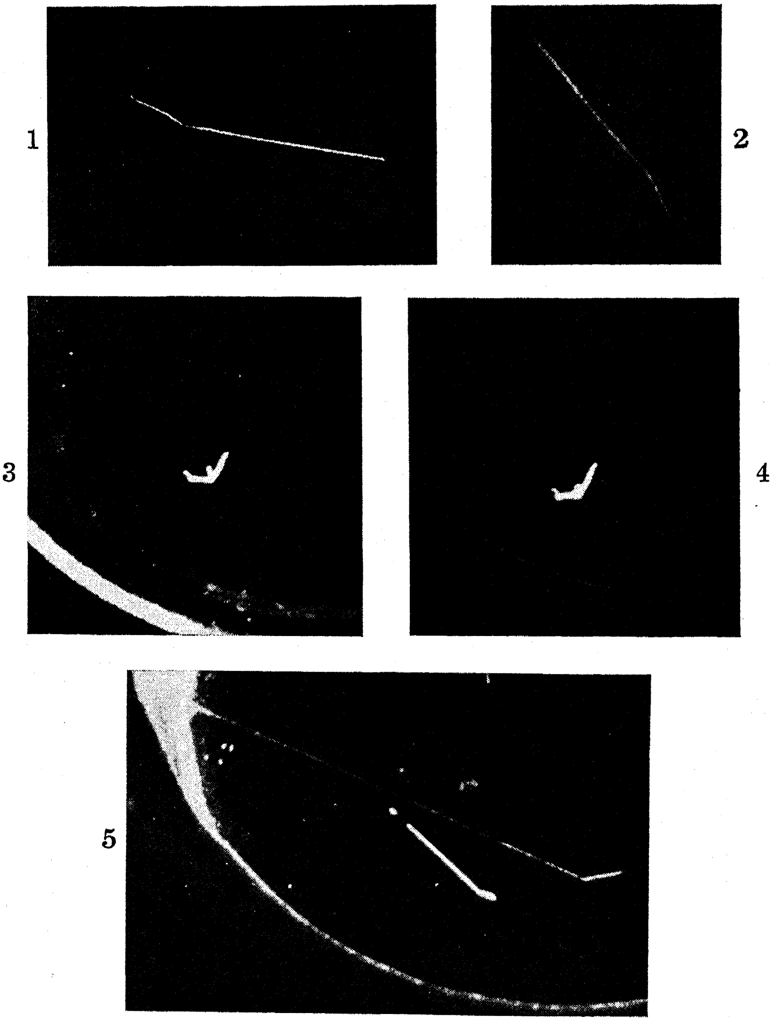


FIG. 12–4. Cloud tracks showing disintegrations produced by neutrons, themselves unseen. 1. A nitrogen atom captures a neutron, then disintegrates into a carbon (short track) and proton or into a boron and alpha particle. 2. An oxygen captures a neutron and disintegrates. 3. A carbon after capture (?) breaks up into three nearly equal atoms (alpha particles?). 4. Same as 3. 5. Two disintegrations in one photograph—very unusual; in one case obviously a proton has been ejected (long track). (Photographs by Feather.) (Courtesy of Chadwick and Feather and Physical Society of London.)

in his early determination of the mass of the hydrogen atom stimulated research which led to the discovery of heavy hydrogen.) Urey and Murphy of Columbia University and Brickwedde of the Bureau of Standards subjected liquid hydrogen to fractional evaporation, then tested spectroscopically the final fraction. It was found that the ordinary Balmer lines [1] of hydrogen, very greatly over-exposed, were accompanied by very near faint lines on the short-wave-length side. The separation of these faint components from the strong lines was in accord with the Bohr idea that accounted for the similar separation of the helium from the hydrogen lines (p. 93). The computed mass of the nucleus was 2 (Bainbridge's mass spectrograph value is 2.0136). Thus was discovered heavy hydrogen. It is now called (in America) *deuterium;* symbol $_1D^2$ or $_1H^2$. Its nucleus is called a *deuteron.* Heavy water may be called deuterium oxide and written D_2O or $H_2{}^2O$, instead of $H_2{}^1O$ for ordinary water. This water [2] weighs 11.2 per cent more than ordinary water, it freezes at 3.8° C. (39° F.), boils at 101.7° C. (215° F.), has a vapor pressure of 721, instead of 760, at 100° C.

Now heavy water is an article of commerce. It is prepared by the electrolysis of ordinary water. In order to obtain one gram of 99.9 per cent of D_2O it is necessary to electrolyze 24,280 grams of ordinary water to a one-gram residue. (The student should compute the cost at 10 cents per kilowatt hour.) Obviously for commercial production it is necessary to utilize large power plants, preferably hydro-electric. Consequently, in Europe, Norway is the chief source of D_2O; in America, large chemical plants specializing in electrolysis.

Having discussed these two great discoveries, that of the neutron and that of heavy hydrogen, we return to the topic of artificial transmutation.

Artificial Transmutation

At first the only projectiles used for this purpose were alpha particles. Then physicists turned to the possibility of using protons, neutrons, and deuterons as projectiles. We shall consider one historic experiment.

[1] See Appendix 12–1 for photographs of the Lyman series of H^1 and H^2.

[2] Since there are three different kinds of hydrogen, H^1, H^2, H^3, and three different kinds of oxygen, O^{16}, O^{17}, O^{18}, there are 18 different kinds of chemically pure water.

The First Completely Artificial Transmutation—by Cockcroft and Walton.

In Chapter 8 it was shown how high-voltage alternating current could be rectified and smoothed out into high-voltage direct current. When, in a discharge tube, hydrogen ions are drawn through an opening in the cathode by a high voltage, we have a stream of protons. Cockcroft and Walton (1932) in the Cavendish laboratory directed protons of only 150,000 electron volts against lithium and observed an astonishing fact—*alpha particles of about seven million electron volts were given off.* This was surprising for two reasons: the energy of the striking protons was small, and the usual process of producing protons by bombarding atoms with alphas had been reversed. Cockcroft and Walton pictured the transformation thus:

$$ {}_3Li^7 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^4 + E. $$

This equation asserts that a proton ${}_1H^1$ enters a lithium 7 nucleus and that the new nucleus breaks up into two helium nuclei which are driven apart with explosive violence, each with an energy of several million electron volts.

Very recently an accurate determination of the energy of each alpha gives it as 8.53×10^6 e volts. Since that of the striking photon is only 150,000 e volts, we may neglect it and make $E = 17.06 \times 10^6$ e volts. Writing in the masses of the nuclei as determined by the mass spectrograph, we have (using "old" masses)

$$ 7.0132 + 1.0072 = 2 \times (4.00108) + 17.06 \text{ M.E.V.} $$

In order to elucidate this equation we now have a lesson in transmutation arithmetic.

Transmutation Arithmetic.

We define [1] a "statom" (generally called a mass unit) as one-sixteenth of a normal oxygen atom or $O^{16}/16$. The hydrogen atom would then have a mass of 1.0078 statoms, helium 4.00216, lithium (7) 7.0148, the electron 0.00054. (We are using here for

[1] The author proposes the term "statom" for mass unit. It might be regarded as an abbreviation of "standard atom" which would condemn it in the minds of purists. But it has a Greek structure, $\sigma\tau\alpha\tau o\varsigma$ = "standing, fixed." This would give it respectability.

illustration purposes the "old" [before 1935] masses of the atoms.) Hence the proton's mass would be 1.0072, the alpha particle 4.00108, and the lithium nucleus (lithium atom minus three electrons) 7.0132. On the left-hand side of the equation above there is a total mass of 8.0204, and on the right-hand 8.00216. Thus a mass of 0.0182 statom has been lost. We change that into energy. The mass of a statom is 1.649×10^{-24} gram. Its energy equivalent is its mass multiplied by the square of the velocity of light and equals 14.83×10^{-4} erg. But one electron volt equals 1.591×10^{-12} erg. Hence 1 statom = 9.31×10^{8} e volts = 931 M.E.V. The mass difference, 0.182 statom, is thus equal to 17.06 M.E.V. and this balances the equation. *This result appears to be an experimental proof of the transmutation of mass into energy.*

We now interpret the equation. The proton of 150,000 volts had sufficient energy, or the "right" energy, to penetrate into the nucleus of lithium (7); a recombination of masses took place; alphas were ejected, the energy of ejection being derived from the excess mass. The lithium (7) nucleus may be pictured as composed of 3 protons plus 4 neutrons. When the projected proton enters, we have the equivalent of two helium nuclei, each of which consists of 2 protons plus 2 neutrons.

The experiment just described stands out as the first in which the atom of an element has been transformed into another kind of atom by very simple artificial means. The voltage necessary for the projection of the protons was obtained by rectified alternating voltage (Cockcroft and Walton have gone up to 600,000 volts). There are two other ways of producing high-speed particles, by the Lawrence and Livingston "cyclotron" and by the high-voltage "electrostatic" generator of Van de Graaff.

New Atom Guns

1. The Cyclotron. Lawrence and Livingston.

The principle of the cyclotron is illustrated in Fig. 12–5. A and B are two shallow hollow half-cylinders. They are made the condenser terminals of a high frequency (about 10^7 per sec.) electric oscillator. By means of an electron discharge from a tube not shown, protons or deuterons or other ions are produced between A and B at the edge of A. When B becomes nega-

tively charged, these ions are accelerated towards B. Inside A or B there is no electric field. A magnetic field at right angles to the plane of A, B, causes an ion to describe a half-circle. If

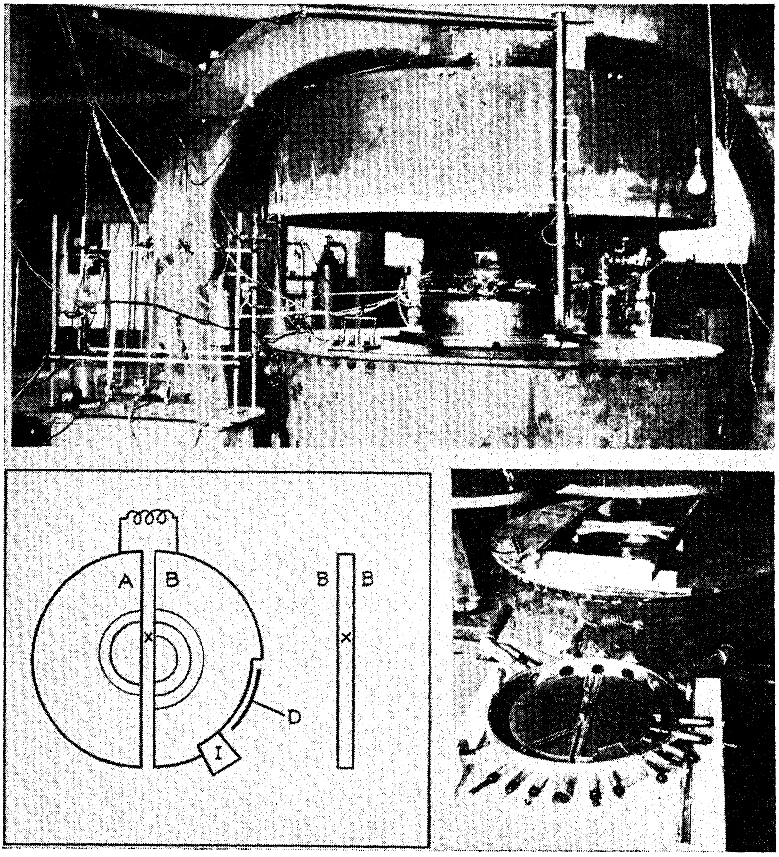


Fig. 12–5. The Lawrence and Livingston "cyclotron" and huge magnet (85 tons). Particles of energies of 3 million volts energy have been obtained by using only 20,000 volts. In their first instrument B was connected to the case; later it was insulated both from the case and from I, the ionization chamber. (Courtesy of Lawrence and Livingston.)

it arrives at the edge of B just as A acquires its maximum negative charge, it will be accelerated towards A, again describe a half-circle of larger radius. It will continue to describe these half-circles of constantly increased radius, getting a boost in its

energy at every half-turn. Thus if the accelerating voltage[1] between A and B is 10,000 volts and the number of half-circles is 200, the final energy of the ion will be two million volts.

This sounds too good to be true, but a number of fortunate conditions make it a fact. Suppose the velocity of the ion as it describes one half-circle is v; then if the magnetic field is H, we have the usual relation $Hev = mv^2/r$ or $v = Hre/m$. Now the time of describing a half-circle is $\pi r/v = \pi m/eH$. This is not dependent on the radius. Hence *the times for all the half-circles are the same.* If H is altered to suit the frequency of oscillation or vice versa, the ion will receive its energy increase at the end of every half-circle. Moreover both the electric and magnetic field have a focussing action on the ions, keeping them in narrow ribbons equally distant from the faces of A or B. The result is that the ions can be made to emerge along the arrow in a thin pencil and can be directed against a target or they can be caught in a Faraday chamber and the total number can be determined.

Photographs of the cyclotron (condenser and huge magnet) which have been used by Lawrence and Livingston in the University of California are shown in Fig. 12–5. The condenser plates are enclosed in a flat metal case which can be placed between the poles of the magnet, then evacuated and filled to the desired pressure with hydrogen or deuterium. The dimensions of this magnet are interesting; *it weighs 65 tons, the diameter of the pole pieces is 80 cm., the magnet spools are wound with 9 tons of copper band.* Its cost? It should be possible to accelerate protons by this magnet to an energy of 10^7 e volts. But a new *large* magnet is in the making to have a diameter of pole pieces 114 cm. and to give protons an energy of 25×10^6 e volts!

One exceedingly attractive feature about the cyclotron is that it produces high energies without high voltages. On this account it is likely to find favor as a device for atom smashing.

Cockcroft and Walton avoided some of the difficulties of direct high-voltage generation by employing a number of specially designed kenotrons or electron tube rectifiers in series. These were placed in a circuit indicated by Fig. 12–6. The voltage produced by the secondary was multiplied by the arrangement shown by a factor of 6 and the final product was smoothed out by the

[1] In the later apparatus both A and B are insulated from the case and from I.

condensers and resistances. Their apparatus was very elaborate and involved the introduction of new methods in laboratory technique.

It is not possible here to describe the apparatus used by Crane and Lauritsen in the California Institute of Technology, nor the early apparatus used by Hafstad, Tuve, and Dahl in the Carnegie Institution of Washington.

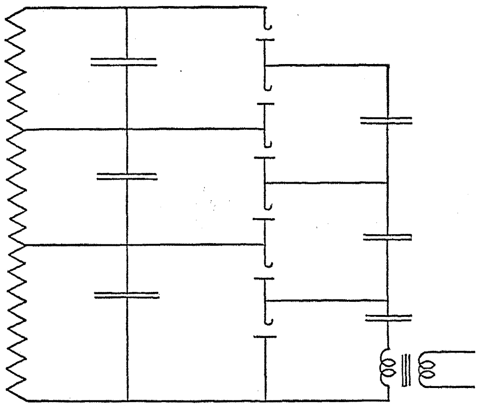


FIG. 12–6. The Cockcroft and Walton method of amplifying voltage by means of thermionic rectifiers and condensers.

2. The Electrostatic Generator.

(*a*) *Van de Graaff*. But superlatives must be used, at least in regard to dimensions, in dealing with the high-voltage generator designed by Van de Graaff (and associates), formerly of Princeton, now of the Massachusetts Institute of Technology. The principle is very old, that of the "influence" machine. To illustrate, let one plate, A, of a condenser (Fig. 12–7) connected to

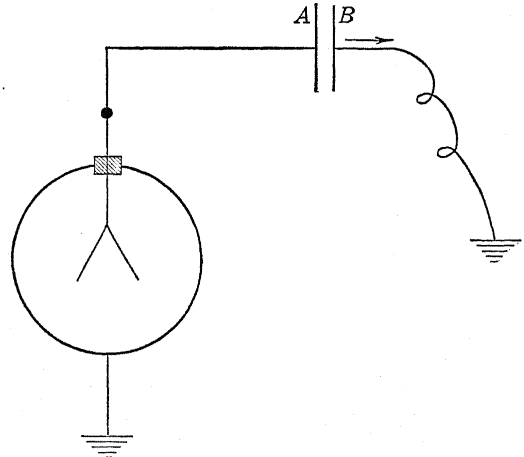


FIG. 12–7. When B, one of two condenser plates, is pulled away from A, work is done and the potential of A increases.

an electroscope be charged, then let the other plate B be drawn away. The leaves diverge more, the potential of A has been increased due to the work which has been done in the separation of the plates. In the Massachusetts Institute of Technology generator, positive and negative charges are "sprayed" by brush discharge from a 10,000-volt generator on silk or paper belts, which are driven by motors and pulleys as shown (Fig. 12–8).

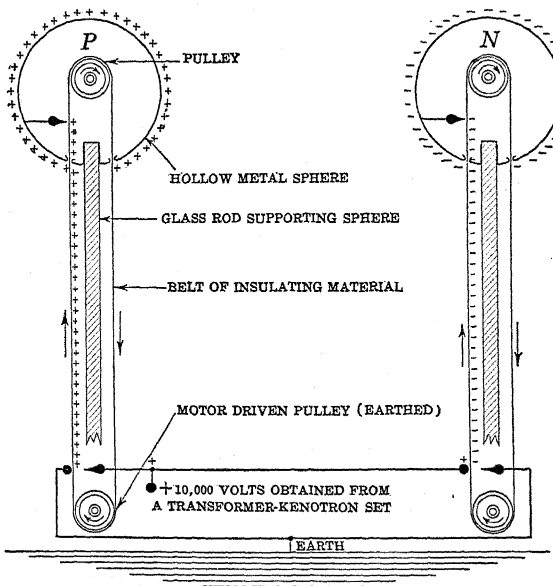


FIG. 12–8. The principle of the electrostatic generator. (Van de Graaff, *Physical Review.*)

The charges are drawn off by points inside the spheres, then pass to the outside surfaces as in the Faraday "ice pail" experiment. As the charges on the spheres pile up, the voltage difference between them increases. It is limited only by the electrical breakdown of the medium between the spheres and the insulating property of the belts and supports. The current that may be obtained depends on this voltage and the power supplied to the motors.

This machine has been in process of development for six years. Just now (May, 1935) it is announced that 7,000,000 volts have been obtained. But no vacuum tube has yet been constructed

to utilize this voltage. The huge dimension can be seen from Fig. 12–9. The tops of the fifteen-foot diameter aluminum

ELECTROSTATIC GENERATORS

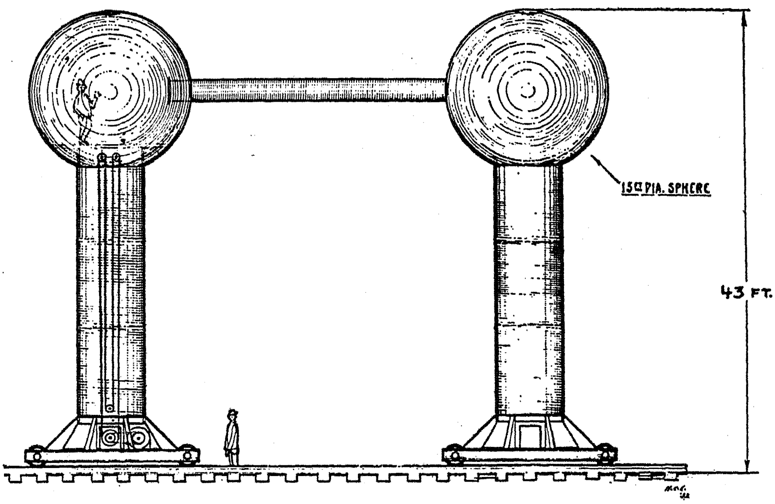


FIG. 12–9. The huge dimensions of the Van de Graaff gun for shooting atoms. (From the *Physical Review*.)

spheres are 43 feet from the floor. The ceiling of the building should be 20 feet higher. It was thought that the interior of one of the spheres would be the only safe place for observers, but neutrons make it unsafe. On account of the great cost of construction and operation, it is rather probable that no machine larger than this or even equal to it will be constructed elsewhere in the near future. However, the Soviet government is building a big one.

(*b*) *Tuve, Hafstad, and Dahl.* But an extremely ingenious and practical modification of the Van de Graaff apparatus has been made by Tuve, Hafstad, and Dahl (Figs. 12–10, 12–11). Two motors are used to drive a long belt which runs over idlers inside metal shells. A charge is sprayed into this belt at one end and the current is doubled, as is done in any ordinary influence machine.

As some charge is taken off each shell, the inner one is higher in voltage than the outer. This voltage difference is utilized in giving the first acceleration to the ions which are generated at the top of the discharge tube. This vertical glass tube, nearly

HIGH VOLTAGE TECHNIQUE

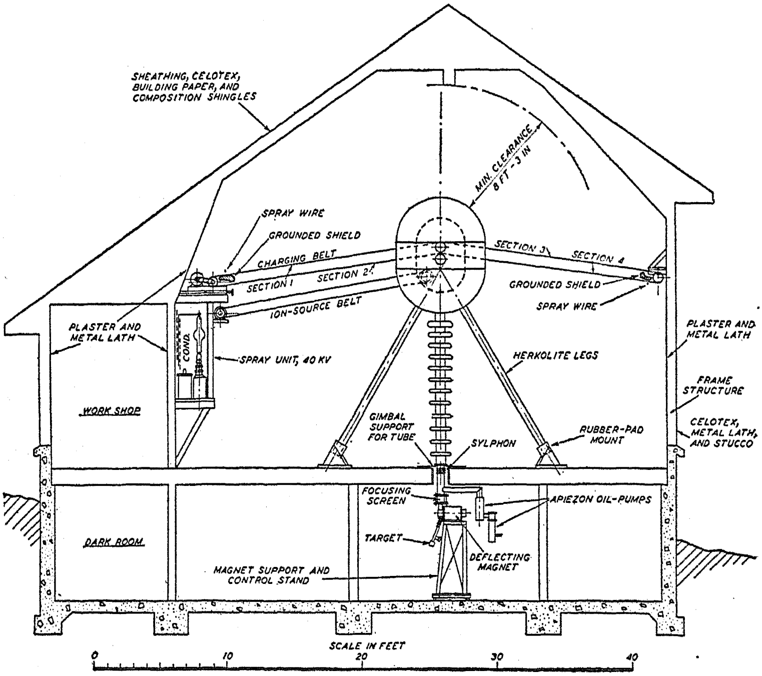


FIG. 12–10. The practical modification of the Van de Graaff idea by Tuve, Hafstad, and Dahl. See also Fig. 12–11. (From the *Physical Review.*)

20 feet long, contains inside it a number (14) of short metal cylinders, coaxial, separated by about 2 inches of "vacuum." These cylinders are connected to metallic rings which are seen outside the discharge tube. The inner shell can be connected to the top cylinder by a corona leak so that the voltage of this cylinder can be made intermediate between the voltages of the two shells. The cylinders below the outer shell, on account of corona discharge, assume voltages intermediate between that of the outer shell and the ground. Hence, as the charged particle goes down the axis of the cylinders, it is constantly accelerated as it passes from one cylinder to the next one. On account of this gradual lowering of voltage by means of the rings, there is no great electric intensity at one point and there is not apt to be lightning flashes down the outside of the tube. This is an important feature of this apparatus.

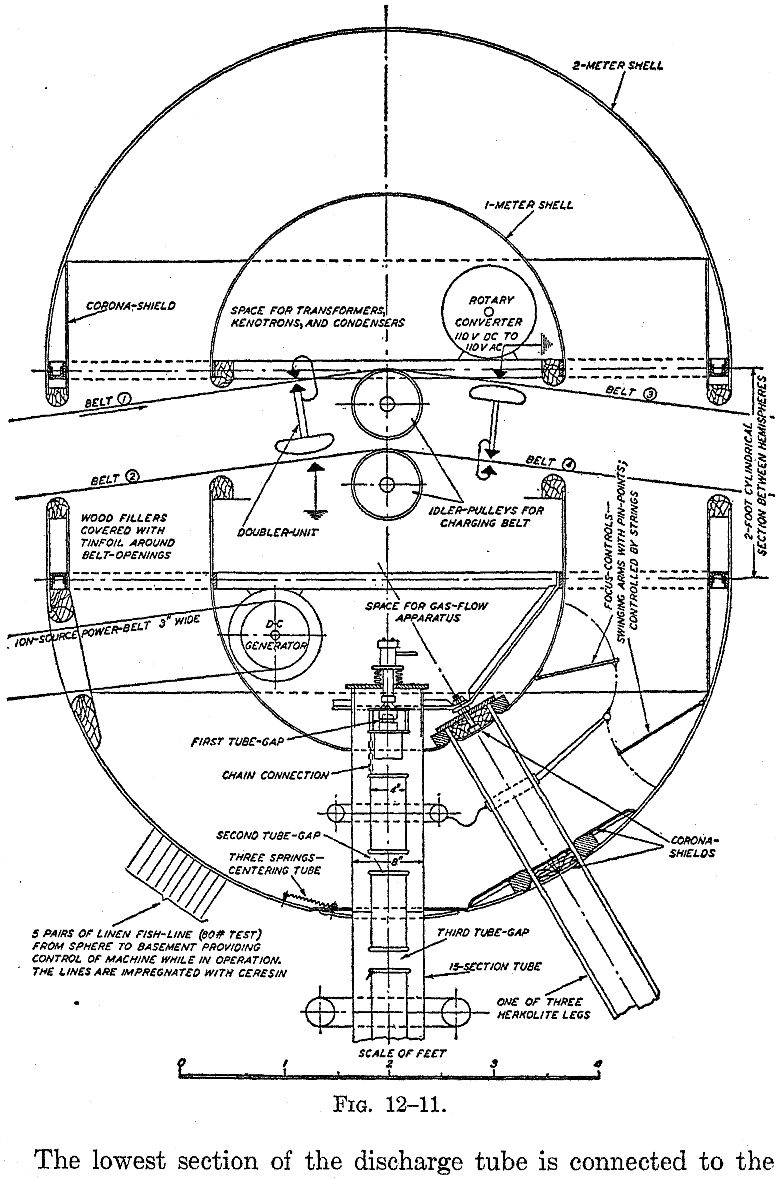


FIG. 12-11.

The lowest section of the discharge tube is connected to the glass part by means of a seamless flexible brass "hose." The target, the material which is to be subjected to ion bombardment, is in the bottom of this section. By imposing a magnetic field

across the tube in the region of the brass hose and by bending the tube accordingly, the ions can be deflected through a definite arc and focussed on the target. Indeed, this is one method of measuring the energy of a stream of known ions.

Voltages very steady, as high as 1,300,000 volts, have been obtained with this apparatus under favorable conditions. But humidity of the air is a limiting factor and in Washington, D. C., the humidity is generally high. Though a separate building very specially designed for this use had to be erected to house this apparatus, it has not been found practicable to air-condition the house. So far the working voltage is 900,000. But the definiteness of voltage and current makes this apparatus of the greatest value in precision measurements. We look for important data to come from the Terrestrial Magnetism Laboratory of the Carnegie Institution.

The erection of new laboratories especially designed for apparatus to be used in the smashing of atoms, the construction of huge machines for this purpose at great cost, the expenditure of hundreds of kilowatts of power for hours, days, months, . . . in the operation of these machines, shows the importance that is attached to this branch of the newest physics—the artificial disintegration of the elements. But we turn aside from this topic to deal with particles of such enormous energies that it seems highly improbable that their equal in energies can ever be obtained by human agencies. We turn to Cosmic Rays.

CHAPTER 13

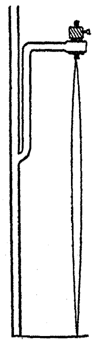
COSMIC RAYS

Here is the story of the modern Argonauts. But the Golden Fleece which the scientific Jasons of the world have been seeking appears to be a new universe, a universe of electrified particles driving through the vast realms of space with enormous energies, energies undreamed of by the physicists of even recent years, a space not visioned by our farthest-gazing astronomers. It is the story of Cosmic Rays.

In the Beginning, 1900–1914.

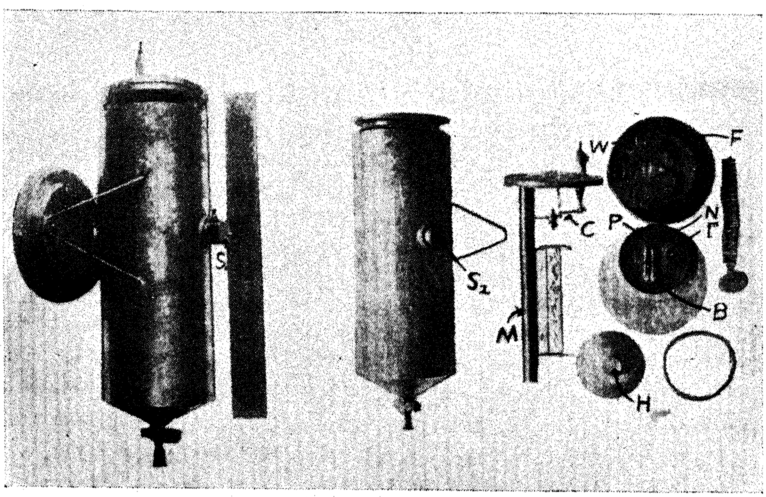
The discovery of radium in 1898 gave a great impetus to the testing of all kinds of matter for radioactivity. It was found that many ordinary materials contained very feebly radioactive sources. Very specially was this true in physics laboratories where radium had been kept. But avoiding all known sources, experimenters found that electroscopes would still very slowly lose a charge. The air was ionized. Where was the radioactive source? Rutherford and Cooke (1903) in Montreal built a wall of selected brick round an electroscope and noted that the discharging rate slightly decreased. About the same time, McLennan and Burton in Toronto placed an electroscope in a large tank of water, later lowered it about 18 feet below the surface of the lake, and made the same observation. McLennan and his associates made tests on bodies of water in various parts of the world. In all of this work, the earth surface or atmosphere was suspected as being the radioactive source. It was not until Wulf on the Eiffel Tower (1909) and Glockel (1910) in a balloon at 4500 meters observed that the radiation increased with altitude that it was known that a new source must be found. Was it the upper air? Hess in Austria (1911) went up to 4000 meters; Kolhörster in Germany (1914) pushed the height to 9000 meters. Both observers, finding that the intensity increased rapidly with height, attributed the ionization to "a penetrating radiation of cosmical origin." Then came the war.

During the war Millikan was chief of the Science and Research Division of the Signal Corps of the United States Army. Not that his experience in that capacity was necessary for him to become acquainted with the performance of sounding balloons, but, at least, soon after the war he made good use of those devices. He constructed (1922) a combined recording electrometer, thermometer, barometer, of total weight of less than half a pound (Figs. 13–1 and 2), and sent it up by means of two balloons to



FIGS. 13–1 and 2. Parts of Millikan's half-pound cosmic ray meter which went up ten miles. In Fig. 13–1 (left) two fine gold-sputtered quartz fibers are supported by a quartz rod. The lower ends of the fibers are attached to a fine quartz rod which acts as a gentle spring pulling them together. When charged up (maximum potential 300 volts) the fibers spread apart. As they discharge due to the ionization produced by cosmic rays, they come together. They are photographed (by shadow) on a revolving film. The distance between them gives their voltage, therefore the rate at which they have discharged.

In Fig. 13–2 (below) are shown parts of the electroscope. Light from the sky enters a fine vertical slit S_1, passes the fibers, then goes through a horizontal slit S_2 to the film which was moved by the main spring of a small watch.



a height of 15,500 meters (10 miles). There one of the balloons burst and the other brought the instrument safely to earth (100 miles away from the starting point, Kelly Field, San Antonio, Texas). He had a fairly complete record of the intensity of the

penetrating radiation as judged by that kind of instrument. (Obviously it was not highly accurate and it was not shielded against soft radiations.) Then he started a very complete program of measuring the intensity of the radiation with shielded electrometers on mountain tops and at various depths below the surface of lakes on mountain sides, in valleys. After the collection of a great amount of data, he set forth (1925) his views: (1) that the radiation in the atmosphere was complex, of four different degrees of penetrating power, the hardest having eighteen times the penetrating power of the hardest known gamma rays, (2) that the radiation was cosmical in origin, (3) *that all the radiation was of the photon nature and could be accounted for only on the assumption that heavy atoms were being created out of hydrogen in the vast regions of outer space,* helium out of 4 hydrogen, oxygen out of 16, silicon out of 28, iron out of 56. The photon energies corresponding to these changes in mass are about 28, 116, 216, 460 million electron volts. Millikan believed that his absorption curve for the cosmic radiation showed the presence of photons of these energies. Compare with 2.62 M.E.V.[1] for the gamma rays from Th C″, the most penetrating radiation from a radioactive substance.

Professor Millikan's announcement produced a sensation in America, at least in the field of journalism, not on account of his experimental findings, though these were of very great scientific importance, but on account of his hypothesis of "continuous creation" of heavy atoms. His view that the radiation consisted of photons was probably the only view held by physicists at that time. The enormous penetrating power of the radiation justified that view. That it was cosmic in origin seemed rather probable since there was no known variation in its intensity as to day, night, or season of the year. But that 56 hydrogen atoms would, at a certain instant, rush together and condense into one atom of iron —to accept that conclusion, the physicists of the world wanted more evidence than had been set forth.

Regener of Stuttgart.

Thus a great stimulus was given to the study of cosmic rays and very soon Millikan's measurements were greatly extended by him and his associates, and by other observers. Regener of

[1] M.E.V. = million electron volts.

Stuttgart sent instruments up three times as far (measured by air density, Millikan's instrument had 11 per cent of the atmosphere above it, Regener's 3 per cent) and nearly three times as deep (Regener sank an electroscope 750 feet below the surface of Lake Constance). Then airplanes were called into service, then stratosphere balloons. Naturally the data obtained showed large discrepancies due in part to the different degrees of shielding and, as we now know, to the different latitudes of the observations.

We turn now to the experimental aspects of this topic. There are *three ways for detecting and measuring cosmic rays:* (1) by the electrometer, (2) by the cloud chamber, (3) by the Geiger-Müller counter.

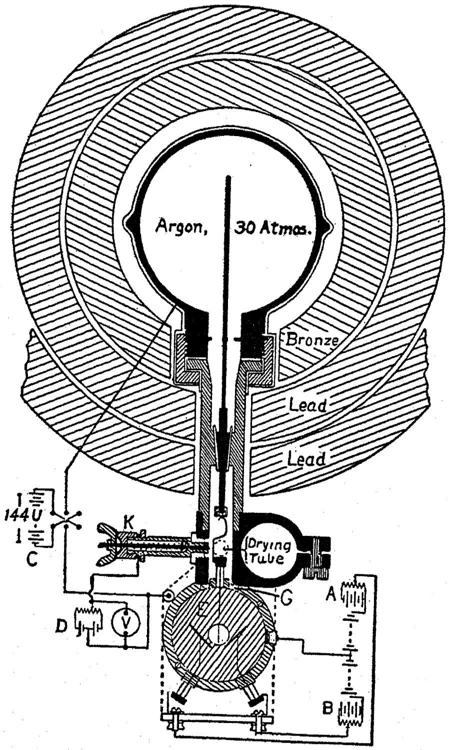


Fig. 13–3. The ionization chamber and electrometer used on the world survey. (A. H. Compton and J. J. Hopfield.)

1. The Electrometer and Ionization Chamber.

In Fig. 13–3 is shown the form of electrometer used by A. H. Compton and his numerous associates in their world survey of cosmic rays (1931–1932). The central metal rod is separated from the steel bomb and this from the heavily shielded case by excellent insulation. The rod may be connected by a fine spring to the electrometer needle and can be charged to a desired voltage. The position of the needle may be read by means of a microscope through the windows (below and to the right of E). The bomb is filled with argon to a pressure of from 30 to 50 atmospheres. At the latter pressure the ionization is nearly 70

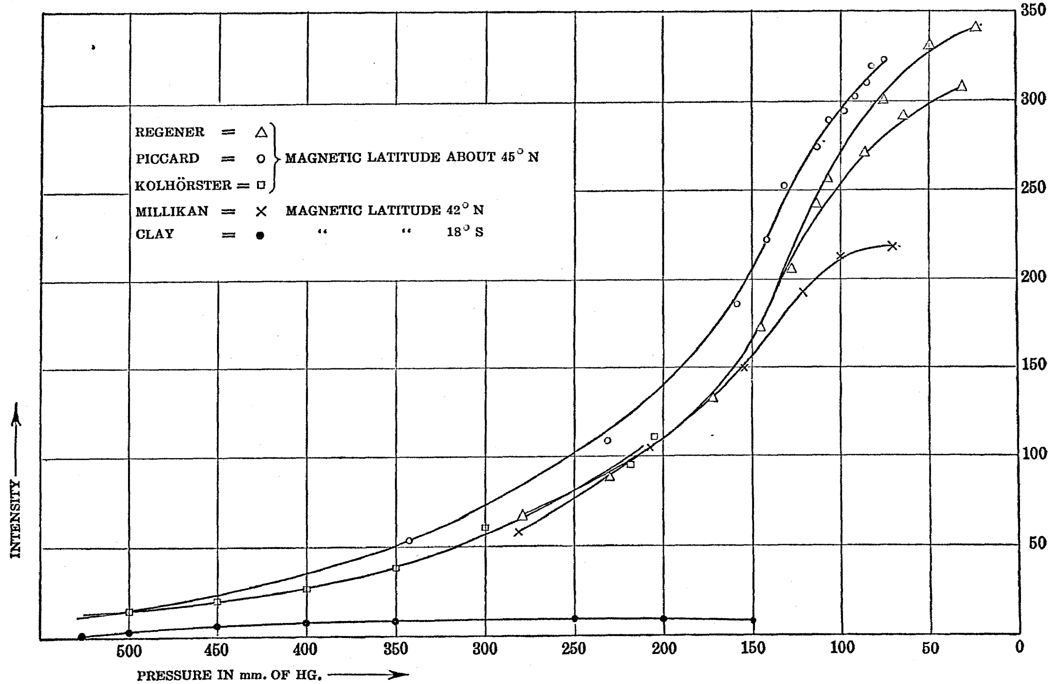


FIG. 13–4. Altitude measurements of ionization; only averages given. Mott-Smith and Howell's values nearly those of Millikan. Clay's measurements shown better in Fig. 13–16.

times that for air at atmospheric pressure. The lead shields, each one inch thick, cut out nearly all earth radiation, and the bronze shell cuts out radiation which might come from the lead. When the instrument had been calibrated, the rate of drift of the needle gave the number of ion pairs produced by the cosmic rays per cm.^{3} per sec. per normal atmosphere. This is the necessary quantity giving the intensity I of the cosmic rays at any point on, above, or below the earth's surface. It is understood, of course, that in the calibration of the instrument it has been taken into a deep mine or tunnel to escape from the action of cosmic rays.

The instrument just described weighs about 500 times as much as the one which Millikan sent up ten miles. In his

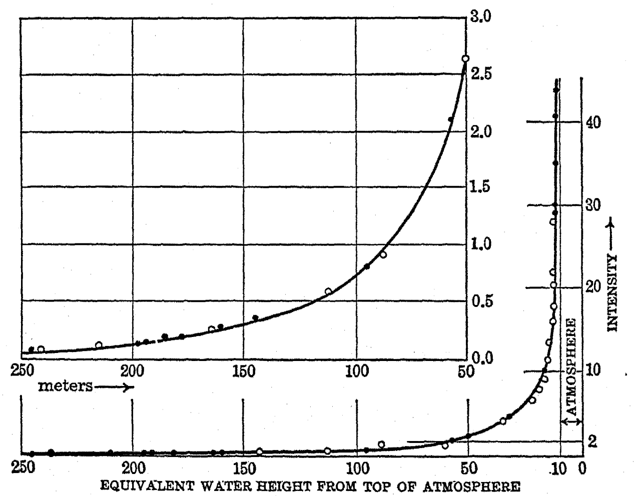
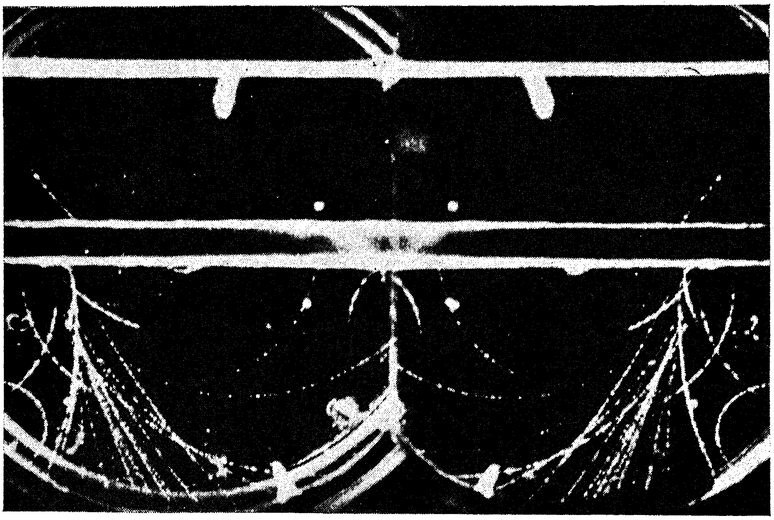


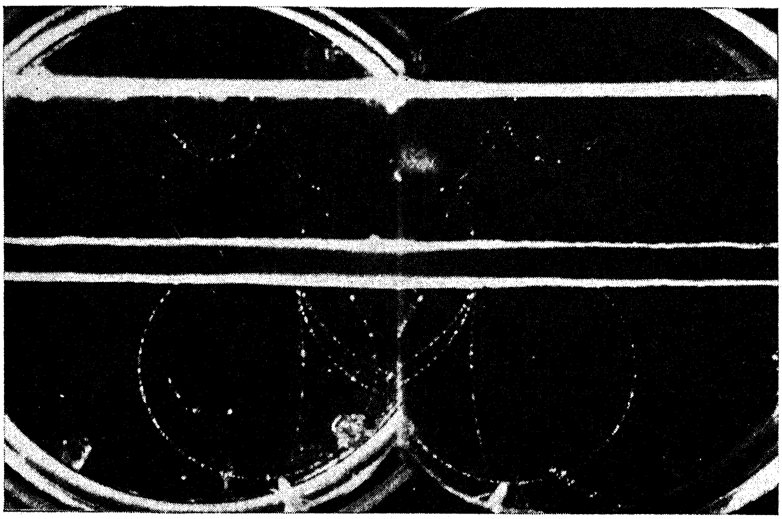
FIG. 13–5. A trace of cosmic ray activity is found 250 meters below the surface of water.

measurements under the surface of lakes and in airplane flights Millikan used shielded instruments. The chief data for nearly unshielded electrometers are shown in Fig. 13–4. Data obtained by Regener and by Clay (Amsterdam) for very deep water are plotted in Fig. 13–5.

The observations made by Mott-Smith and Howell during airplane flights with shielded and unshielded chambers fall between the Millikan and Regener curves; those made by Compton and Stephenson in stratosphere balloons are shown in Fig. 13–16.



a



b

FIG. 13–6. The picture on the left is a direct picture; the one on the right is by reflection. Magnetic field = 17,000 gauss. a. Shower of 28 electrons, positive and negative, coming from the lead bar; total energy = 2.5×10^9 volts. b. The production of single electrons in lead and of pairs + and −.

The work of Victor H. Neher who designed and, in part, constructed the excellent recording ionization instrument used by Millikan and Neher in all their measurements should be recorded.

2. The Cloud Chamber.

The cloud chamber method does not differ in principle from that of the Wilson chamber for making evident the tracks of α or β rays. But when there is imposed on the moving particles an intense magnetic field, 10,000 to 20,000 gauss, (nearly) at right angles to their paths, and when cameras are properly placed to take photographs and are synchronized with a high intensity light flashed at the right instant after the expansion of the chamber, the whole set-up is elaborate—and costly. As has been stated in previous chapters, *ordinary* photons (X-rays, γ-rays) do not of themselves produce tracks, though they may release electrons which write their beaded paths. But it will be seen that there are photons associated with cosmic rays, secondaries not primaries, which are of such great energies that they smash atoms to smithereens; a whole burst of particles, positive and negative, diverge from a point, with their motions on the whole in a definite direction indicating the direction of the unseen bombarding photon (Fig. 13–6).

The appearance of the track, whether beaded like that of an electron or continuous and heavy like that of a proton or an alpha, indicates the nature of the particle. The curvature of the track for a known magnetic field gives the energy (Appendix 13–1). If, to one looking along the magnetic field, the track is curved clockwise, the particle is negatively charged; if counterclockwise, positively.

3. The Geiger-Müller Counter.

In the Geiger-Müller (later called G-M) counting method (Appendix 13–3), the small brass cylinder C of Fig. 11–5 is generally replaced by a seamless cylinder of thin copper; this fits closely the inside of a thin-walled pyrex glass cylinder. A fine tungsten wire (diameter = 0.1 mm.) is sealed so as to be axial for the cylinder (Fig. 13–7). After the tube has been exhausted and baked out, it is filled with argon to a pressure of about 7 cm. A potential of about 800 volts is placed on the electrodes. When this cylinder, lying horizontally, replaces C of Fig. 11–5, the count at latitude 45 and

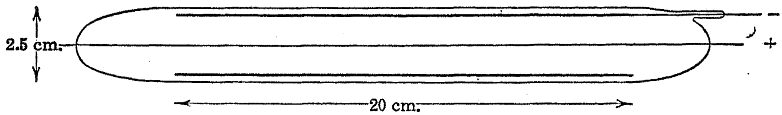


FIG. 13-7. When a potential of several hundred volts is applied between a thin-walled copper cylinder and a fine wire along its axis, we have a counting cylinder for cosmic rays.

sea level will be about 200 per minute. However, we can arrange two or three of these cylinders in parallel (Fig. 13-8) in circuits (Appendix 13-2) so balanced that there will be no count unless both or all these are triggered off at the same time. This will occur when a cosmic ray passes through all G-M cylinders. This is called a coincidence; a triple coincidence, if there are three cylinders in line; a fourfold or quadruple if there are four. The rays therefore will be limited to the solid angle subtended by the cross-section of one cylinder at the mid-point between the top and bottom cylinders. Thus we have a direction finder, a *telescope for cosmic rays.* When the line joining the centers of the circles is vertical, we measure the number of rays coming to us in a nearly vertical direction. For this case, with the distance between the axial lines about 2 inches, we may get a count of 10 per minute. When the line is at 45° to the vertical, we may measure the number coming from the East downward towards the West. The count would be about 6 per minute. Thus we may compare the E-W 45° with the W-E 45°, etc. When the line is horizontal and the axial lines are a few cylinder diameters apart, the count is zero except for accidentals. Cosmic rays arriving in the horizontal direction must traverse a great air thickness. Thus the rays come chiefly from the vertical.

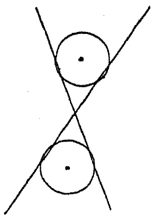


FIG. 13-8. A telescope for cosmic rays. A circuit can be arranged so as to count only those rays which pass through both cylinders.

Comparison of the Three Methods.

We may summarize now the points of excellence of these three methods: (1) The shielded electrometer gives the total intensity (number) of all the cosmic rays coming to it above the horizontal plane, that is, for the hemisphere above it. It makes possible the comparison of the intensities at various points on, above, or below

the earth's surface. When a differential instrument is used, one which offsets the cosmic ray ionization by that due to a definite amount of uranium, we have an instrument of great precision, one capable of making evident small variations of cosmic ray intensity. (2) The cloud chamber gives us the energies of the individual particles, the number of positives and negatives, the number of *bursts*. *To this method alone we owe the discovery of the positron.* (See Fig. 13–13.)

When a strip of lead or other metal is placed inside the cloud chamber, we can measure the loss of energy of a particle due to its passage through the metal; this by measuring the radius of curvature before and after. Thus Anderson has found that a cosmic ray electron, positive or negative, loses about 50 M.E.V. per cm.[1] of lead. The energy of the particles which traverse 750 feet of water, therefore, if electrons, must be of the order of 7×10^{10} volts.

(3) The G-M counter gives us the number of particles coming in a certain direction. By it Bothe and Kolhörster (1928), using the coincidence method, were led to the important conclusion that the rays which penetrated the two cylinders were electrified particles—they could not be photons. This placed in doubt the photon nature of the incoming rays and suggested the possibility that there would be a variation in cosmic ray intensity connected with magnetic latitude, this on account of the deflection of incoming electrified particles by the magnetic field of the earth. By means of this magnetic effect, using G-M counters, we have established the fact, practically all physicists believe, that the particles of greatest energy are positives.

And now we have opened up a tremendous topic. *Are the incoming rays photons or electrified particles?* Let us approach this question gradually.

Why cannot coincidences be due to photons? Suppose a photon passes through a G-M cylinder. Unless it releases an electron or smashes an atom, it will not disturb the cylinder. (The ionization in every one of the three methods is due to electrified particles, shooting at least part way through the chamber.) Consequently a photon would have to eject an electron in every one of the cylinders through which it passes in order to produce a coincidence. The probability of its doing so would be extremely

[1] This has been variously estimated as 20, 35, 50 M.E.V. See page 300.

small. But an electrified particle of sufficient energy would leave a line of shattered atoms all along its path. A "coincidence" would necessarily result when such a particle would pass through two or more cylinders.[1] But, it may be argued, this electrified particle might be a secondary, an electron, or other particle ejected from an atom by a photon. This is true. A considerable percentage of the ionizing particles, at sea level, even of great energy, are secondaries, or tertiaries. But we can prove by direct experiment that electrified particles of enormous energies are passing through these G-M cylinders. Let us arrange four cylinders vertically and place above them a sheet of lead 10 cm. thick. A certain number of coincidences per hour will be obtained. Then increase the lead thickness to 35 cm. The number of coincidences will be cut down only a few per cent.[2] From the particle point of view this means that most of the electrified particles, coming down vertically, after penetrating the atmosphere (equivalent to 10 meters of water or 90 cm. of lead) still have sufficient energy to penetrate 30 cm. of lead. If we accept Anderson's value of 50 M.E.V. as the loss per centimeter of lead, this would mean that most of the particles at the top of the atmosphere have an energy of at least six billion (6×10^9) volts. If then at the top of the atmosphere we have electrified particles, either primaries or secondaries, driving down with an energy of 6×10^9 volts, we can account for some of the phenomena which we observe. But if they are primaries, they will be influenced by the earth's magnetic field before they arrive at the atmosphere; if they are secondaries they will be uniform in number all around the earth at the atmosphere top; and if they have these energies they cannot be greatly influenced by this field. This topic is discussed later.

Can we have a rain of secondaries of such energies? If so, what must be the energies of the primaries, whether photons or electrons? When, in 1925, Millikan announced that the great penetration of the rays established them as photons, the maximum energies of which were 450 million volts, many physicists thought that he was stretching his imagination to the breaking point. But now with far more definite knowledge in regard to the

[1] Millikan, in *Electrons* (+ *and* −), 1935, Chapter XVI, presents strong arguments against the view here given.

[2] The authority for this statement is Clay's recent (1934) experiment.

properties of these rays, we must multiply his maximum energies by a few thousand.

From the point of view of photons as primaries, we might interpret the result of the above experiment as follows. A photon of great energy enters our atmosphere. It collides with an electron driving it forward. In our discussion of the Compton effect we showed that a very high energy photon would, in a head-on collision, give up practically all its energy to an electron. Such a collision therefore is a catastrophe for the photon; it practically ceases to exist. This collision might take place at the top of the atmosphere or just above our instrument. In the former case the electron would have to penetrate the atmosphere, losing energy all the way and suffering deflection by the earth's magnetic field. As before, we can show that its original energy must have been greater than six billion volts. One question which arises is, which would lose the more energy coming down through the atmosphere, a photon or an electron? There is no question regarding the ordinary radioactive photons (γ rays) and electrons; the former are about one hundred times as penetrating as the latter for equal energies. But now it appears probable that for the enormous energies in the cosmic rays the electrified particle has the greater penetration. So far then as energies are concerned, the electrified particles might be either primaries or secondaries.

The Rays Are Deflected by the Earth's Magnetic Field.

We return to the topic of the influence of the earth's magnetic field on these particles. As early as 1927 and in succeeding years Clay and his associates (Berlage, Woltjer, *et al.*) in sailings from Amsterdam to Batavia found that the cosmic ray intensity at the magnetic equator was less than at 45° N. by nearly 20 per cent. Millikan looked for a difference as between Pasadena and Churchill and found none. (Geographic latitude 34, magnetic latitude 41, compared with geographic latitude 59, magnetic latitude 70.) This confirmed him more than ever in the belief that the primaries were photons. But Arthur Compton organized (1931) a world survey. All of the observers were equipped with instruments similar to the shielded electrometer, Fig. 13–3. The data brought in by these observers, from seventy different stations scattered over the world, together with recent data by Clay (1934), are very strongly in favor of the view that the primaries, all pri-

maries, are electrified particles. The data by Clay and Berlage, Compton and his many associates, Millikan, Bothe and Kolhörster,

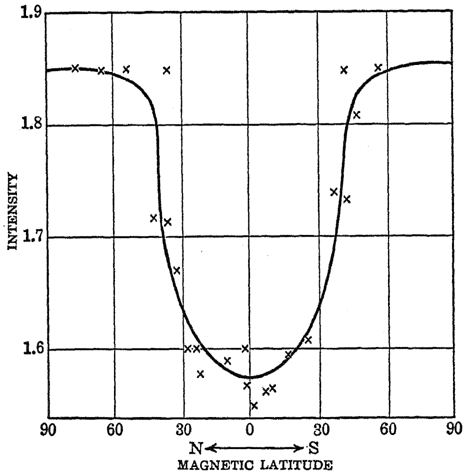


FIG. 13–9. The sea-level cosmic ray intensity is about 18 per cent greater towards the magnetic poles than it is at the magnetic equator.

are plotted in Fig. 13–9. The sea-level results show that the ionization is constant from 50° to 90° magnetic latitude, but that at the magnetic equator it is only 85 per cent of the polar value. But when altitude data are plotted (Fig. 13–10), we see that the difference rapidly increases. Finally Clay's recent (1934) measurements by pilot balloons (see Fig. 13–4) show that at an altitude of 15 kilometers above the magnetic equator the ionization is only a small fraction [1] (perhaps 1/25) of that at an equal altitude at Amsterdam (54° N.). To state it in other words, at the top of the atmosphere near the magnetic poles the ionization produced in an electrometer is between 500 and 2000 I; at sea level it is 1.87 I. The corresponding figures [2] for the equator are 20 I and 1.58 I. The rays at the equator are very few but very penetrating in comparison with those at the pole. The fact that the equatorial rays are very penetrating is further confirmed by the data (Fig. 13–11) obtained by Clay at Colombo (0° Lat.) and Genoa (45° N.). The curves

[1] There is great need for this data to be checked by a straosphere flight above the equator.

[2] I is the symbol for ion pairs per cubic centimeter in normal air per second.

show that the fraction of sea-level intensity which penetrates 7 meters of water is larger at the equator than at 45° N.

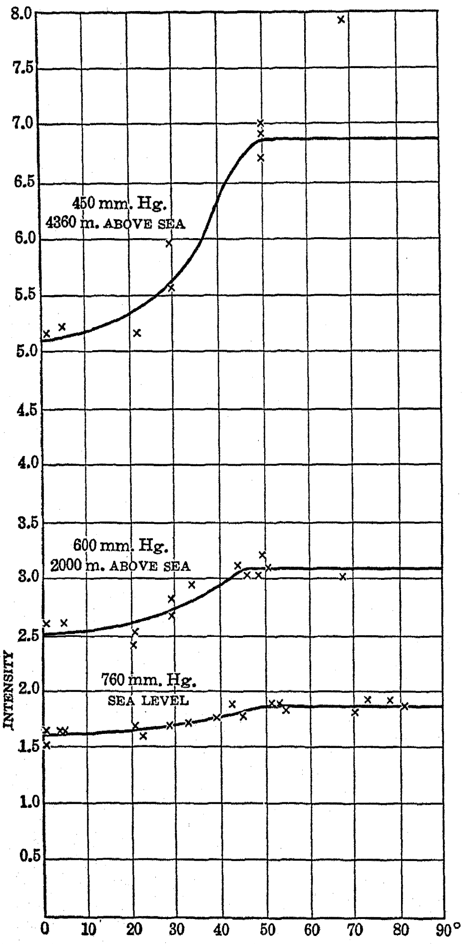


FIG. 13–10. The variation of cosmic ray intensity due to magnetic latitude increases rapidly with altitude.

If cosmic rays were photons, the ionization at the top of the atmosphere in shielded instruments would be constant around the earth (this on the assumption that there is no definite cosmic origin). This is very far from being a fact. *The conclusion is therefore unavoidable that a very considerable fraction of the primary rays are of the electrified particle nature. It seems highly probable that all are of that nature.*

It is true that all data are not as favorable to this conclusion as those here presented. Using less favorable data, Millikan in his latest text, *Electrons* (+ *and* −) [1935] argues strongly for the view that the chief part of the magnetic variation is due to the action of the earth's field on secondaries. But on that basis the earth's field could not influence the intensity of ionization at the top of the atmosphere, and Figs. 13–9, 13–10, and 13–11 show that in that region there is a large magnetic effect. But Fig. 13–16 shows this effect in most striking form, and supporting evidence is given in the pages to follow.

How Do We Account for This Magnetic Effect?

The earth is a great magnet but, unfortunately for computational purposes, not a symmetrical one. The so-called poles [1] are about 79° N., 80° W., and 72° S., 155° E. A glance at a magnetic map of the earth will show that it is difficult to compute with accuracy the magnetic latitude and longitude of any point on the earth's surface. Then what is the magnetic intensity at a point a few thousand miles above the earth's surface? But we can smooth out the irregularities and compute the approximate intensity of the magnetic field on or above the earth's surface.[2] We then find the effect of this field upon electrified particles driving more or less towards the earth from all directions in space. We continue to follow these particles as they are deflected by the field and we determine the energies they must have in order to reach the earth at a certain point. The mathematical operations are too complicated for presentation here. They were partly carried through by Carl Störmer in his attempts (for the past thirty years) to account for phenomena connected with the aurora borealis, by Epstein of the California Institute of Technology (1930), and especially by Abbé Lemaitre of Louvain and Vallarta of Massachusetts Institute of Technology (1933).

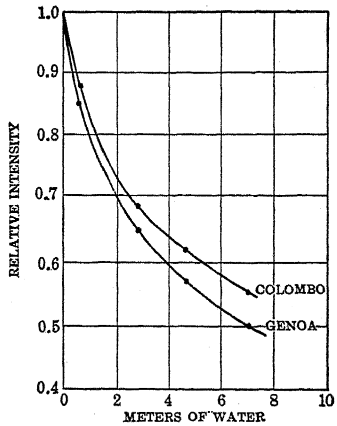


FIG. 13-11. The rays which arrive at the earth's surface at the equator are more penetrating than those towards the magnetic poles.

One general principle may be restated. A magnetic field does not change the energy of electrified particles moving through it. The component of the motion along the field is not altered; that at right angles is only changed in direction. In a uniform field a particle would describe a helix around the field. In a converging field these helices would tend to converge as narrowing corkscrews.

[1] The "magnet" is nearer to America than to Europe.

[2] We can simplify the problem by regarding the earth as a dipole of magnetic moment 8×10^{25} e.m.u. with the line of the poles near the center of the earth.

Let us suppose that we are placed 4000 miles above the earth at a point in the magnetic equatorial plane. To us looking "north," electrons going towards the earth would be turned clockwise from east towards the west. As they would get further in, they would come to stronger fields and would be turned more. Finally they might be moving at right angles to the radius vector or parallel to the earth's surface and after that they would be going away from the earth. The problem is a little like that of the magnetron. Electrons would go out radially from a filament F towards a cylindrical anode P and would strike P with an energy of V volts if V is the potential between P and F. But if a magnetic field is imposed parallel to F, the particles would describe a curved arc. If the magnetic field is increased or V decreased, this curved arc might just *not* touch P. The electron then would never arrive at the cylinder. Given a certain field and a potential just greater than this critical voltage, all the particles would arrive at P; for a potential just less, none would arrive. Similarly for electrified particles coming in towards the earth, we may find the limiting energies greater than that at which all particles will arrive at the earth's surface at a certain latitude or less than that at which none will arrive. The approximate relation is that the energy of particles (of electrons + or −) which arrive at latitude θ is $1.9 \times 10^{10} \cos^4 \theta$ volts.

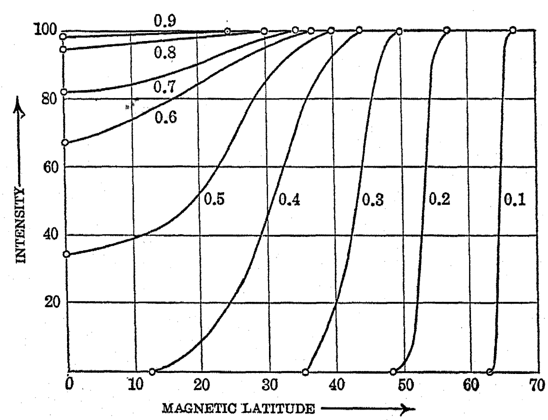


FIG. 13–12. Curves derived from theory indicate that of electrons having an energy of 10^{10} volts (curve 0.4) all would reach the earth's surface at 43°, none would arrive at 10° magnetic latitude.

An important part of the result of this mathematical analysis is set forth in Table I and in Fig. 13–11. Let us consider the curve $x_0 = 0.4$ in the figure. We see that electrons of energy 0.95×10^{10} volts or protons of 0.86×10^{10} or alphas of 1.56×10^{10} would all arrive at any point between latitudes 44 and 90 but none would arrive at points between latitude 10 and the equator. (This of course leaves out of account absorption by the earth's atmosphere.)

TABLE I

Equivalent electron voltages corresponding to various values of x_0

x_0	Electrons (10^{10} Volts)	Protons (10^{10} Volts)	α-Particles (10^{10} Volts)
0.1	0.0596	0.01722	0.01842
0.2	0.238	0.1618	0.2308
0.3	0.536	0.449	0.760
0.4	0.954	0.861	1.564
0.5	1.490	1.397	2.625
0.6	2.145	2.050	3.928
0.7	2.920	2.823	5.46
0.8	3.821	3.719	7.25
0.9	4.830	4.729	9.27
1.0	5.96	5.85	11.52

By assuming that the incoming particles were electrons of energies lying between 0.5 and 4×10^{10} volts, Arthur Compton showed that the data obtained in the world survey fell fairly well on the curve given by Lemaitre and Vallarta's theory. But an inspection of Table I shows that instead of electrons of 0.5×10^{10} volts, he might have substituted protons of 0.42×10^{10} volts or alphas of 0.75×10^{10} volts. Similarly for the particles of greater energy. In other words, there are numerous solutions. Then, too, the ranges of these particles must be considered. Thus the world survey did not definitely answer the question as to the nature of the cosmic rays.

But it did show by the magnetic effect that they are partly of the nature of electrified particles. And the associated theory shows that these particles cannot originate in the earth's atmosphere. For the *maximum* magnetic effect would be found in the magnetic equator where the intensity is 0.3 gauss. Using our frequent relation $Hev = mv^2/R$ and making v nearly equal to the velocity of light, we have $R = 6 \times 10^{30}$ cm., where R is the radius

of curvature of the path and m is the greatly increased mass of the electrified particle (due to its great energy). But, if we can extend the law holding in radioactivity to particles of such great energy, the range r in air is 0.2×10^{30} cm. *Thus the radius of curvature is 30 times the particle's range.* The greater the range, the greater the radius of curvature, or the less the departure from a straight line path. Thus, if a particle had a range of a mile in air, it would have a radius of curvature of 30 miles and in its one mile of air it would be bent from its original direction only 1/30 of a radian or 2 degrees. If the atmosphere were uniform, increasing the range to 10 miles would not change this angular deflection. But with our rarified atmosphere extending up to over a hundred miles, the deflection would be increased, not sufficiently however to account for the magnetic effect which has been found.

Summarizing this, we state—theory shows that if the primary rays consist of electrified particles lying between certain energies, the intensity at sea level for points between (say) 40 and 90 magnetic latitude should be constant but below 40 should decrease. The experimental facts are entirely in accord with this conclusion. Moreover, theory shows that this magnetic effect cannot be due to particles originating in our atmosphere, that is, released by photon or neutron bombardment. (Regener shows (page 303) there are no neutrons at the top of the atmosphere.)

More Particles Come from the West than from the East—Especially near the Equator and in High Altitudes.

There is another effect which has a bearing on this question. Let us arrange a threefold or fourfold G-M coincidence counter to find the number of particles coming in a certain direction. At sea level in latitudes 45–90 the maximum number come down vertically and fall off symmetrically, on all sides of the vertical, to zero for the horizontal direction. But as we approach the magnetic equator the West-East count is greater than the East-West. Especially is this true as the altitude increases. Now we have already shown that if we have positively electrified particles coming in from outer space, they should be given a west-to-east deflection. We conclude therefore that the most penetrating of the rays are positively electrified particles, protons or *positrons*,[1]

[1] We here anticipate the account of the discovery of a new particle, a positron. See Fig. 13–13.

rather certainly not alpha particles. Some of the experimental results are shown below.

T. H. Johnson of the Bartol Institute, Swarthmore, Pa., has been one of the most active workers in this field. He has made measurements on Mount Washington, in Mexico City, in Peru,

TABLE II

Observer	Magnetic Latitude	Height above Sea (Meters)	Inclination to Vertical	$\frac{100 \times (W.-E.)}{E.}$			Hours for One Observation
Johnson	29° N. Mexico City	2250	30°	6.8 ± 2			12 hours E.
			45°	10.2			16 W.
				6.6			14 periods
			55°	10.3			each
Alvarez, Compton (1933)	29° N. Mexico City	2310	15°	3 ± 2			14 hours E.
			30°	5.5			14 hours W.
			45°	12			28 periods each
Stevenson (1933)	48° N. Colorado	3100	30°	2.2 ± 0.6			
			45°	2.3			
Johnson (1933)	0° Peru	0	15°	6	6.5	7	
		3500	30°	8	10.	13	
		4500	45°	10	13.	14	
Rossi (1934)	11° N. Eritrea East Africa	2370	15°	1 ± 2.			15–30 hours
			30°	15.7			(shielded)
			45°	18.8			2 cm. lead
Johnson, Stevenson (1933)	51° N. Swarthmore, Pa.	0	15°	1. ± 0.7			73 hours E.
			30°	3.			90 hours W.
				2.5			
Hull (1933)	56° N. Hanover, N. H.	180	45°	1. ± 1			70 hours E.
				0 ± 1			50 hours W.
							100 periods each
Clay (1933)	18° S. Bandoeng, Java	760	15°	0			2–50
			30°	15			12 cm. lead
			45°	10			
				no lead	30 cm. lead		
Korff (1934)	29° N. Mexico City	2175	45°	9.	6.		Note large experimental error
	3° S. Peru	2325	30°	9 ± 2	2 ± 3		
			45°	14 ± 1.4	9 ± 3		
	2° S. Peru	4300	50°	16 ± 3	9 ± 3		

and in various other places. Some data obtained by him and other observers are shown in Table II.

These data gathered in various parts of the world, though slightly chaotic, tell the same story—the most penetrating radiation is due to positively charged particles, for these are the particles, as judged by the deflection due to the magnetic field, that are most in evidence at the equator. But data obtained by coincidence methods are subject to large statistical variations. Consequently, the final result depends partly on the way in which the data are treated. This is illustrated by the result for Mexico City at 45°; we have the numbers 6.6, 10.2, 12.0; yet each of these numbers was obtained by taking several thousand counts. The author took about 15,000 counts west and 20,000 east in about 100 periods each (alternating) and found that, by ordinary averaging, the west was greater than the east at 45° by 1 per cent; but, if the data had been weighted by quantities proportional to the times of counting, the difference was practically zero. Attention is called to the fact that the data obtained by Rossi, Clay, and Korff are not logically consistent. It would appear that at least one of these observers did not take enough counts to offset statistical variations. Korff argues from his data that less than half the particles causing the west excess are able to pierce 30 cm. of lead—therefore that these particles did not come from outside the atmosphere. But it is dangerous to base an argument upon absorption since the law for these particles is unknown.

Labor—and Tragedy.

Perhaps the reader, considering the data of this table and the curves which represent the results of the world survey, can get some appreciation of the labor spent to solve this aspect of the cosmic ray problem. Hundreds of pounds of apparatus have been carried, frequently by the observers themselves, to remote regions; tropical heat and arctic cold have been endured; in one case the utmost of tragedy came—both observers on the Mount McKinley expedition, Allen Carpe and Theodore Koven, lost their lives by falling into a glacier.

But the search continues. Airplane ascents to great heights have given place to risky, uncertain, stratosphere balloon journeys. There also has been tragedy and near tragedy. The earth and all its atmosphere has become one great physics laboratory.

THE CLOUD CHAMBER METHOD

The Discovery of the Positron.

As has been stated earlier, when a strong magnetic field is imposed upon a cloud chamber, we are able not only to identify the nature of the charged particle whose track is photographed but also to measure its energy. In the photographs here shown (Figs. 13–6 and 13–13) the field is away from the reader, hence electrons will be turned clockwise. The more nearly straight the track, for a definite particle, the greater the energy. How then

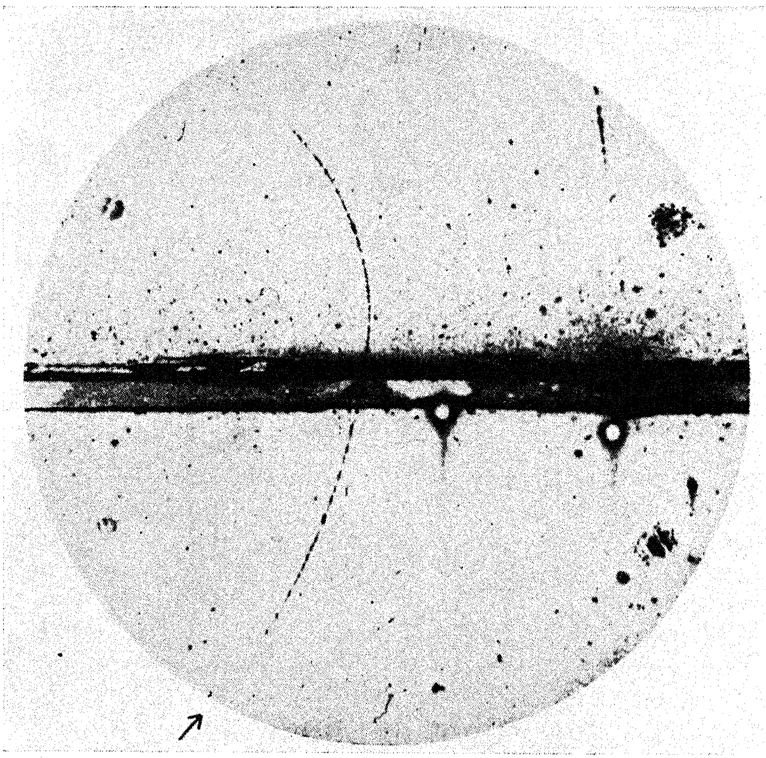


FIG. 13–13. An historic photograph, announcing the discovery of the positron, 1932. A 63 million volt positively charged particle enters the cloud chamber as shown by the arrow, passes through 6 mm. of lead, and emerges with an energy of 23 million volts. The length of the latter path is at least ten times greater than the possible length of a proton path of this curvature. ($H\rho$ for the lower curve was 2.1×10^5; to change to volts, if an electron or positron, multiply by 300.) (Carl D. Anderson, *Physical Review*.)

would we interpret the tracks of Fig. 13–13? Would it be possible for an electron of 23 million volts, curving down and clockwise, to pass through a lead plate (6 mm. thick) and emerge with an energy of 63 million volts? Or must we assume the existence of a new kind of particle, like an electron in every way except that it is oppositely charged, and allow it, with an energy of 63 million volts, to move up, counter clockwise, and pass through the lead plate, then emerge with an energy of 23 million volts? This seemed a revolutionary idea, but finally there seemed to be no escape from it. In *Science*, 1932, Anderson, of the California Institute of Technology, announced the discovery of *the positron*. Since then positrons have been obtained in a variety of ways, some of which will be indicated in the next chapter.

The physicists of the world had become accustomed to think of the electron as the ultimate unit of negative electricity and the proton or hydrogen nucleus as the smallest mass associated with a positive unit; then came the *neutron* and three months later the *positron*. *These discoveries changed our views concerning the ultimate units of the universe and increased its complexity manyfold.*

Information Derived from Cloud Chamber Photographs—and Conflicting Conclusions.

The vertical cloud chamber, in an intense horizontal magnetic field, besides leading us to the discovery of the positron and besides giving us the energy of individual particles, has made visible to us the extraordinary universe in which we live—this universe of particles of enormous energies. An inspection of Fig. 13–6 will make this clear. These photographs were taken in the California Institute of Technology by Anderson, Millikan, Neddermeyer, and Pickering. The student should consult their paper[1] for a full discussion of these and several other photographs. The intense magnetic field, 17,000 gauss, which causes these particles to describe curved paths, was produced by a current of 1600 amperes, at 250 volts, in water-cooled copper tubes! (Cost per hour?) For these photographs G-M cylinders were placed one above and one below the cloud chamber and the mechanism was so arranged that every time there was a coincidence an expansion of the chamber took place and a photograph was taken. If an electrified particle must pass through both G-M

[1] *Phys. Rev.*, Vol. 45, p. 352, 1934.

cylinders in order to produce a coincidence, it must have passed through the chamber and its track should be seen as a vertical beaded line. But no such track is ordinarily seen in photographs at the California Institute of Technology laboratory. This is perplexing. But so is the explanation given by the authors. Their argument is that if a primary electrified particle were responsible for a coincidence, its vertical path should be seen in the cloud chamber. But the same argument would hold for a secondary particle—unless so many secondaries were produced in their special arrangement as to blank a single vertical streak. Moreover, if there are present all the electrified particles seen in Fig. 13–6, we ought to have coincidences all the time in cylinders near one another, however placed, whereas if the cylinders are in the same horizontal plane we get no coincidences. Moreover, if they are above one another the number is proportional to the solid angle.

But the authors lose sight of one very important fact. When a sheet of lead is placed in or around a cloud chamber, new particles are produced which are not necessarily part of the coincidence phenomenon. This point has been placed beyond question by an important experiment which has just been reported from the Harvard laboratory.

However, there is no doubt regarding one point emphasized by the California Institute of Technology workers. Showers of electrified particles are seen, all the individuals of the shower, positrons and negatrons [1] having initially nearly the same direction. This gives the direction of the bombarding photon. In general there is no electrified particle track (only 1 in 1000 cases) leading up to the origin of the shower. Apparently a photon is the source.[2] (But the photon which gave rise to the shower in Fig. 13–6 could not have passed through the G-M cylinders; the direction is not right.) So, too, low energy photons must be responsible for the ejection of the negatrons or positrons which describe circles in Fig. 13–6. *Hence there must be showers of photons!* Unseen! But those who claim that the primary cosmic rays are electrified particles hold that all these photons are second-

[1] The term "negatron" is occasionally used to indicate the original electron. But usage is now inclining to the old term "electron" for the negative unit and "positron" for the positive unit of electricity.

[2] But as this text goes to the publishers, the author learns that Stevenson and Street have definitely proved, by cloud tracks, that electrons may produce showers.

aries produced by the collision of the primary protons or electrons or alphas with atomic nuclei.

Coincidences Are Due to Electrified Particles Passing through All Cylinders.

In the *Physical Review,* June 1935, Street, Woodward, and Stevenson of Harvard show that when a threefold vertical counter is used with 4.0 cm. of lead between the upper two cylinders and a cloud chamber between the lower two, controlled by the coincidences, between 90 and 94 per cent of all the photographs show a single vertical track. Moreover, practically no coincidences occurred when the central cylinder was slightly out of line.

Another result obtained by the Harvard authors is an estimate of the absorption of rays of various energies. This was done by changing the thickness of the lead plate between the upper pair of cylinders. The results are:

For Particles of Energy Range in M.E.V.	0–680	680–1150	1150–1900	1900–2500
Average Loss per Cm. of Lead	45	31	25	20

It is seen that these values are considerably less than Anderson's recent value of 57×10^6. The explanation is that the latter observer considered chiefly particles of only a few hundred M.E.V. energy.

The reader will observe that the Harvard workers have found that the greater the energy of the particles, the less the loss [1] per cm. of lead. This is in line with other estimates. It supports the view that, so far from photons being necessary for explaining the high penetration of the rays, compared with electrified particles of great energy they are highly absorbed.

A Strange Phenomenon—Bursts.

There is one obscure phenomenon connected with cosmic rays that must have an important place in the physics of the future. It is that of "bursts." Ordinarily a ray releases about 120 ion

[1] The reader with communistic leanings will conclude that cosmic rays belong to the capitalistic class—the greater the wealth, the easier it is to avoid life's obstructions. This might be an argument for taxing the rich—or relieving them of taxes!

pairs per cm. of path in an ionization chamber, but a few times per day there are released suddenly a few million. What has happened? No present-day theory of the atom can account for this phenomenon. Thus we see that these bombarding particles of enormous energies are opening up a new world for exploration.

Do the G-M Counter and the Ionization Chamber Measure the Same Radiation?

It has been argued rather strongly by some observers that the G-M counter does not register accurately the primary rays, that it is especially influenced by secondary radiation. But the experiments which have just been described, together with very recent experiments by Regener and his associates, dispel that view.

The Stuttgart workers have been sending up more electroscopes and of various kinds. But especially *they have sent up to a height of 28 kilometers a G-M cylinder together with a complete amplifying and recording outfit.* The cylinder was of a very special design

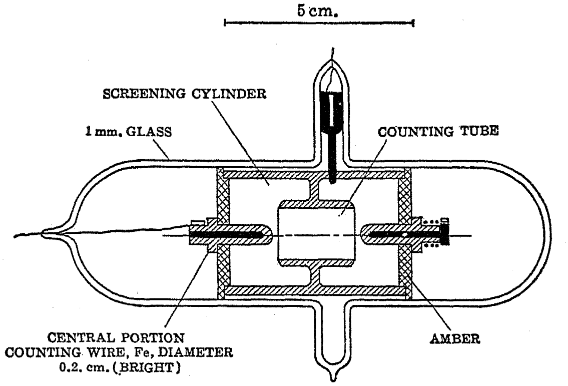
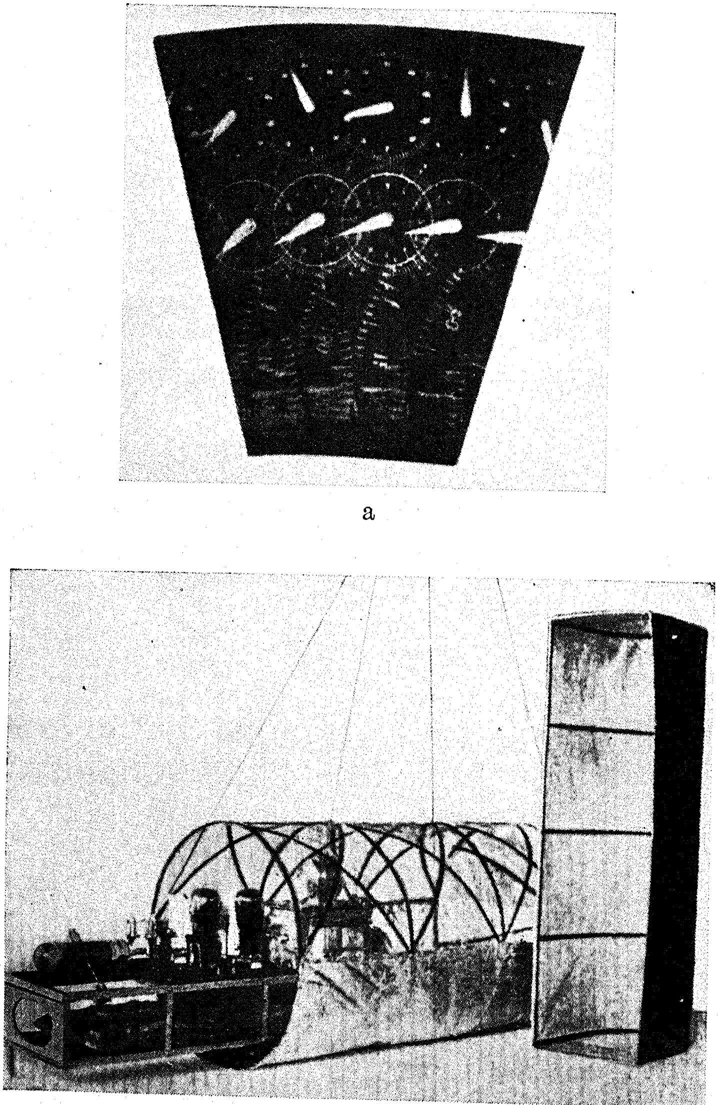


FIG. 13–14. For this tube used by Regener the count was independent of orientation.

(Fig. 13–14). It was constructed so as to present practically the same cross-section for all directions. It was tested in this regard and found satisfactory. The high voltage battery, 1200 volts, weighed only 1.2 kg. It was specially shielded from temperature changes. (The voltage of dry cells may change considerably with temperature change.) The ionization impulse due to a cosmic ray operated, by means of a small electromagnet, the escapement of a



a

b

FIG. 13–15. This whole mechanism, Geiger counter of Fig. 13–14, amplifier, camera, 1200 volt battery, barometer, was sent up to a height of about twenty miles—and functioned up and down. (The Regeners of Stuttgart.)

watch. This was photographed (Fig. 13–15) every 4 minutes. Thus the total count and the rate of change of count were determined. The barometric pressure and temperature were also recorded.

When the counts per 4 minutes were plotted against the height of the balloons and a smooth curve drawn through the points, it was found that this curve agreed practically at all points with the corresponding curve for the ionization chambers, if the two curves were made to agree at one point. This up to a height of 28 kilometers. Hence the important conclusion was drawn that the G–M cylinder and the ionization chamber measure the same disturbances. The same conclusion was reached when G–M cylinders were sunk below the surface of Lake Constance. But here the count was small and the conclusion doubtful.

One other important result came out of the recent Stuttgart experiments. Ionization chambers which had previously been sent up to the top of the atmosphere were now lined (a) with cellophane, (b) with paraffin, and sent up. The object was to test if there are any neutrons in the cosmic rays. Had there been, both of the lined chambers, being now filled with protons, would have shown a large ionization. But the curves for the various chambers agreed. *The cosmic rays are not composed of neutrons.*

Convincing Evidence that Primaries Are Electrified Particles.

We desire to give another point of view regarding the intensity of the rays. The quantity I plotted in Fig. 13–4 measures the total ionization due to the rays coming to the instrument from the hemisphere above. By means of the curves we are able to compute the intensity coming in a definite direction within a certain small conical angle. Let us call the vertical component V. Then when V is plotted against height (measured by pressure) we have the curves of Fig. 13–16. Here we see that Clay's curve gives $V = 12$, nearly, at the top of the atmosphere (or at least at a height of 15 km., and the curve is flat at that height); this at magnetic latitude 18 S. For the Bowen-Millikan curve $V = 200$ at the top of the atmosphere; magnetic latitude 42° N. For Compton-Stephenson-Millikan at 52° N., $V = 550$. Compton estimates that above the magnetic pole the value of V would be about 2000. This would compare with $V = 12$ above the equator. Stratosphere flights above the magnetic pole and above the equa-

tor are needed to check these values. (Perhaps some courageous navigator will fly around the earth at a height of 20 miles [1] with an ionization chamber aboard and include the north and south magnetic poles in his journey!) In any case it appears highly probable that the ratio 30 to 1 that we now have for the intensity at the top of the atmosphere for latitudes 50 and 18 may be easily changed to 100 to 1 for higher latitudes. *We thus feel that it is safe to claim that not more than one per cent of the intensity of cosmic rays is due to incoming photons.*

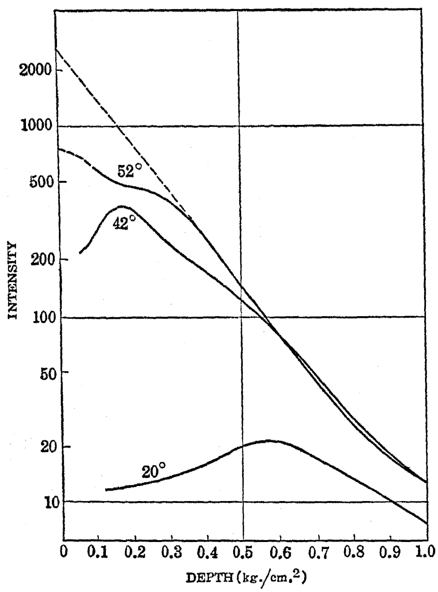


FIG. 13–16. The *vertical* intensity (computed) of the radiation at 52° latitude at the top of the atmosphere is about 600; at 20° about 12. Overwhelming evidence that cosmic rays are electrified particles. (Compton.)

The above method of analyzing the rays tells us something about the nature of the incoming particles. The humps in these curves show "ranges" of certain components of the rays. (It will be recalled that α rays from polonium have a definite hump near the end of their range (Fig. 11–7). However, the hump is unnecessary for an explanation of the effect we are considering. Merely the ending of the path will do.) Consequently we are led to the view that there are three kinds of electrified particles in the primary rays—probably α rays which are the most easily absorbed, then electrons + and −, then protons.

One puzzling feature of the rays is that it must be the more easily absorbed component which is responsible for the "showers." They increase with altitude. What kind of mechanism must it be that will release millions of ions in one shot? Where do the ions come from?

[1] The pressure of the air would be about 10 mm. Is it probable?

What Are the Cosmic Rays?

The primary rays are not neutrons, they are not photons. In the author's opinion the best view inclines to the belief that they consist of alphas, electrons and positrons, protons. The secondaries contain all the above, together with ions and photons. But there seem to be no neutrons.

Whence Come the Cosmic Rays?

The outstanding view is that they are electrified particles of energies lying between 10^9 and 10^{12} volts; that they fill space with an energy greater than that from all the stars; that, apart from the magnetic influence of the earth, they arrive uniformly from all directions; that there is therefore no astronomical body which is a source.

There is some experimental evidence that there is a slight variation, 0.1 per cent, of the intensity of ionization in phase with sidereal time. This might be interpreted as due to the rotation of our galaxy which gives us a velocity of 300 km. per sec. towards declination 47° N., right ascension 20 hrs. 55 min. The source of the rays would therefore be beyond our galaxy, perhaps at a distance of 10^{10} light-years! This number is an easy one to write. But when we remember that a light-year is 63,000 times the distance between the earth and the sun, or nearly 6×10^{12} miles; that α Centauri, the nearest fixed star, is 4.35 light-years away, about 25×10^{12} miles, we see that 10^{10} light-years is a sizeable distance, about 6×10^{22} miles or 6×10^{14} times the distance from us to the sun. (The reader, of course, may try to visualize this by letting one inch or one millimeter represent a million miles—or the distance to the sun—or the distance to a star!)

Are there super-radioactive atoms in outer space? Or were they there 10^{10} years ago? Or were there some really violet happenings in the universe in the very distant past, happenings of which we are now learning by means of this cosmic dust?

Dilemmas.

If these rays come from sources 10^{10} light-years away, would they not be absorbed by the matter in space? Astronomers answer, no.

X-rays produce mutations in plant and animal life (see Chapter 17), probably by their action in energizing electrons in atoms. Cosmic rays, through the action of either primaries or secondaries, produce bursts in atoms. Are cosmic rays, as a result of constant bombardment through all the ages, responsible for the evolution of life on our planet?

The spectral lines that we observe originating in the distant visible parts of our universe are shifted towards the red; the photon energy is decreased—the greater the distance, the greater the decrease. Do cosmic rays coming from vast distances continually lose energy?

If alphas, electrons, protons are flying about in space, would we not have collisions resulting in the formation of atoms as Millikan proposed? But there are no appreciable quantities of atoms in space.

Would we not have a Maxwellian distribution of energies among the particles due to bombardments? But space is vast and, to one of these particles, empty until the particle encounters matter. Then it may hit something.

At present we do not know how or why cosmic ray particles originate.

Poets at times glimpse mysteries and see visions. Thus wrote Browning:

> ". . . Oh, never star
> Was lost here but it rose afar!"

> ". . . If I stoop
> Into a dark tremendous sea of cloud,
> It is but for a time; I press God's lamp
> Close to my breast; its splendor soon or late
> Will pierce the gloom. I shall emerge one day."

CHAPTER 14

TRANSMUTATION OF THE ELEMENTS

INDUCED OR ARTIFICIAL RADIOACTIVITY

The story of transmutation has been interrupted in order to introduce the reader to the positron and the manner of its discovery. It will be noted that that discovery was brought about by utilizing cosmic rays as bombarding particles; by making visible in a strong magnetic field the paths of the primaries and secondaries; by an ingenious trick of putting into the cloud chamber a strip of lead, thus definitely giving the direction of the particle and its charge. After the clear proof of the existence of this strange particle, positrons appeared very frequently.

In radioactivity we met with alphas, electrons, as particles of atomic disintegration: we now must be prepared to add to the list protons, neutrons, positrons.

The Joliots Discover Artificial Radioactivity.

In Chapter 12 we described the experiment due to the Joliots which led to the discovery, by Chadwick, of the neutron. The Joliots missed the discovery of the neutron by misinterpreting the phenomena. Incidentally it may be recorded that they also missed the discovery of the positron. Looking back over their plates, they found tracks which looked like those of electrons going in the wrong direction, as judged by the curvature in the magnetic field. They could not understand why electrons should go in that direction, but they appeared to be doing so (there was no lead strip in the chamber). After just missing two great discoveries in 1932, the Joliots apparently resolved to let nothing go by.[1] In January 1934 they made a great discovery—that of *induced radioactivity*.[2]

[1] Also they discovered the "materialization of radiant energy" or the changing of photons into matter. This will be discussed later. And they proposed the "antineutrino." This will not be discussed.

[2] For this discovery they have been awarded the Nobel Prize in chemistry, 1935.

They had been bombarding aluminium with alphas from polonium: they obtained neutrons which they were now able to identify by putting a sheet of paraffined paper or cellophane in front of the Geiger counter. It will be recalled that neutrons knock protons out of material rich in hydrogen. But a new phenomenon appeared. Placing before the Geiger counter only the aluminium foil which had been irradiated by alphas, they obtained counts which decreased exponentially with time. There was a definite half-life of three minutes, fifteen seconds. *The aluminium had been made radioactive.*

The transformations involved in this experiment may be represented[1] thus (energy changes not represented):

$$_{13}\mathrm{Al}^{27} + {_2}\mathrm{He}^4 \rightarrow {_{15}}\mathrm{P}^{30} + {_0}n^1.$$

But the phosphorus is unstable; it changes to silicon with the emission of a positron as shown[2] by

$$_{15}\mathrm{P}^{30} \rightarrow {_{14}}\mathrm{Si}^{30} + {_{+1}}e^0.$$

It is easy to write down these relations. How do we know that they are correct?

Instead of a Geiger counter we may use a cloud chamber. The aluminium foil covers a small opening in the wall of the chamber: then the tracks of the positrons are seen; their energies can be measured; successive photographs after withdrawal of the polonium alpha ray source show that the number of positrons decrease exponentially with time. Before the withdrawal of the alpha source, the neutrons drive out protons; electrons also escape, due to other transformations. All of this is shown by the photographs of Fig. 14–1 a, b, c, taken by von Meitner and Philipp of the Kaiser Wilhelm Institute, Berlin.

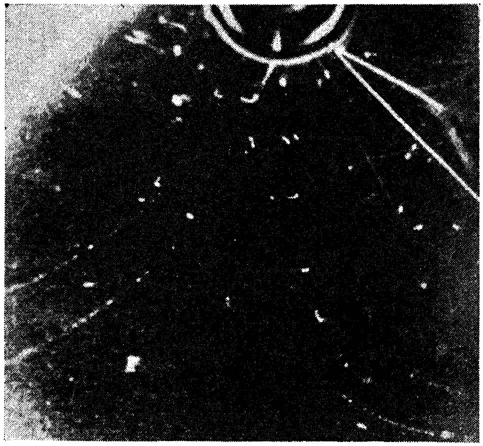
The Joliots observed that positrons did not appear immediately upon exposing the aluminium to alpha rays. In fact the

[1] We know now that an isotope of aluminium is itself artificially radioactive and it can be obtained in at least five different ways.
$_{13}\mathrm{Al}^{27} + {_0}n^1 \rightarrow {_{13}}\mathrm{Al}^{28} + \gamma$; $_{13}\mathrm{Al}^{27} + {_1}\mathrm{H}^2 \rightarrow {_{13}}\mathrm{Al}^{28} + {_1}\mathrm{H}^1$; $_{15}\mathrm{P}^{31} + {_0}n^1 \rightarrow {_{13}}\mathrm{Al}^{28} + {_2}\mathrm{He}^4$;

$$_{14}\mathrm{Si}^{28} + {_0}n^1 \rightarrow {_{13}}\mathrm{Al}^{28} + {_1}\mathrm{H}^1;\ {_{12}}\mathrm{Mg}^{25} + {_2}\mathrm{He}^4 \rightarrow {_{13}}\mathrm{Al}^{28} + {_1}\mathrm{H}^1.$$

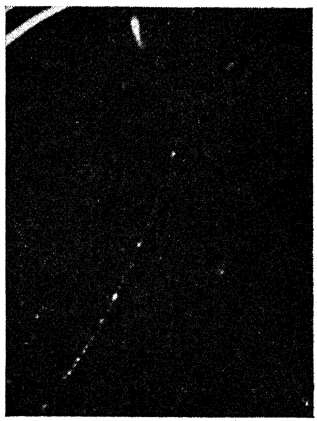
[2] The symbol for the positron in those equations is $_+e$. To balance the equation it should be written in the form $_{+1}e^0$. But the simpler symbol is retained. Obviously $_-e$ is the electron.

FIG. 14–1. Aluminium bombarded with α particles changes over to radioactive phosphorus, and ejects (a) protons, positrons, electrons; (b) after bombardment ceases, the phosphorus continues to send out positrons; (c) ten minutes later. The number of positrons decreases exponentially with time. (von Meitner and Philipp, *Naturwissenschaften*, 1934.)



a

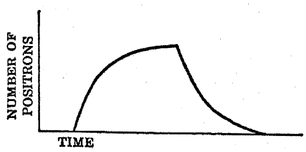
b



c

curve showing the rise and fall (Fig. 14–2) is similar to that for the accumulation of radon from radium. It is the characteristic curve of radioactivity.

FIG. 14–2. Aluminum bombarded by alphas from polonium produces radioactive phosphorus. The rise (and fall after bombardment ceases) of the number of positrons given off are characteristic of radioactivity.



The Joliots carried out chemical tests. The irradiated aluminium foil was quickly dissolved in HCl; the salt, dried, was inactive. The gas in a thin-walled tube contained the activity. It presumably contained hydrogen phosphide, identifying phosphorus as the active material. Similar tests were carried out by them for other elements—boron, magnesium. Here are two relations:

$$_5B^{10} + {}_2He^4 \rightarrow {}_7N^{13} + {}_0n^1; \qquad {}_7N^{13} \rightarrow {}_6C^{13} + {}_+e$$

$$_{12}Mg^{25} + {}_2He^4 \rightarrow {}_{13}Al^{28} + {}_1H^1; \qquad {}_{13}Al^{28} \rightarrow {}_{14}Si^{28} + {}_-e.$$

Rush Hour in Twentieth Century Physics.

Thus it was announced early in 1934 that light elements might be transformed not only into new elements but even into radioactive elements by bombarding them with alpha particles. Then the hunt was on. All the physics laboratories in the world that had access to radium or that possessed "guns" for projecting light particles, alphas, deuterons, protons, turned their special kind of gun on the atoms of chemical elements. Fermi, Amaldi, and their numerous associates in Rome, using neutrons as bombarding particles, went through nearly the whole chemical range within a year. The Cambridge workers, Rutherford, Chadwick, Cockcroft, used alphas and protons of various energies. Lawrence and Livingston with their cyclotron outdid the world with their high-energy protons and deuterons. Crane, Lauritsen, and their associates in the California Institute of Technology; Hafstad, Tuve, and Dahl in the Carnegie Institution, Washington; Dunning and Pegram in Columbia University; all of these were skilled and untiring workers. A vast amount of data has been collected in one year. Let us examine some general results.

In natural radioactivity only the heaviest atoms, with a few feeble exceptions, disintegrate, expelling alphas or electrons. In artificial radioactivity the lightest atoms are the easiest to break up, the particles expelled being positrons, electrons, neutrons, protons, occasionally alphas. This at least is true when the bombarding particles are protons, deuterons, or alphas. The greater the energy of these projectiles, the greater the yield of transmuted atoms. But neutrons are strange projectiles. They are very effective against heavy atoms and, within limits, the

less their energy the more effective, as will be shown later.

The amount of radioactive phosphorus, P^{30}, produced by the Joliots was very small; the activity was only about one five-millionth of that of the polonium source. It might appear, therefore, that this phenomenon of induced radioactivity is a thing of no consequence. But the recently discovered properties of neutrons and the new energies produced by the cyclotron are being used to produce radioactive bodies comparable in intensity with radium itself.

Attention ought to be called to the fact that the phenomenon of induced radioactivity might have been missed by the Joliots had it not been for the *extreme sensitivity of the Geiger counter.* For the ionization-electrometer combination requires a minimum current of about 10^{-15} ampere, or 6000 electrons per second. But a Geiger counter detects one per second or a few per minute. Moreover, it operates almost instantly. Hence its value in detecting the half-life of an element like P^{30}, the half-life of which is about 3 minutes.

It is now evident that *induced radioactivity is a special case of transmutation.* The stable nucleus of an element is transformed by bombardment by one of the four projectiles, protons, deuterons, neutrons, alphas. (Occasionally a fifth projectile, a high energy γ-ray, is effective. More may be heard of this in the future.) A new nucleus is formed. It may be stable or unstable. In the latter case it changes over to another nucleus by the ejection of a proton, neutron, or electron. Why it should or should not change over is discussed later.

Let us consider the simplest possible cases. Can anything happen when protons bombard hydrogen? Or what is the right-hand side of the equation ${}_1H^1 + {}_1H^1 \rightarrow \quad ?$ The rules for this game have already been stated (Chap. 12). The sum of the subscripts on one side of the equation must equal the corresponding sum on the other. Similarly for the superscripts. The mass (including energy equivalent) must also balance. The question above is left to the reader.

Let us now bombard heavy hydrogen with heavy hydrogen nuclei (deuterons). We might have the relations

$$ {}_1H^2 + {}_1H^2 \rightarrow {}_1H^1 + {}_1H^3 + E \qquad (1) $$

$$ {}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + E. \qquad (2) $$

If (1) is possible, we have a new hydrogen ${}_1H^3$; if (2), a new helium ${}_2He^3$. It has been shown experimentally that we have one or the other of these cases. We then adjust the unknown masses ${}_1H^3$ and ${}_2He^3$ to balance each equation. *In other words, we have a means of measuring these unknown masses,* this on the assumption that we can measure the kinetic energies of the particles and the energy of a gamma radiation, if observed.

Unfortunately at the present writing the masses of the light nuclei are in doubt owing to an error found by Aston in his previous (before 1935) measurements. He has just announced that ${}_1H^1$ (the neutral atom) should be 1.0081 instead of 1.0078, and ${}_2He^4$ should be 4.0041 instead of 4.00216. As many of the other light atoms had been measured relatively to H and He, these also should be changed. The author is adopting tentatively the masses as given in the table in Appendix 14–1. These values are in part determined by the mass and energy relations as illustrated in (1) and (2).

Since 1 M.E.V. = 0.00107 statom, we have in these equations, provided we can measure all the energies involved, an extremely accurate method of measuring differences in atom masses. Thus we have a check on the values as obtained by the mass spectrograph. This is a new procedure in physics—to measure mass by measuring energy. For example, the difference between the old (4.00216) value for helium and the new (4.0041) represents an energy difference of 1.9 million volts; this is very far from being an insignificant quantity.

When deuterons were fired into heavy hydrogen as in (1), the range of the proton ${}_1H^1$ was found to be 14 cm. The known range-energy relation gave the energy of the protons as 3.0 M.E.V. Now the striking deuteron has a small momentum and kinetic energy; the combined forward momentum of the ${}_1H^1$ and ${}_1H^3$ must equal that of the ${}_1H^2$; the momentum of ${}_1H^3$ to the right (Fig. 14–3) must equal that of ${}_1H^1$ to the left. As it can be shown that these momenta are large compared with that of the forward component, the two directions are practically in a straight line. Since the masses are as 1 to 3, the energies of the particles ${}_1H^1$ and ${}_1H^3$ are as 3 to 1. Hence the total energy on the right-hand side of (1) is 4 M.E.V. or 0.0043 statom.

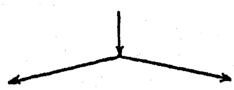


FIG. 14–3. The forward momentum of the two exploded components must equal the momentum of the bombarding particle.

(We have omitted the energy of the bombarding deuteron.) We take ${}_1H^1 = 1.0081$, ${}_1H^2 = 2.0142$; then ${}_1H^3 = 4.0284 - 1.0081 - 0.0043 = 3.0160$. It will be seen that this value agrees with that in the table, Appendix 14–1.

Thus we measure the mass of an atom that has never been measured in any other way, except in very rare cases by the mass spectrograph.

A beautiful cloud chamber photographic proof by Dee of the validity of the above reasoning is seen in Fig. 14–4. The long

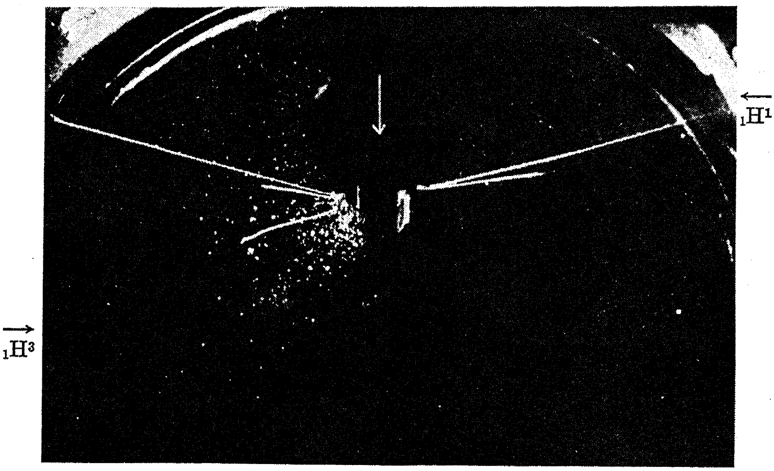


FIG. 14–4. A deuteron coming down in the direction of the white arrow, bombarding a deuteron, produces a proton ${}_1H^1$ and a new hydrogen ${}_1H^3$, thus ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^1 + {}_1H^3 + E$. (Dee, *Proc. Roy. Soc.*, 1934.)

track is due to ${}_1H^1$, the other to ${}_1H^3$. The lengths of the tracks are in accord with the energies 3 and 1 M.E.V. But if relation (1) is true, can relation (2) also be true? It has been found that even for a deuteron energy of 20,000 volts, there are a large number of neutrons in evidence. In this case the energy (En) of the neutrons must be measured indirectly by measuring the energy of the recoil particles. Thus En came out equal to 2.0 M.E.V. Therefore, in (2), $E = 2.67$ M.E.V. $= 0.00285$ stat-om. Hence

$${}_2He^3 = 4.0284 - 1.0085 - 0.00285 = 3.0170.$$

This agrees with the value in the table, Appendix 14–1. Again

we measure the mass of an atom new [1] to the scientists of the world. Again cloud chamber photographs show recoil tracks due to neutrons and add support to the validity of equation (2).

We have seen that heavy hydrogen bombarded by deuterons breaks up as shown in (1) and (2). But so far no disintegrations of heavy hydrogen have been found when the bombarding particles were alphas or protons.[2] Then neither helium nor hydrogen should disintegrate when bombarded by deuterons. Experiment confirms this. (There is, however, some experimental evidence that an alpha bombarding a deuteron breaks it up into a proton and neutron, and then escapes, thus:

$$ {}_1H^2 + {}_2He^4 \rightarrow {}_1H^1 + {}_0n^1 + {}_2He^4. $$

Only one observer has claimed to have found this reaction.)

Probably it is now evident that a complete discussion of all the transformation products which may be produced by bombarding the 92 elements (with their 250 atoms) by the four kinds of projectiles—neutrons, protons, deuterons, alphas—would run into volumes. We may illustrate the complexity by considering the case of lithium bombarded by protons and deuterons. We might have the following: [3]

TABLE I

(1)	${}_3Li^6 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^3 + E$	Dee and Walton
(2)	${}_3Li^6 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^3 + \gamma + E$	Lauritsen and Crane
(3)	${}_3Li^6 + {}_1H^2 \rightarrow {}_2He^4 + {}_2He^4 + E$ (Fig. 14–5)	Dee and Walton
(4)	${}_3Li^6 + {}_1H^2 \rightarrow {}_3Li^7 + {}_1H^1 + E$	
(5)	${}_3Li^7 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^4 + E$	Cockcroft and Walton
(6)	${}_3Li^7 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^3 + {}_0n^1 + E$	Oliphant, Kempton, and Rutherford
(7)	${}_3Li^7 + {}_1H^1 \rightarrow {}_4Be^8 + \gamma$	Crane and Lauritsen
(8)	${}_3Li^7 + {}_1H^2 \rightarrow {}_2He^4 + {}_2He^4 + {}_0n^1 + E$	Oliphant and Kensey
(9)	${}_3Li^7 + {}_1H^2 \rightarrow {}_4Be^9 + \gamma$	
(10)	${}_3Li^7 + {}_1H^2 \rightarrow {}_4Be^8 + {}_0n^1 + E$	Bonner and Brubaker
(11)	${}_3Li^7 + {}_1H^2 \rightarrow {}_3Li^8 + {}_1H^1 \rightarrow {}_4Be^8 + {}_{-1}e^0 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^4 + {}_{-1}e^0 + {}_1H^1$ (energies omitted in 11)	Crane and Lauritsen, Delsasso, Fowler

[1] Not quite. Bainbridge had measured it with his mass spectrograph.

[2] As far as arithmetic and masses are concerned, we could have

$$ {}_1H^2 + {}_1H^1 \rightarrow {}_1H^3 + {}_1e. $$

It can be computed that the positron would have an anergy of about 8 M.E.V.

[3] E, which is equal to the kinetic energy of the particles on the right, is equal to the energy of the mass difference (left minus right) plus the kinetic energy of the particles on the left. γ is the energy of the gamma photon, as measured by its absorption, all in mass units or statoms.

While there is doubt about some of these transformations, all are believed to be possible and every one has been put forth as probably established by the experimental results. No. (5) of course is the pioneer experiment of Cockcroft and Walton (Chap. 12). A cloud chamber photograph (Fig. 14–5) shows clearly

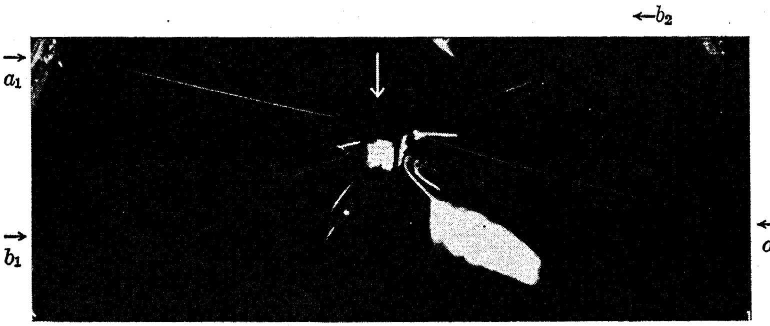


FIG. 14–5. A deuteron bombarding lithium produces two alphas which are ejected in nearly opposite directions a_1, a_2 : b_1, b_2.

$$_3Li^6 + {}_1H^2 \rightarrow {}_2He^4 + {}_2He^4 + E.$$

(Dee and Walton, *Proc. Roy. Soc.*, 1933.)

that the tracks occur in pairs, the two members of a pair equal in length (unless one has penetrated the target) and oppositely directed along the same straight line. A similar photograph by Dee and Walton shows the same kinds of tracks when deuterons are the projectiles as in (3). In (1), (2), and (6) we again meet the new isotope of helium, $_2He^3$, and again we can get a check on the mass of this atom. In (7), (10), and (11) we have a new atom of beryllium, $_4Be^8$. Crane, Delsasso, Fowler, and Lauritsen suggest that it breaks up into two alphas as indicated in (11). They found (June, 1935) clear evidence that there was present a radioactive element as a result of the transformation process. They measured the half-life of the radioactive atom by causing the chamber to expand and taking photographs 1/4, 1/2, 3/4, and 1 second (50 for each time) after the ion beam was shut off. The average number of tracks per photograph, 7.08, 4.84, 3.70, 2.45, gave the exponential law of decay, with half-life = 0.5 sec.

The above authors claim that the γ rays of (7) have an energy of 16 M.E.V. It will be remembered that in (5) the total energies of the two alphas was 17.06 M.E.V. Hence it appears that

in (7) the γ ray contains all the energy that was divided between the two alphas in (5). Note that 17 M.E.V. is twice as great as the maximum α ray energy of the naturally radioactive elements (Thorium $C^1 = 8.76$ M.E.V.), *and six times the energy of the maximum γ ray.* In (7) the bombarding protons had an energy of between 400,000 and 10^6 e volts. For a fuller discussion, see Fig. 14–14.

In (11) there is proposed another new atom, ${}_3Li^8$, an isotope of lithium. However, there must appear other supporting evidence before it can be accepted as established.

Nothing has been said above regarding the energies of the projectiles; nor how the phenomena change as these energies change. Moreover, it is not possible here to indicate the labor involved or the experimental skill that has been employed in measuring the energies E and γ. Just one item among many to show how physicists conceal their troubles. Bonner and Brubaker at the California Institute of Technology took more than 9000 cloud chamber photographs, made all their measurements and computations, and reported their findings in about half a page.

We have been considering the transformations which may take place when lithium, the simplest element (simplest except for those compact units, hydrogen and helium) is bombarded by only two of the four projectiles. We now select a few high points from the great amount of already known data.

Radioactive Sodium.

Lawrence, Livingston, and their associates have shown that sodium, under energetic deuteron bombardment, emits protons, neutrons, alphas, and becomes radioactive! The relations which have been proposed are:

(1) $${}_{11}Na^{23} + {}_1H^2 \rightarrow {}_{10}Ne^{21} + {}_2He^4$$

(2) $${}_{11}Na^{23} + {}_1H^2 \rightarrow {}_{12}Mg^{24} + {}_0n^1$$

(3) $${}_{11}Na^{23} + {}_1H^2 \rightarrow {}_{11}Na^{24} + {}_1H^1$$

(4) $${}_{11}Na^{24} \rightarrow {}_{12}Mg^{24} + {}_{-}e + \gamma + E.$$

Lawrence, stepping the energy up to 2.15 M.E.V., was able to get the deuterons out of the cyclotron chamber through a thin window into the air. He was thus able easily to expose different

materials to the deuteron stream. A piece of rock salt (clean surface) exposed for an hour was radioactive, at first in a complex way suggesting two or more different kinds of disintegrating atoms, but 20 hours after the bombardment had ceased the decay became exponential with time. The transformed product appeared to be a simple radioactive material. The half-life was 15.5 hours. *This is a convenient half-life. Moreover, the amount of radioactive sodium,* $_{11}Na^{24}$, *obtained may be very large.*

Lawrence computed that 1 microampere of 2.17 M.E.V. deuterons produced enough radiosodium to send out 10^7 electrons per second. (However, that means that only one radioactive atom was produced for 1.7×10^6 deuterons.) As the activity increases very rapidly with the voltage of the deuterons, he expects to raise this to about 10^{10} electrons per second. (The student might compute how long it would take to produce 1 gram of radioactive sodium or 2.6×10^{22} atoms!) It will be recalled that radon tubes are used for therapeutic purposes. *Since sodium is a constituent of food and of body material, there is reason for believing that radioactive sodium may have an important place in hospitals as well as in physics laboratories.*

Lawrence proved by chemical test that the radioactive material was sodium. Moreover, the same product was obtained by Fermi when he bombarded aluminium by neutrons. Thus:

$$_{13}\mathrm{Al}^{27} + {}_0n^1 \rightarrow {}_{11}\mathrm{Na}^{24} + {}_2\mathrm{He}^4.$$

The manner in which radioactive sodium, $_{11}\mathrm{Na}^{24}$, breaks up is shown above. The energy of the γ ray is 5.5 and of the electrons 1.2 M.E.V. It is seen that the gamma radiation is of great penetrative power, greater than that from ordinary radioactive material. This is one of the characteristics that gives to radiosodium [1] the importance to which reference has been made.

Two Alphas from Lithium, Three from Boron.

Referring back to the eleven ways in which lithium may be transformed by protons and deuterons, we see that there are

[1] The announcement has just been made (August, 1935) that a cyclotron may manufacture radiosodium equivalent to 50 milligrams of radium per day, and that that would be worth several thousand dollars. But that may be very misleading. For the half-life of radium is 1600 years, that of radiosodium 15.5 hours. What is meant is that the radiosodium is equivalent to the radon in equilibrium with 50 mg. of radium. It would have to be manufactured every day, whereas for radon one would merely have to draw it off the radium.

two, No. (3) and No. (5), in which alpha particles are the only products. Figure 14–5 shows two alphas hurled away from one another by the explosion. We have a similar but more complex result in the case of boron. The relations ${}_5B^{10} + {}_1H^2 \rightarrow 3\,{}_2He^4 + E$ and ${}_5B^{11} + {}_1H^1 \rightarrow 3\,{}_2He^4 + E$ would cause us to look for three alphas in either of these cases. The three alphas would most probably have equal energies and therefore should separate at angles of 120°. Kirchner of Leipsic has verified this conclusion. His cloud chamber photographs show three such tracks. *The alpha particle is obviously a strongly knit unit.*[1]

Continuing our analysis of the eleven cases above, we note that in all cases in which the proton is captured, a disintegration occurs and a new particle is ejected—except in case (7). But in that case it seems highly probable that ${}_4Be^8$ breaks up at once into two alphas. There are other reactions which have been proposed, as: ${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + \gamma$. (This has been established.) But again ${}_7N^{13}$ breaks up into ${}_6C^{13} + {}_+e$. So far as our present observations go, it appears to be impossible to form a stable heavy atom out of a light one by firing into it a proton, without the ejection of a neutron, deuteron, or alpha. In other words, *we cannot make heavy atoms out of hydrogen.*[2] It will be recalled that one of the theories vigorously set forth regarding cosmic rays was that these rays are the gamma radiations due to the creation of heavy atoms by the sudden union of the necessary number of hydrogen atoms. That theory receives no support whatever from the known results in transmutation of the elements. Moreover, later in this chapter there will be presented modern ideas regarding the building up of atoms—that they are built up of protons and neutrons. True, inside a nucleus one of these units may transform into the other by the ejection of the appropriate electric particle. But outside the nucleus they appear to be definite, independent building blocks. However, we cannot put them together in any ratio we please, as will be shown.

[1] There is evidence that groups of alphas are rather strongly bound together as in C^{12} and O^{16} but in natural radioactivity (see Table I, Chapter 11) we have three or four alphas ejected successively from atoms. For example,

$$\begin{array}{ccccccccc} U_{II} & \rightarrow & \mathrm{I}_0 & \rightarrow & \mathrm{Ra} & \rightarrow & \mathrm{Rn} & \rightarrow & \mathrm{Ra\ A.} \\ & \searrow & & \searrow & & \searrow & & \searrow & \\ & \alpha & & \alpha & & \alpha & & \alpha & \end{array}$$

Yet we cannot write ${}_{92}U_{II}{}^{234} \rightarrow {}_{84}RaA^{218} + {}_8O^{16}$.

[2] This statement may be proved to be incorrect in the near future.

Disintegrations by Neutrons

Neutrons are strange projectiles. Alphas, protons, deuterons, are like tracer bullets—they leave a line of ions in their wake and by this line their paths can be followed. But neutrons leave no path. When the alphas, protons, deuterons, have high energies, they disintegrate nuclei, chiefly of the lighter elements. But neutrons are most effective against heavy atoms and increase in efficiency (within limits) as their energy decreases.

Intensifying the Efficiency of Neutrons.

The technique of dealing with "streams" of neutrons is new to physics. Here is the method due to Dunning, Pegram, Fink, and Mitchell of Columbia. They increased the supply of neutrons several thousand times over that used by the Joliots, Chadwick, and others by filling a platinum capsule with powdered beryllium and from 500 up to 2000 millicuries of *radon*. The gas comes in contact with a very large surface of the powder and the alphas sent out by the radon have free access to the beryllium atoms. Neutrons and γ rays are emitted. With the capsule surrounded by 0.75 mm. of lead, the softer γ rays are nearly all absorbed. The neutrons emerging from the lead are slowed down in their passage through a sphere of paraffin. (Fermi had shown that neutrons are slowed down by material containing hydrogen.) Of the protons ejected from the paraffin, only those originating in the outer surface can emerge from the sphere and these cannot reach the ionization amplifier *A*. To prevent neutrons which have been scattered back from the walls of the room from reaching the amplifier, sheets of *cadmium* are placed as shown in Fig. 14–6 (a cylinder of cadmium would be better).

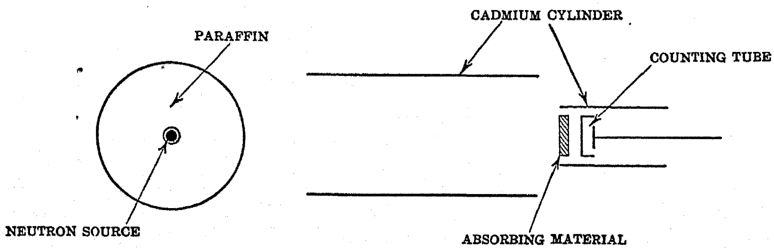


Fig. 14–6. Radon mixed with beryllium in a small tube is an intense neutron source. The neutrons are slowed down by a paraffin sphere. The counting tube is lined with lithium. Cadmium cylinders shield this tube from scattered neutrons. (Modification of method used by Dunning, Pegram, *et al.*)

For it has been found that 1 mm. of cadmium absorbs practically all neutrons of a certain low range of energy. The walls of A are lined with *lithium*, the disintegration products of which produce the ionization. This expedient greatly increases the count.

There ought to be inserted here a few facts regarding neutron sources. A gram of radium (chloride) costs about $60,000. The Columbia workers are able to obtain from the Memorial Hospital the radon supply from 5 grams of radium. This yields daily about 650 millicuries of radon. As the half-life of radon is just less than 4 days, the maximum the Columbia men can use is about 2000 mc. The radiation from radon, RaA, RaC, is then equivalent to that from 6000 mc. of polonium (Chadwick used 10 or 25 mc. of polonium). The high energies of these alphas and the complete intermingling of gas and beryllium brings about the high neutron yield. About 2×10^{11} alphas per second strike the Be and about 4×10^6 neutrons/sec. are emitted.

As has been stated, lining the ionization chamber with lithium (or boron) greatly increases the count. The interaction of Li and slow neutrons (energies only about 1/20 e.v.) results in two groups of particles, one of about 1.5 cm. range and the other about 5.5 cm. This reaction is proposed:

$$_3\mathrm{Li}^6 + {}_0n^1 \rightarrow {}_2\mathrm{He}^4 + {}_1\mathrm{H}^3 + E.$$

It appears that a slow neutron traversing 1 mm. of Li probably trips off the above reaction, thus having several thousand times the efficiency of fast neutrons.

But it seems probable that a cyclotron operating at a few M.E.V. will produce more neutrons than could be produced by the entire radium supply of all the laboratories of any country gathered in one place.[1] For Kurie, using the cyclotron in the

[1] The approximate amounts of radium in hospitals, laboratories, etc., in the world are given below. These data have been supplied by Mr. Frank Hess, principal mineralogist, Bureau of Mines.

U.S.	Grams	Elsewhere	Grams
287 hospitals	90	Belgium	160
414 physicians	35	Czechoslovakia	55
9 laboratories	6	France	51
Private, industrial	107	England	42
		Sweden	8
Total	238	Russia	8?
		All others	134
		World total	700

University of California, bombarding beryllium with deuterons at 2 M.E.V. computes that he obtains 10^9 neutrons per second. Unfortunately the great mass of the magnet causes these to be scattered in all directions. Even then, a cloud chamber at 50 cm. from the source is traversed by about one million neutrons per second. With higher voltages the neutron supply will probably be greatly increased.

Since radon and its products send out at least three different kinds of alphas (different energies), the neutron emission from the capsule is complex in energy. The absorption due to placing a sheet of cadmium 0.8 mm. thick in front of the chamber *A* is shown in Fig. 14–7. The curves show that when the paraffin thickness is less than 2 cm., the absorption due to the cadmium is practically zero. With no paraffin around the neutron source, the amplifier (lithium lined) gives 20 counts per minute; with 10 cm. thickness of paraffin around it there are 200; but a thin sheet of cadmium which did not absorb any of the original neutrons now absorbs nine-tenths of all falling on it. This is slightly perplexing. *By interposing a thickness of 10 cm. of one material we multiply the count by 10; by adding a thickness of less than a millimeter of another material, we get back to the first value.* We must watch our step when working with neutrons.

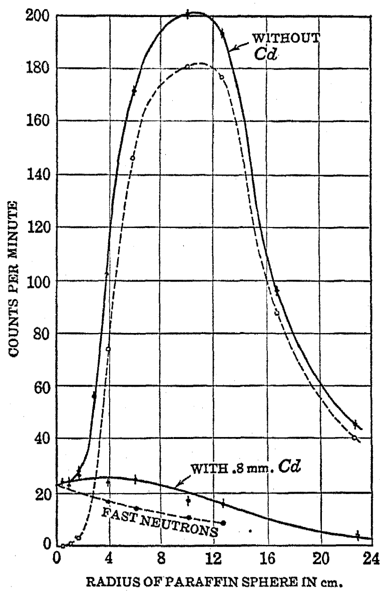


Fig. 14–7. Neutrons which have passed through 10 cm. of paraffin produce 10 times the count that direct (fast) neutrons produce. But 0.8 mm. of cadmium absorbs nine-tenths of these slow neutrons. The fast neutrons are only slightly absorbed. (Dunning, Pegram, Fink, and Mitchell, *Physical Review*, June, 1935.)

We may "explain" these phenomena in part as follows. High-energy neutrons in passing through material containing hydrogen experience elastic collisions with hydrogen nuclei, thus losing kinetic energy. (We might suspect that if the neutrons

were made to pass back and forth often enough, the hydrogen nuclei would be increased in energy and the neutrons decreased until they would be in thermal equilibrium. But so far complete equilibrium has not been observed.) Slow-moving neutrons have difficulty in passing or avoiding certain atomic nuclei. Thus there is greater absorption for the slow neutrons. The "absorption coefficient" may be stated in terms of the combined effective cross-section of the neutron and nucleus. Here are some values as obtained by the Columbia workers. The effective cross-sections are in 10^{-24} cm.^{2} H—31.; D—5.3; Li—49; Be—3.8; B—600; C—2.8; Cd—3300; Sm—5000; Hg—430; Pb—6; U—100. These are perplexing data. *Why should the probability of a neutron's striking a nucleus be 6 times as much for hydrogen as for heavy hydrogen; or 200 times as much for boron as for carbon; or 1200 times as much for cadmium as for carbon?* There must be some very special characteristic of one nucleus as compared with its neighbor in the chemical scale that accounts for the data above.

For fast neutrons the story is very different. There the cross-sections vary downwards rather gradually from 1.68×10^{-24} cm.^{2} for uranium. The combined radius or the sum of the radii of nucleus and neutron for fast neutrons computed on the basis of the classical theory is 7×10^{-13} cm. for light atoms and nearly twice as great for heavy atoms. Presumably this would give a radius of 3.5×10^{-13} cm. for light nuclei; this is of the same order as values obtained by other methods.

Naturally Radioactive Uranium Changes Very Slowly; Artificially Radioactive, Very Fast.

We have shown that by causing neutrons to pass through several centimeters of paraffin or water, the ionization count in a lithium-lined chamber may be increased ten times, as compared with neutrons going directly to the chamber. Similarly for radioactivity induced by neutron bombardment. Silver is one interesting example. It becomes strongly radioactive, with two periods of 20 seconds and 2 minutes. But it is not possible to list all elements made radioactive by neutrons. Fermi has found about 50 between fluorine (No. 9) and uranium (No. 92). Perhaps the most surprising cases are those of thorium and uranium. Both are naturally radioactive with enormously long periods (about 10^{10} years), both send out alphas. But under

neutron bombardment they become radioactive with short periods (uranium 15 and 40 seconds, 13 and 100 minutes).

Element 93 Is Discovered.

The reaction for uranium was presumably as follows:

$$_{92}U^{238} + {}_0n' \rightarrow {}_{92}U^{239} + \gamma$$
$$_{92}U^{239} \rightarrow {}_{93}X^{239} + {}_{-}e.$$

The claim made by Fermi, Amaldi, and their associates that element No. 93 had been obtained was challenged by other workers. But the Rome experimenters made as exacting chemical tests as the short life-times of the atoms would permit and all tests pointed to the conclusion that the new atoms were not members of the uranium family. They are called transuranic elements.

We ought to review the steps by which this result has been achieved. We start with the heaviest element, uranium. Occasionally atoms eject an alpha particle and change over to a new element; disintegration continues, step by step, until radon is reached. We enclose radon with one of the lightest elements, beryllium. Alphas which are still shooting out from radon and its products cause a transformation in the light atoms. New particles, neutrons, emerge with great energy. They are slowed down; they strike uranium nuclei and transform them and the new atoms become unstable. They break up in a hurry! We use the products of "slow" uranium to produce a "quick" uranium. But a light atom and a new particle (neutron) intervene and a new element is discovered.

We Ask a Question.

We have laid stress on two points. Alphas, protons, deuterons are most effective against light atoms, and their efficiency in producing transformation increases rapidly with their energy. Neutrons are especially effective against heavy atoms and are most effective when slowed down. (But they must have energies greater than 50,000 e.v.) Can we account for this?

We Try to Answer.

Clearly the forces between nuclei and positives must be repulsive, up to a certain point; between nuclei and neutrons they must be nearly zero at a distance, then mildly attractive.

We recall Chapter 3. A particle charge $+e'$ and velocity v approaches a nucleus of charge Ze, where Z is the atomic number. If the particle has not too great an energy and is headed straight for the nucleus, it stops at a distance d from the nucleus so that its potential energy $Zee'/d = \frac{1}{2}mv^2$. Then $d = 2Zee'/mv^2$. If the particle, initially of zero velocity, had been accelerated by an electrical field, we would have had $\frac{1}{2}mv^2 = Ve$ ergs, where Ve is the work done. From this we can find the distance of nearest approach. The greater the atomic number of the nucleus, the greater is d. The greater the driving energy of the particle, the less is d. If we picture the nucleus as having a definite radius r_0, then the electron volts necessary to drive the particle up to that boundary is proportional to Ze'/r_0. This may be called the *potential barrier*. If the particle has an energy greater than this, it crosses the boundary. *Then the law of force changes.* There must be an attraction for an alpha or a proton inside the nucleus (for we know that all nuclei retain in equilibrium a positive charge up to an amount Ze). Then the potential rapidly changes. All of this may be illustrated by Fig. 14–8. The potential of a proton approaching a nucleus is shown by the hyperbolic curve Ze/d until it arrives at a distance r_0; then it drops rapidly. If it does not have an energy great enough to

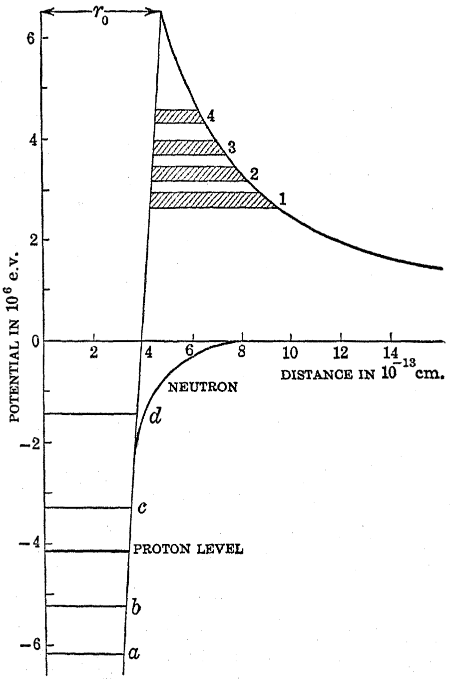


FIG. 14–8. A positively electrified particle, driving in towards a nucleus, may enter at energy levels 1, 2, 3, ⋯, lower than that indicated by r_0, the energy required to surmount the potential barrier. The neutron gracefully falls in.

bring it to the nucleus boundary, it is scattered away, as Rutherford found in 1912 when he was led to the idea of the nuclear atom. On this point of view then we can understand why the most energetic alphas that we know, those of thorium C′ of 8.76 million volts, cannot disintegrate atoms heavier than potassium,[1] as Rutherford found years ago. (But with a great supply of alphas or protons of high energy, say 5 M.E.V., as can be given by our new devices, we may find that heavy atoms are disintegrated.)

On the classical view here presented, a proton or alpha cannot disintegrate a nucleus unless it has energy sufficient to carry it over this potential barrier. But wave mechanics comes to our aid. It finds possible "resonance levels" of lower potential indicated by 1, 2, 3, . . . through which an alpha may enter. Experimentally this is verified by the fact that the computed potential barrier of aluminium is about 8×10^6 volts, whereas particles of energies lying between 4 and 5.3×10^6 volts produce disintegration. If an alpha of energy W enters the nucleus, we picture it as dropping to one of several levels, such as (*a*), (*b*), (*c*), (*d*). If this brings about the ejection of a proton from another definite level, the latter departs with an energy $W = E_\alpha - E_p$. Here E_α is the energy of the alpha in the level in which it rests, E_p is that of the proton in the level from which it escaped. We would expect then to find groups of ejected protons of various energies. The experimental results are indicated in Fig. 14–9.

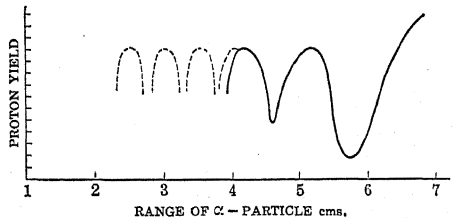


FIG. 14–9. Alphas of 5.2 cm. range produce more protons than those of 4.6 or 5.8 cm. range. One of 5.6 cm. range is less effective than one of 4.2 or 5.3 cm.; showing "resonance levels" in the nucleus. (After Chadwick and Feather.)

Alphas of 4 cm. range cause protons to be ejected from aluminium. As the energy of the alphas is increased, the proton yield increases

[1] On this simple view we may compute a limiting value of the radius r_0 of the potassium nucleus. $Z = 19$, $V = 9 \times 10^6 = 3 \times 10^4$ e.s.u. And the charge on an alpha $= 2e = 9.55 \times 10^{-10}$ e.s.u. $r_0 = 6 \times 10^{-13}$ cm. Or thus, 9×10^6 e.v. $= 9 \times 10^6 \times 1.6 \times 10^{-12}$ ergs. This equals

$$\frac{2 \times 19 \times (4.75 \times 10^{-10})^2}{r_0} \text{ ergs.}$$

Find r_0.

but falls again, showing that an alpha of a 5.6 cm. range is much less effective than one of 4.2 or 5.3 cm. But beyond the 5.6 cm. range the proton yield increases rapidly to a large value. The curve of Fig. 14–9 is suggestive of the Frank and Hertz curve for electrons bombarding mercury atoms. That was the experiment, it will be recalled, which demonstrated the existence of "resonance potentials" for electrons outside the nucleus. These experiments in transmutation are establishing corresponding levels of protons, alphas, and neutrons inside the nucleus.

All that has been said above regarding potential barriers and energy levels applies to alphas, protons, deuterons. Concerning neutrons—as has been stated—there is no potential barrier. There must in general be mild attraction between nucleus and neutron; for some nuclei, cadmium, samarium have strong attraction. The indicated line (Fig. 14–8) shows the neutron potential.

This leads us to discuss the *structure of the nucleus*. After the discovery of the nuclear nature of an atom by Rutherford in 1912, there was developed the view that a nucleus consisted of protons and electrons, the former in excess, and it was this excess which determined the chemical properties of the atom—it became identified with the atomic number. The helium nucleus, the alpha particle, contained four protons and two electrons; the aluminium nucleus contained 27 protons, 14 electrons, with atomic number 13. The older phenomena in radioactivity were explained in terms of this picture. But the after discovery of the neutron and positron in 1932, the picture changed.

Only Protons and Neutrons in the Nucleus.

According to the present view, there are no electrons as definite units in the nucleus—only protons and neutrons. Table II shows the composition of a few nuclei. After we pass hydrogen and the rare and unstable helium isotope $_2He^3$, we generally change to a new atom by adding 1 deuteron, that is, 1 proton and 1 neutron. But as we get towards heavy atoms, the number of neutrons increases. The number of protons fixes the atomic number. To get an isotope we change the number of neutrons. Some of the unstable isotopes are indicated by an asterisk. Obviously stability for light atoms is secured by making the ratio of neutrons/protons equal to unity. However, that simple rule does not hold for heavy atoms.

TABLE II

	At. No., Z Protons	Neutrons
$_1H^1$	1	0
$_1H^2$	1	1
$_1H^3$	1	2
$_2He^3$	2	1
$_2He^4$	2	2
$_3Li^6$	3	3
$_3Li^7$	3	4
$(_7N^{13})$*	7	6
$_7N^{14}$	7	7
$(_7N^{15})$	7	8
$_{13}Al^{27}$	13	14
$(_{13}Al^{28})$*	13	15
$_{48}(Cd)^{110}$	48	62
$_{48}(Cd)^{111}$	48	63
$_{48}(Cd)^{112}$	48	64
$_{48}Cd^{113}$	48	65
$_{48}Cd^{114}$	48	66
$_{48}Cd^{116}$	48	68
$_{92}U^{238}$	92	146

But how do we account for radioactivity on this basis? When an alpha is ejected, the answer is easy—2 protons and 2 neutrons, in the close combination which forms the alpha particle, leave the nucleus. But when an electron is ejected as happens at times in natural and induced radioactivity, we picture a neutron being transformed into a proton and an electron, the latter leaving the atom. Induced radioactivity, however, differs from natural in two very important respects: in the former, electrons or positrons may be emitted, seldom an alpha; in the latter, only electrons or alphas. But in the former, in the case of heavy atoms bombarded by neutrons, electrons only are found.

The nucleus as we have pictured it here is so simple in structure that we are induced to ask simple questions. Can we not manufacture heavy hydrogen by bombarding hydrogen with neutrons? Can we not change almost any atom into its heavier neighbor by bombarding it with deuterons? According to one observer, Lea, the first question has been answered in the affirmative. But the answer needs confirmation. The second question has been answered for special cases in the affirmative by various observers.

But either the bombarded or the formed atom is unstable. And what would happen if we bombarded uranium with deuterons? and what would be the product? Element 93? 94?

Concerning Deuterons as Projectiles.

A deuteron may be regarded as a dumbbell, a positively charged mass, a proton, connected by a slender rod to a neutron. The positive mass affords us a handle by means of which we are able to hurl the dumbbell. When the deuteron comes to a heavy nucleus, the positive charge may not be allowed to enter; there is less restraint for the neutron. There may then be a breaking up of the deuteron. The proton may escape at once. The neutron may enter and remain in the bombarded nucleus permanently or the loaded nucleus may later break up. For example, we have

$$_{13}Al^{27} + {}_1H^2 \rightarrow {}_{13}Al^{28} + {}_1H^1 \qquad \text{and} \qquad {}_{13}Al^{28} \rightarrow {}_{14}Si^{28} + e.$$

Concerning the Density of the Nucleus.

The mass of the gold atom being nearly two hundred times that of the hydrogen atom is about 3.3×10^{-21} gm. If the diameter of the gold nucleus (see page 47) is 3×10^{-12} cm., its volume is 1.4×10^{-35} cm.^{3}. Therefore its density is 2.3×10^{14} gm./cm.^{3}. *Four thousand million tons per cubic inch, a mere nugget!* The result of a computation like this for density or temperature is expanded into a volume by our writers of popular science. Such a volume may become a best seller!

Gamma Rays, Indirectly, May Produce Radioactivity.

A new chapter will be inserted here in the near future. For it has very recently been found that when an iodine compound is spread on or surrounds beryllium [1] and the whole is irradiated with γ rays, the iodine, $_{53}I^{127}$ becomes radioactive. The γ rays obviously eject neutrons from beryllium and the neutrons operate on iodine to form a radioactive atom. There are at least two such atoms. (The student should trace out all the interesting possibilities in this case.) With our new devices for producing high voltage X-rays this discovery promises to open up an important field for research.

[1] It might be thought that ordinary beryllium Be^9 $(4\,p + 5\,n)$ is broken up into Be^8 $(4\,p + 4\,n)$ and a neutron. But is this possible? See Appendix 14–1.

The Game of Transmuting the Elements.

And now the student can play a game of transmuting the elements. Let us take an element X of atomic number Z and atomic mass A and bombard it with γ rays. Then we obtain relations like these:

$$
\begin{aligned}
{}_zX^A + \gamma &\rightarrow {}_zX^{A-1} + {}_0n^1 + E \\
{}_zX^A + \gamma &\rightarrow {}_{z-1}X^{A-1} + {}_1H^1 + E \\
{}_zX^A + \gamma &\rightarrow {}_{z-1}X^{A-2} + {}_1H^2 + E.
\end{aligned}
$$

Then we replace the γ by ${}_0n^1$ and obtain another set of five or six equations, then ${}_1H^1$, ${}_1H^2$, ${}_2He^4$, ... And after we have written them all out, we try to find how many of them represent impossible cases. But if we increase the energies of the γ's and n's, how many of them are then impossible? And presently we shall have discovered some of the rules of the game. (And then we will wonder what Mendeleeff thinks of this business!)

The rules of the game will be seen to be the following. As has previously been stated, the addition of a deuteron, $(p + n)$, to a light element will generally give the next higher element. We then arrange the elements in horizontal rows (but rising on tiers as we go to large atoms). Isotopes of atoms will be in vertical columns. Then bombarding an atom with a deuteron will tend to carry it one step to the right; with a neutron, one step up. Since a proton, p, equals $d - n$, bombarding with a proton carries the atom one step to the right and one step down. A proton is the "resultant" of these two lines. Consequently, representing an atom by a square, we have the upper right-hand corner of Fig. 14–10.

Evans and Livingston have been playing this game but with caution. For they have assembled (April, 1935) what are believed to be the most conspicuous of the reactions that have been experimentally obtained; this at least for the first 30 elements. This is put in the form of a chart (Fig. 14–10). Of the 90 known elements and 280 isotopes about two-thirds are charted in Fig. 14–11, and in this figure a number of reactions due to neutron bombardment are indicated.

In Fig. 14–10 the deuteron ($2\ Z$ when $Z = 1$) bombarded by a γ ray causes a neutron to be ejected and the atom reverts to light hydrogen. Also a deuteron bombarding a deuteron carries the atom over towards helium, but a neutron may be ejected, giving

an end product of $_2He^3$, or a proton may be ejected giving the atom $_1H^3$. Helium is an end product of several reactions but no bombardment disturbs it. Lithium 7, bombarded by a deuteron, ejects a neutron, then an alpha, and lands in $_2He^4$.

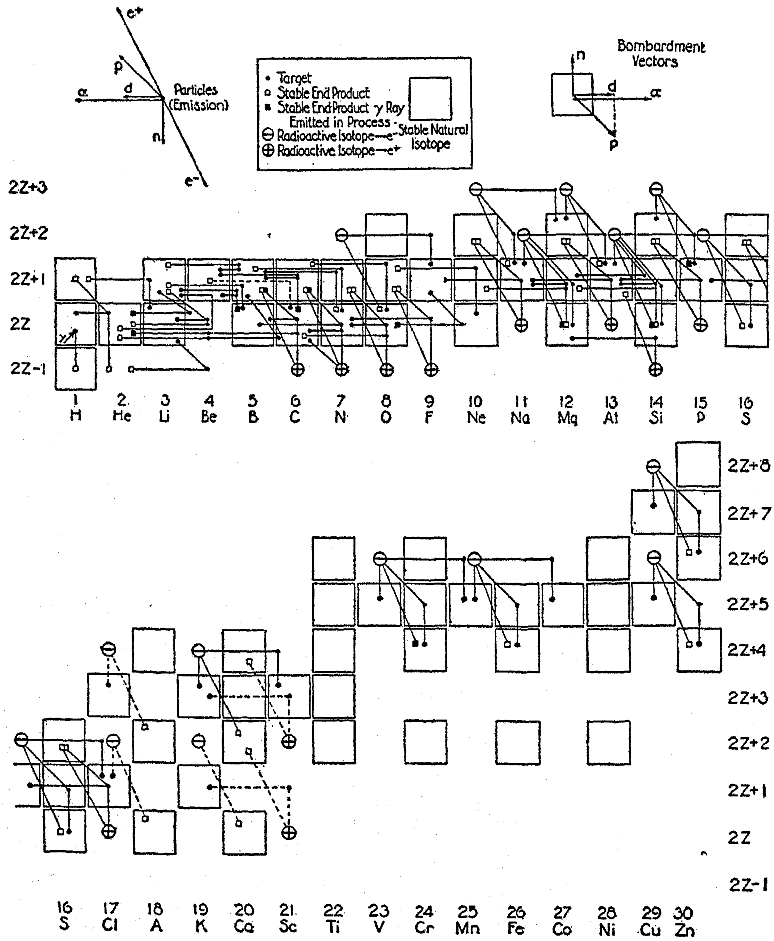


FIG. 14–10. A partial chart of the lighter elements showing transmutations produced by bombardment. A bombarding deuteron carries an element one step to the right, an alpha two steps, etc.

But this chart (Fig. 14–10) illustrates effectively the feverish haste with which data regarding these reactions are accumulating. Prepared in April 1935, the chart was believed to contain all the

acceptable data; but at the time of this writing (July, 1935) several new reactions are known or have been claimed as reasonably certain. Compare the five reactions in the chart due to protons and deuterons operating on lithium with the eleven of Table I. Again it is instructive to note that, whereas all previous ages of the human race were necessary to lead up to the simple Mendeleeff table, it required only about ten years (1919–1929) to accumulate the data of Fig. 14–11, a few years for Fig. 14–10.

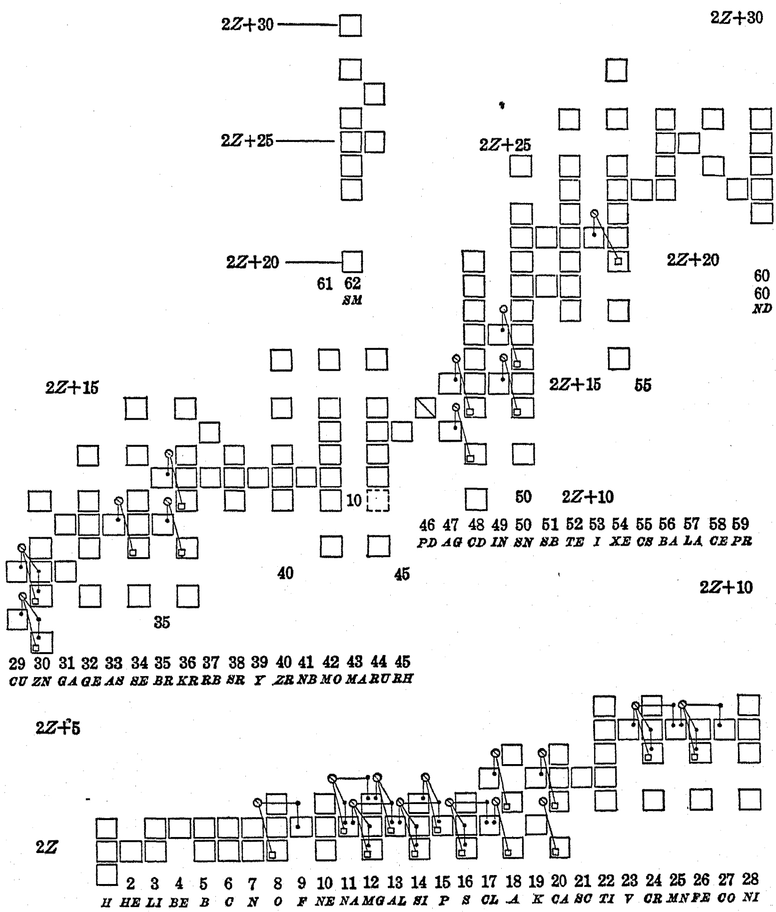


FIG. 14–11. A partial chart showing chiefly transmutations due to neutrons. Note the large number of isotopes; tin has eleven. Note, too, the increasing percentage of neutrons.

All the radioactive reactions of Fig. 14–10 and many not shown there have been accumulated in one year.

Here are two reactions which appear as reversible:

$$\left.\begin{array}{l} {}_5B^{11} + {}_2He^4 \rightarrow {}_0n^1 + {}_7N^{14} + E \\ {}_7N^{14} + {}_0n^1 \rightarrow {}_2He^4 + {}_5B^{11} + E \end{array}\right\} \quad \text{Are both possible?}$$

And here is a reaction which, once started, seems to run around in a circle:

$${}_{12}Mg^{24} + {}_0n^1 \rightarrow {}_1H^1 + {}_{11}Na^{24}; \qquad {}_{11}Na^{24} \rightarrow {}_{12}Mg^{24} + {}_{-1}e^0.$$

We start with magnesium and a neutron, we end with magnesium and an electron; meanwhile a proton has escaped. But a new particle is wanted in this reaction. It is called the *neutrino*.[1] And what is a neutrino? At present it is an hypothesis. It is supposed to have about the mass of an electron and to be electrically neutral. Nobody has seen one, though many are looking for it. Naturally it will be hard to find—if it exists. Its existence is demanded by the theorists in their attempts to explain the transformation of a neutron (while in the nucleus) into a proton, which remains, and an electron and neutrino which escape. Similarly when the transformation is the other way—a proton transforms into a neutron, positron, and *antineutrino.*

While we are dealing with these mysterious quantities, let us add another, the *negative proton.* It is also being sought.

Returning to our discussion of Fig. 14–10, we observe that all the radioactive atoms which send out electrons, in returning to a stable atom, are *above* the stable groups and all those which send out positrons are below. Of course it could not be otherwise if the stable atom occupied a central range. The symbols $\ominus$ and $\oplus$ mark the limits of stability of the proton + neutron combination.

A few years hence (this is being written in July, 1935) by means of the cyclotron, at least, protons will be given an energy of 20 million volts. What will happen to atoms struck by such missiles may be left to the imagination—though theory might help to indicate the results. Surely we will be able to produce many more radioactive atoms. And Figs. 14–10 and 14–11 will become very complicated.

[1] The author's symbol for it is ${}_0n^0$. If the electron is ${}_{13}Al^{28} \rightarrow {}_{14}Si^{28} + e$ has not energy enough, we might balance the equation by adding ${}_0n^0$ on the right.

Packing Fraction Curves.

In Fig. 14–12 are shown packing fraction curves (see page 35), plotted with the best data available at this date (July, 1935). The masses have been determined from transmutation relations as explained in this chapter. In curve I are shown elements of smaller mass than $_7N^{15}$. It is seen that all atoms lie on or very near the smooth curve except $_1H^3$, $_2He^3$, and $_3Li^6$. These are rare atoms, the rarity indicating instability or difficulty of formation. It is seen too that if the mass of the neutron, n, be increased by that of an electron 0.0005 it will lie on curve I. (The masses of H^1, H^2, etc., are those of neutral atoms.)

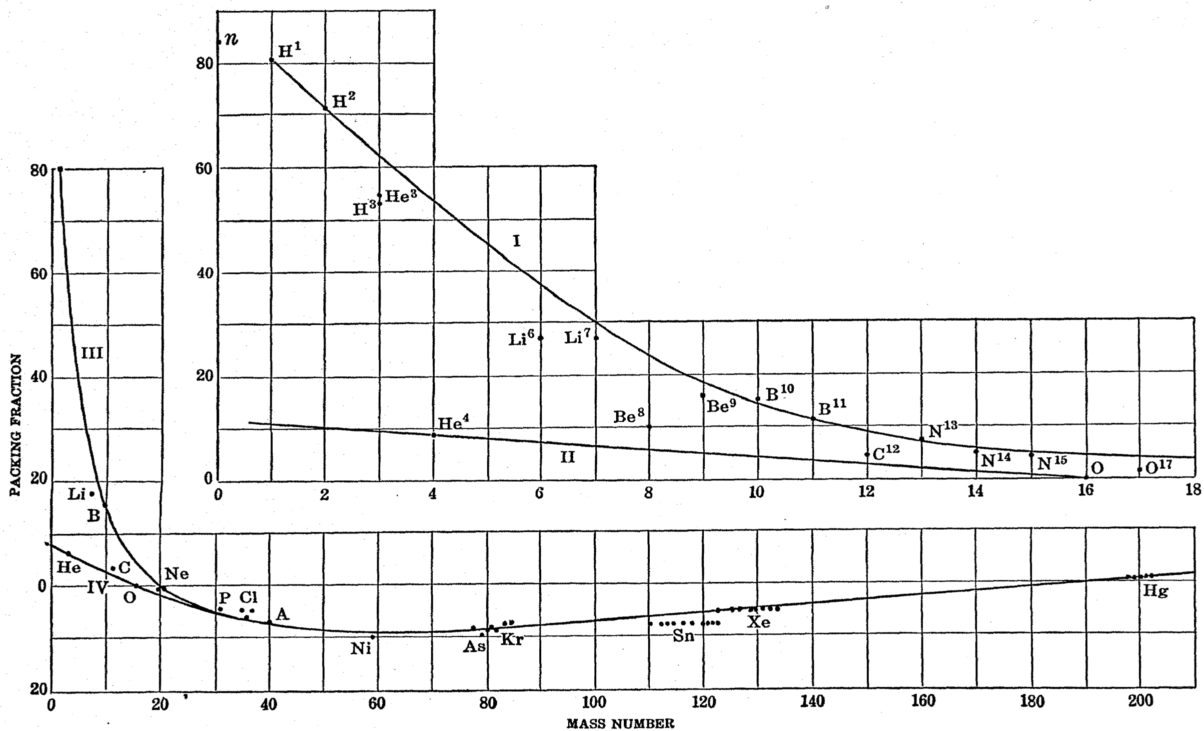
Curve II shows that the masses 4, 8, 12, 16 belong together—at least that they are quite different from the light elements of curve I, though Be^8, a rare atom, departs from the curve.

Curve III, of importance for masses heavier than oxygen, shows that the atoms between oxygen and mercury lie below the zero line, that the fractions are negative. Above mercury the fractions become positive and the atoms approach the instability of the radioactive class.

The Materialization of Radiant Energy

In all the transmutations which we have considered, there has constantly been the necessity of changing excess mass over into energy. Generally this energy has taken the kinetic form of motion of some of the ejected particles, but frequently it has taken the form of γ radiation. In the latter case *matter* has changed to *light*. The question arises, does the reverse phenomenon take place?

In 1933 the Joliots showed that one form of penetrating radiation excited by alphas in beryllium gave rise to positrons; that these emerged chiefly from heavy elements and that γ rays (of 5 M.E.V.) were the exciting agents. Then going back to more direct methods they showed (as did Anderson and Neddermeyer) that γ rays from ThC″, the most energetic (2.65 M.E.V.) from all the ordinary radioactive materials, were also able to drive positrons out of heavy atoms. Electrons were also driven out. The mode of exhibiting these results is shown in Fig. 14–13. A polonium disc is placed near a sheet of beryllium and this near lead, which forms part of the wall of a cloud chamber. A magnetic field at right angles to the cloud chamber shows the two



PACKING FRACTION
MASS NUMBER
n
H¹
H²
H³
He³
He⁴
Li⁶
Li⁷
Be⁸
Be⁹
B¹⁰
B¹¹
C¹²
N¹³
N¹⁴
N¹⁵
O
O¹⁷
I
II
III
IV
Li
B
He
C
O
Ne
P
Cl
A
Ni
As
Kr
Sn
Xe
Hg

FIG. 14–12.

kinds of electrons. By the curvature of the paths we are able to compute their energies.

How do we account for the appearance of these positive and negative electrons? When we direct ordinary X-rays at atoms, we find that only negative electrons emerge—no positives. Here is where a "good theory," as Eddington calls it, comes to aid the experimentalist. Dirac had proposed that in or near the nucleus of a (heavy) atom an intense γ ray could be transformed into a pair of electrons, one plus, one minus. The Joliots accepted this view—and they found experimental reasons justifying it.

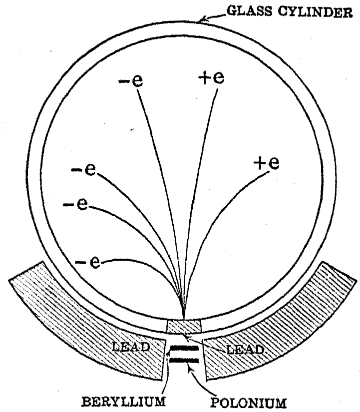


FIG. 14–13. A γ ray of 5 million electron volts from beryllium driving into lead are transformed into a positron and an electron (so it is believed). The first experiment showing radiation (γ rays) changing into matter. (Irene Curie-Joliot and M. Joliot.)

An electron (of either sign) has a known mass m. The energy required to bring it into existence is mc^2, which is nearly 81×10^{-8} ergs = 510,000 volts. Hence in order to generate a pair of electrons we would have to use a photon of 1.02×10^6 volts. However, that would leave no energy to take the kinetic form. Consequently it would be necessary to use photons of energies greater than 1.02 M.E.V.

Now it had long been known that the nucleus has peculiar laws of absorption for photons. Whereas the electron cloud around the atom absorbs chiefly low frequencies, the nucleus seems to have a high absorption for large frequencies. The reason now seems clear. The high-frequency photons accept the privilege, when near the nucleus of a heavy atom, of changing over into electrons.

Crane and Lauritsen and their associates have investigated this matter very critically. Using their high-voltage gun they have bombarded beryllium with deuterons or lithium with protons and have obtained some rather amazing results. We recall that in equations (2) and (7) γ rays were obtained when lithium was

bombarded with protons. Crane and Lauritsen directed these γ rays against lead and by means of a cloud chamber measured the energies of the electrons. They found positive and negative electrons, pairs, in what might be called broad band spectra or energy levels. The maximum band had an energy of 16 M.E.V. (This included $2\,mc^2$ or 1.02 M.E.V.) Hence, they reasoned, the γ rays, giving rise to these pairs, had energies of 16 million volts. Let us emphasize this point. The most intense γ radiation from a radioactive source is the 2.65 M.E.V. from thorium C''. Here are γ rays of six times that energy and they are obtained by hurling million-volt protons against lithium. What may be expected of that 16 M.E.V. radiation in its action on atoms? And if 16 M.E.V. are obtained with 1 M.E.V. protons, why not 20 or 30 M.E.V. with higher proton energies?

Here then is a new region for exploration; first, the production of γ rays of great energies; and second, the use of these rays in many fields.

We have asked the question—*does radiation change to matter?*—and the answer is, some radiation does. But the smallest element of matter (that we now know) is the electron. The energy required to produce it is 500,000 volts. Fractions of this photon energy cannot change over. Red light, for example, can never change to matter. Now it is possible that a single electron cannot be produced in this way; we must create a pair. In this case the lowest energy photon which can be transformed is one million volts. And photons of that energy are rather rare in our ordinary world.

CHAPTER 15

WAVES, PARTICLES, NEW ATOM PICTURES

Photons have energy, momentum, frequency, wave length. Particles have energy, momentum; have they frequency and wave length?——Is there evidence of Economy in Nature, a Principle of Least Effort, of Least Time, and does this Principle determine the energy states of atoms?——May we have waves of great speed combining to produce particles of small speed, and may these waves guide the particle in its motion?——(The de Broglie Atom.)—Is a mathematical equation a machine into which we feed a simple law of Nature and get out a multitude of regulations?——(The Schroedinger Equation.)

In an earlier chapter we described an experiment by means of which the pressure of light was measured, and we showed how we could account for that pressure by one of two points of view: that of waves, or that of photons. In the latter case we accepted as established the Compton effect. The theory of this effect made use of two relations: the energy E of a photon is hf, and the momentum M is hf/c. Remembering that $c = f\lambda$ where λ is the wave length associated with the photon, we have $\lambda = h/M$. Let us extend this to apply to any particle in motion. Then a mass m moving with velocity v ought to have associated with it a "wave length" which would be equal to Planck's constant h divided by the momentum, or $\lambda = h/mv$.

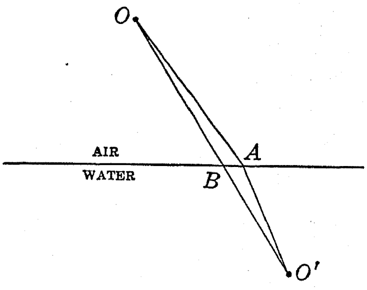


FIG. 15–1. Light passing from a point O in one medium to O' in another medium does not follow the shortest geometrical path but that requiring the least time. Fermat's Principle.

This would suggest an analogy between a photon and a particle in motion. But there is another reason for setting forth such an analogy. Let us consider light passing from a point O in air to O^1 in water. We know that it is bent, as shown in Fig. 15–1. We can easily prove that the total time required for the light to pass along OA in air and AO^1 in water is less than the time along any other path from O to O^1, even less

than along the straight line OBO^1. For though OB is less than OA, O^1B is greater than O^1A and it is the greater time-excess along O^1B that justifies the above statement. This illustrates a great general principle [1] which is the basis of Geometrical Optics. It is known as Fermat's *Principle of Least Time.* The paths of "rays" of light through a microscope, a telescope, the human eye, of light coming to us from the sun and bending down towards the earth through atmospheric layers of increasing density, are all in accord with this principle.

But there is a somewhat similar principle concerning the motion of a particle. Let us consider a particle describing an orbit passing through two points O, O^1, in such a way that the total energy is conserved. Then the *kinetic* energy at any point P of the path multiplied by the time required to pass over a small element of the orbit at P, and the total product added up—the sum is a minimum. This is a celebrated principle, Hamilton's *Principle of Least Action.* It is to be observed here that the sum involves *energy* multiplied by *time.*

In earlier chapters we took pains to point out that Planck's constant h, which originally was only a constant in the relation $E = hf$, is dimensionally equal to energy $\times$ time, or momentum $\times$ length. Hence h is a constant of *Action* as Hamilton used the term.

It was considerations like these that led Louis de Broglie to write several theoretical papers (1922–26) on topics like this: "On the Parallelism between the Dynamics of a Material Particle and Geometrical Optics." The mathematical operations were not of great difficulty but the physical concepts appeared—well, rather fanciful. But de Broglie felt that it was necessary to depart from orthodox methods. At the beginning of one of his articles he admirably states the case. "*Under pressure from the results of experiments,* physicists have been obliged to admit that the old dynamics, even when enlarged by relativistic ideas, could not interpret phenomena involving quanta. Today it appears necessary to create a new mechanics closely connected with the theory of waves."

[1] To prove this principle we assume the law of refraction $\sin i/\sin r = n = c/v$ where v is the velocity in water. Or conversely, assuming the principle, we may derive the law of refraction.

The Electron Becomes Wave-like.

Perhaps theoretical physicists might have played with the above analysis for a long time without causing much excitement in the world of physics, but when, in 1927, Davisson and Germer, utilizing the great resources of the Bell Laboratories, showed experimentally that a stream of electrons was scattered from a surface of a nickel crystal as if the electrons possessed "wave lengths" and that the wave length (very nearly) satisfied the relation $\lambda = h/mv$, doubting physicists realized that there was something very fundamental in the analogy with which de Broglie had worked. A rough diagram of the apparatus used by Davisson and Germer is shown in Fig. 15–2. Very soon after the

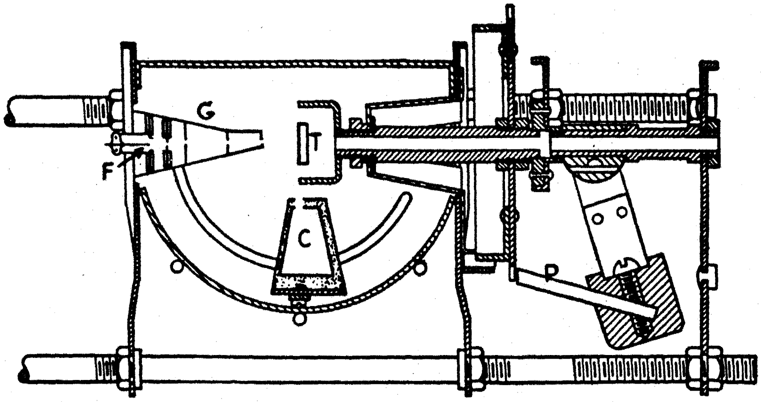


FIG. 15–2. The Davisson and Germer apparatus, the first to show (1927) the wave character of electrons. Electrons from a hot filament F are fired by the "gun" G at a crystal T. They are scattered back in special directions as detected by C. All of this in a glass bulb exhausted to the highest possible vacuum.

announcement of the above result, G. P. Thomson (son of J. J.) published a paper containing photographs clearly demonstrating the wave character of electrons.

It will be recalled that the wave-like character of X-rays was discovered by passing a narrow beam of the rays (general, heterogeneous radiation, not monochromatic) through a crystal and registering the effect on a photographic plate. The "Laue spots" (Fig. 7–4) are interpreted as due to the scattering of the X-rays from atoms in definite crystal planes. If X-rays (homogeneous, not general) pass through a thin film (or a powdered

crystal) in which crystals are oriented in all directions, then the direction of a maximum will make a constant angle with that of the X-rays, and in place of a spot we shall have a circle, as shown in Fig. 15–3. It was this kind of photograph (Fig. 15–4) that was

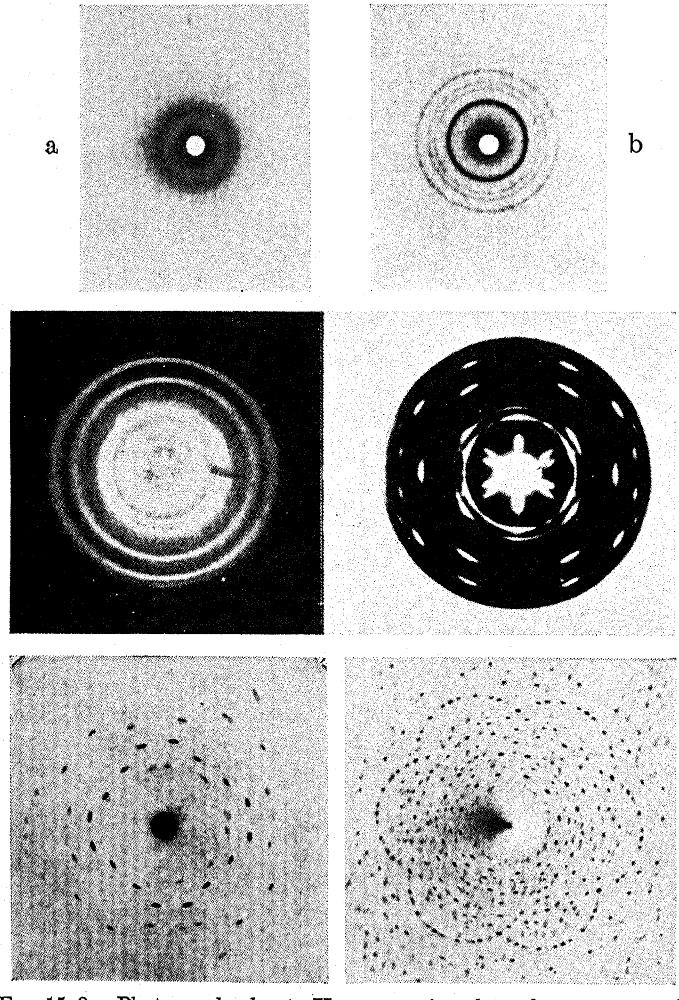


FIG. 15–3. Photographs due to X-rays passing through: a, an amorphous material, vitreous silica; b, material whose crystals have a completely random direction ((a, b) Randall); c, aluminium in powder form, crystals in nearly random direction (A. W. Hull); d, drawn aluminium wire, rather definite structure (G. L. Clark); e, crystal of zincblende (Knipping); f, crystal of beryl (Lehmann). Exposure times, minutes or hours. See Figs. 15–4 and 16–2.

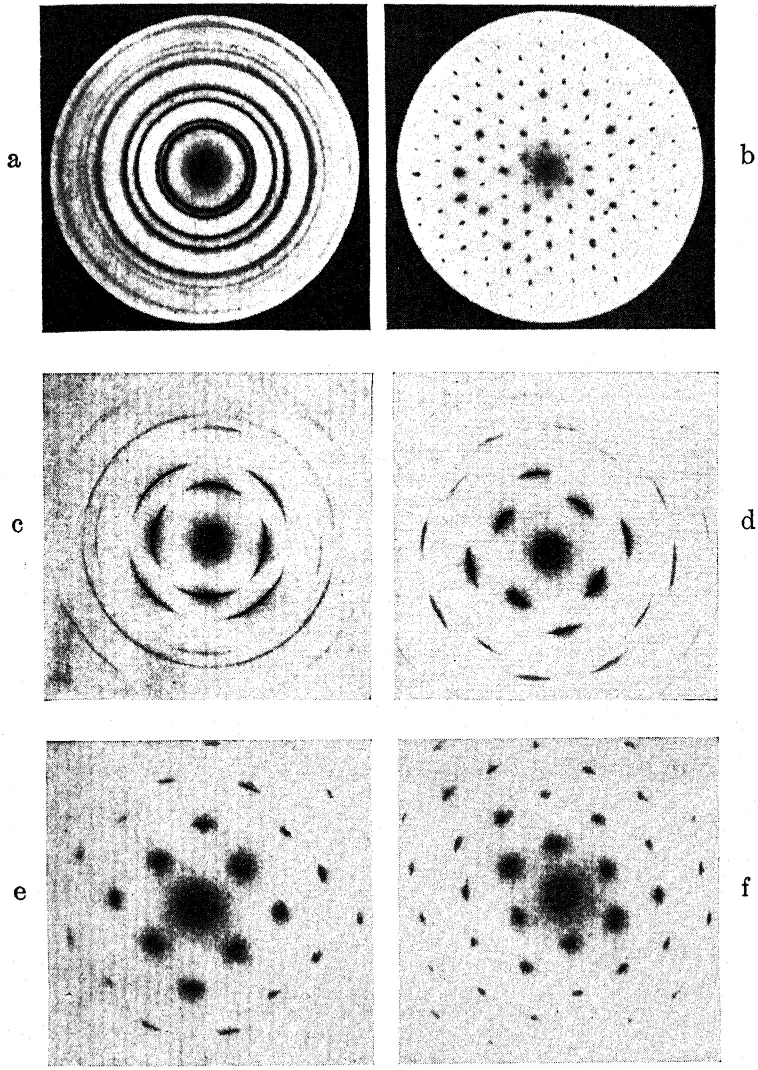


Fig. 15-4. Photographs due to electrons passing through: a. gold foil (10^{-6} mm. thickness), completely random direction of crystals, 58 kilovolts (Eisenhut and Kaupp); b, mica (10^{-7} to 10^{-6} mm.), complete structure, crossed gratings, 43 kilovolts (Eisenhut and Kaupp); c and d, gold foil partially crystallized by tempering; e, platinum, thin crystal, electron stream parallel to [100]; f, same as e. Electron stream parallel to [110] c, d, e, f, due to Trillat and Hirsch. Exposure times from 1/50 of a second to 1 second. Compare with Figs. 15-3 and 16-2.

taken by Thomson. The procedure now is somewhat as follows. Electrons from a hot filament,[1] oxide coated (Fig. 15–5), are accelerated by a high potential. Some of them pass through a small hole in the anticathode, then through a very thin film to a fluorescent screen or photographic plate. (Obviously the tube must be highly exhausted and the photographic plate must be shielded from light.) If the film is of beaten gold, the crystals are likely to lie in random directions and we have circular rings. From the geometrical dimensions of the tube, the radii of the rings, and the distances between the crystal planes, we can compute the wave length of the electrons; from the accelerating

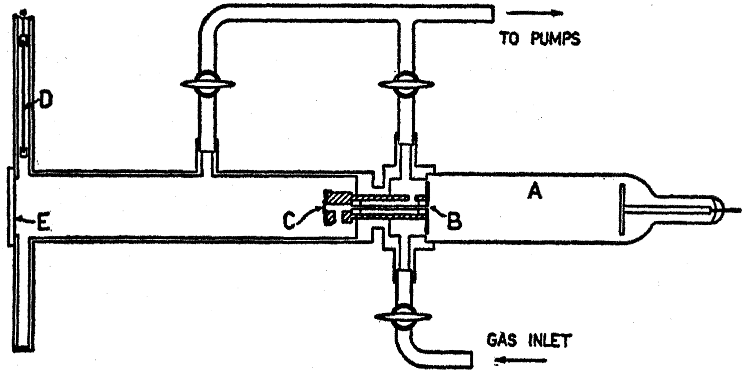


FIG. 15–5. Cathode rays—electrons, streaming through a hole in the anode B, pass through thin gold foil C (10^{-6} mm. thickness) to the photographic plate D (when in central position).

potential we get their momentum; then we can test the relation $\lambda = h/mv$.

The announcement in 1927 that electrons possessed wave-like characteristics came as rather startling news to most physicists. For thirty years the idea had been developed that an electron was the ultimate particle of matter. We supposed that we knew its mass and dimensions. We had seen the track that it formed in a cloud chamber. We had used it as a particle of definite energy when we projected it against a target for the generation of X-rays. Now it became clear that a *stream* of electrons passing through a crystal was diffracted in exactly the same way as a stream of ether waves, perhaps we should say a stream of photons.

[1] The figure shows the tube used by Thomson, the cathode ray type.

Nor was this all. A little later, Otto Stern (formerly of Hamburg, now of Pittsburg) showed that a stream of *atoms* of definite mass and velocity behaved in the same way and that the same law held, $\lambda = h/mv$.

Comparison of X-Rays and Electron Waves.

Let us now consider the quantitative relations for X-rays and electron waves. In Fig. 15–5, if V is the potential between anode and cathode, the kinetic energy of the electrons which have passed through the opening in the anode is given by $Ve = \frac{1}{2} mv^2$. Substituting the known values for e and m, we find $\lambda_e = h/mv = \sqrt{150/V}$ angstroms where V is in volts. The corresponding relation (Chapter 7) for photons is $\lambda_p = 12{,}345/V$ Å. (It would appear that λ_e varies inversely as the square root of V, and λ_p inversely as V. Obviously the constants must compensate for the apparent difference in dimensions.) From the above we see that electrons of 100 volts have the same wave length as photons of 10,000 volts, or 1.23 Å. This would be a rather long wave length for interference patterns, and the energy of the electrons would be very small. But at least we see that it is easy to obtain electron wave lengths of the right order of magnitude for interference and diffraction experiments, with the atoms in crystals as scattering centers.

Thomson used a potential of 30,000 volts. Assuming the relations for X-ray and electron wave lengths, he computed the size of the unit cube for different metals as measured by X-rays and by electron waves. (See Table I.) The two values differed by only about 1 per cent for films of aluminium, gold, platinum, silver, ... Thus he established, as did Davisson and Germer, the complete similarity between electron waves and X-rays.

But what kind of waves are these which are associated with an electron? For that matter, what kind of waves are associated with a photon? To the latter question we answer, or we used to answer, ether waves. We are more or less satisfied with that answer. But to the former question—well, the student must now be prepared for a headache.

A Particle of Small Velocity v Has Waves of Large Velocity u.

We return to the two great principles, Least Action and Least Time. In the former we assert that the sum of $\frac{1}{2} mv^2 \times$ time

TABLE I

Size of unit cube, a, in angstroms or 10^{-8} cm.

Metal	Measurements	
	X-Ray	Cathode Ray
Aluminum	4.046	4.06; 4.00
Gold	4.06	4.18; 3.99
Platinum	3.91	3.88; 3.89
Lead	4.92	4.99
Iron	2.87	2.85
Silver	4.079	4.11
Copper	3.60	3.66
Tin (white)	2.91	2.86

must be a minimum. In the latter, that the sum of l/u must be a minimum, where l is an element of path and where u is the wave velocity. These may be brought together if $v^2t = lK/u$ where K is some constant of proportionality. Since $l/t = v$, we have $uv = K$. It is obvious that since the left-hand side of the equation has the dimensions of a velocity squared, we may put $K = c^2$ and we have $u = c^2/v$. Here we take c equal to the velocity of light; then the velocity of the waves of a photon would be equal to that of the photon itself.

Thus we have derived in a very crude way a relation set forth by de Broglie. It pictures a particle of velocity, say 186 miles per second, having waves associated with it of velocity 186,000,000 miles per second! Then our respect for the vanishing ether leads us to assert that at least these waves are not ether waves. We are perhaps easily reconciled to the picture of a photon having waves tied to it moving with it, waves and photon having the speed of light. But to picture a particle of a speed of one-tenth that of light having waves associated with it possessing a speed 100 times that of the particle—that is a picture that man hath not seen on land or sea! Yet experiment establishes the relation $\lambda = h/mv$.

But what is the "frequency" associated with a free electron? We may obtain this from the relation $\lambda f = u$ or from the equivalent, $mc^2 = E = hf$. Putting in the values for m, h, and c, we get

$f = 1.3 \times 10^{20}$; this for any electron, provided that its speed does not approach that of light.

We have been discussing an important common property of electrons and photons. Now let us deal with contrasts.

A Contrast, Photons *vs.* Electrons.

A photon has a transient existence. It is generated in one place, it moves with the speed of light to another place and is there absorbed. It cannot stand still. Having been born, it rushes with vast speed to its death. But an electron can remain at rest, or move with any speed less than that of light. We are accustomed to think of it as having a continuous existence, though recently discovered phenomena cause us to accept this view with reservations. But at least it lives forever or it is completely destroyed. There are no half-way measures for an electron. A photon, however, may be partially or almost completely destroyed, as in the Compton effect, when it strikes an electron.

An electron may have an elastic or an inelastic collision with an atom. In the former case it is scattered without loss of energy, and if the atoms are arranged symmetrically as in a crystal, we have Laue spots or diffraction patterns. The electron waves and the crystal structure control the pattern. In the other case, we have X-rays generated and they go out in all directions. Thus it is that when we pass a beam of electrons through a thin film, the energy is large and we can secure a good photograph, as in Fig. 15–4, in a few seconds. The corresponding time for X-rays might be a few hours. But X-rays can penetrate large thicknesses as compared with electrons. Consequently, using the greater thickness, there is apt to be a sharpness to the X-ray interference pattern that is not found with electrons.

Of course there are the obvious contrasts. An electron in motiön can be deflected by electric and magnetic fields; the number of electrons going to a certain area may be measured by the electric charge collected. Thomson showed that the particles producing the curves in Fig. 15–4 were electrons, not photons. When he imposed a magnetic field across his tube, the whole interference pattern was shifted. There would have been no shift for photons.

How do we account for the diffraction patterns of Fig. 15–4? In the case of X-rays, we may invoke the century-old explanation

for similar phenomena due to visible light; X-rays are waves in the ether. The wave-like character of X-rays was discovered before the idea of the photon had become firmly established. But even if we allow the photon or particle idea to prevail, we can still acquiesce in the explanation that a photon has a definite frequency, therefore there is a definite wave length associated with and moving with it, and it is this wave length that determines the direction in which the photon must go—or the direction in which the most photons of that frequency go. *The photon is guided by its waves.* But an electron? Almost shamefacedly we use the same words. Electrons are guided by their waves. Perhaps it would be more direct to say that *we* are guided by the waves; we compute the wave length from $\lambda = h/mv$ and we then find the direction in which those waves have a maximum intensity. There we find the maximum number of electrons. Again we note that when electrons are passing through small holes or crystallized sheets, their wave-like qualities are in control; when they strike a screen they are particles. The number is measured by the electric charge collected.

The student should recall some of the many ways in which single electrons or photons act as quanta or particles—the photoelectric effect, the Bohr atom, the Compton effect, the Raman effect, the Geiger counter, the cloud track, the oil drop, ... Then the ways in which these particles are guided by their waves. The author's suggestion that a photon may be likened to a sunfish helps to bring together the two characteristics—a particle and periodicity. But is an electron a sunfish? In a photon sunfish the skeleton (waves) moves with the fish. But an electron of one-tenth the speed of light has waves moving with a speed ten times that of light, therefore one hundred times its speed. No, a sunfish electron will not do. In this respect, an electron is more slippery than an eel!

The relation that has been discussed, $\lambda = h/mv$, holds for atoms, particles, an automobile. The student should compute the wave length of a hydrogen atom at room temperature; of a neutron which has been ejected from beryllium. In the latter case we have an interesting combination. Since the waves of a particle are rather ghost-like antennae and since a neutron itself is an almost ghost-like particle, its waves become ghost-like to the second power! It is rather difficult to obtain a beam of

neutrons, but by means of paraffin spheres and cadmium sheets (Fig. 14–6) it can be done. There should then be maxima and minima directions for neutrons emerging from a very fine slit in cadmium, or scattered from the right kind of crystal. The experiment awaits performance.

Group and Phase Velocity

To one other point of view developed by de Broglie we now direct attention. It will be recalled that in Chapter 4 we considered two trains of waves of the same frequency but of different wave lengths, the shorter waves traveling slower than the longer. Then we saw that a heaped-up crest due to the two sets of waves would travel slower than either train. This was the *group* velocity; the velocity of a wave train was the *phase* velocity. De Broglie showed that if v is the velocity of a particle and u that of its waves, the relation $u = c^2/v$ could be obtained. The picture that results from this is that a particle is the heaped-up crest due to two trains of waves moving in a dispersive medium with a speed perhaps one hundred times that of the particle. Mathematically this is attractive, but as a model it is not satisfactory.

There is, however, one application of this idea that is misleading. In the Huyghens' explanation of the refraction of light as it passes from air to water, the velocity (u) in water is less than in air (c) and the ratio c/u is n, the index of refraction. Newton wanted the corpuscles of light to be attracted towards the *denser* medium, in which case the velocity v of the corpuscles in water would be greater than in air and v/c would be the index of refraction. From this it would result that $v = c^2/u$ and it might appear that there is a group velocity, v, greater than that of light. But Michelson by direct experiment showed that the group velocity in water, or carbon bisulphide, is less than the wave velocity. Hence the above argument is misleading.

It is true, however, that the *momentum* of a photon in water is greater than in air. This follows from the following argument. The energy of a photon is hf; its velocity in water is hf/v or nhf/c. (This assumes that the photon travels with the waves or vice versa.) Hence there is an increase in the momentum as the photon crosses the boundary and we come to Newton's aid in accounting for the law of refraction!

De Broglie Pictures of an Atom

Let us recall Hamilton's Principle—that the summation of kinetic energy × time or momentum × distance between two points on an orbit must be a minimum. Now these products are of the same dimensions as h, Planck's constant, and this is a constant of nature. Hence we restate the Principle and say that the sum of such products must be a whole number of h's; that the minimum value in a minimum complete orbit is h. If the orbit is a circle, the momentum is obviously constant and we have the relation: momentum × circumference $= h$, or $2\pi rmv = h$. This is Bohr's well-known assumption defining the smallest circular orbit. If we join this to the relation $\lambda = h/mv$, we see that *the circumference of the first circular orbit is the wave length of the electron as it describes that orbit.* In the second circular orbit [1] there are two wave lengths of the electron in that orbit; in the third orbit, three, etc.

Reviving the old Bohr picture for this purpose, we see the circular orbits as a strange group of standing waves. As the electron moves around the n^{th} orbit with velocity v_n, the phase waves move [2] with velocity $u_n = c^2/v_n$. Now the frequency ϕ_n of the waves in this orbit is u_n/λ_n and, *mirabile dictu,* comes out equal to mc^2/h. That is our old friend $hf = h\phi_n = E = mc^2$! Did we put that into the picture somewhere? (The student might regard this as a picture puzzle: he is requested to find the bird. It will be seen of course that we have reversed one of de Broglie's operations.) From this it appears necessary to distribute the electron all around the orbit and to set up in it phase waves of frequency mc^2/h. At first sight it would seem that the frequency is the same in all the orbits. It would be so if the total energy were given by m_0c^2. But we must now remember that the total energy includes the mass energy, m_0c^2, plus the kinetic, plus the potential. Since the potential energy at infinity is taken as zero, for any closed orbit it is negative. Now it

[1] In an elliptic orbit what is v? and the periphery? and λ?

[2] The exacting student has observed that this presentation is not correct. The total energy is $mc^2 + V$ where V is the potential energy. Since λ equals h/mv and $f = u/\lambda$, we have

$$u = \frac{c^2}{v} + \frac{V}{mv}.$$

We ought therefore to take account of the change in potential energy. This is left as an exercise for the advanced student.

has been shown in Chapter 6 that when an electron or a planet describes a circular orbit, the kinetic energy in that orbit is just one-half of the potential loss. Consequently the above sum just equals m_0c^2 − K.E. In other words, the energy, and therefore the frequency, decreases as we go to smaller orbits. *The energy radiated is the energy loss divided by* h or it is *simply the difference between the phase frequency in the larger and that in the smaller orbit.*

In some texts the following argument is given. The energy is m_0c^2 + K.E. The value of the K.E. is then found from the Bohr relation $v = 2\pi e^2/nh$. Of course it follows that the photon frequency is the difference between electron frequencies. But as the electron goes into a smaller orbit it must not only acquire a new larger frequency, it must also radiate a photon frequency equal to the amount acquired. This is a good deal to require of an electron. But as we have treated the matter in the previous paragraph, the electron merely discards, as a photon, unwanted frequencies as it goes to its new abode.

With the interpretation of total energy as given above, it becomes clear that for any orbit, elliptical or circular, the frequency is constant and is simply E/h. But the wave length equals h/mv. Consequently in an elliptical orbit the wave length is constantly changing, being large when the electron velocity is small, therefore at the greater distance from the nucleus. However, we need not worry about the wave length, nor about the rotational frequency in the orbit: the electron frequency is the thing! Bohr or Sommerfeld might have said that an electron in any position near a nucleus has a frequency equal to E/h. Then to find the frequency radiated as the electron changes from one energy state to another, it is necessary only to find the difference in the electron frequencies. How easy, if a formula would give us the energies! The spectral frequency may again be regarded as a beat note between two phase frequencies, one of which is dying, the other coming into being.

This is a curious, an amazing result. The phase frequencies are of the order of 10^{20}; the photon frequencies (for visible light) of the order of 10^{15} per second. We are asked to accept the view that a photon, something that is radiated through space with the speed of light, is due to a beat note in two "vibrations" which are not radiated and which have frequencies about 100,000 times that

of the photon. Moreover, this beat note can only be obtained when the atom changes from one energy state to another. Therefore one of these phase vibrations must cease and the other start before any energy will be available for radiation. It seems to be an impossible picture—but it helped to draw (1929) a Nobel prize.

In illustration of the above relations, textbooks frequently give the following. Let f_1 and f_2 be the frequencies of two supersonic notes, neither of which can be heard separately by the ear, but the beat note, $f_1 - f_2$, is within the hearing range. Then when the two frequencies are received, the ear hears the difference frequency. This is a common error. The ear must be sensitive to both components, otherwise a beat note is not heard. The same statement may be made regarding a radio receiver. And from the standpoint of experimental physics it may be held that a beat note photon cannot be radiated unless its components are radiated, this unless Nature has modes of operation at present undreamed of.

De Broglie's great contribution was not in giving us the picture which we have just discussed but in introducing into microscopic phenomena the idea of waves, of periodicity, of assigning to the total energy E of every particle a characteristic frequency f given by the relation $E = hf$. It was this idea that intrigued Erwin Schroedinger,[1] who in a few brilliant papers (1926–1927) gave to the world what is probably the most important theoretical contribution since the time of Maxwell.

Schroedinger's Equation

If one is asked, "What is Schroedinger's theory?" one must reply as in the case of Maxwell's theory, "His theory is his equation." Now the reader may be quite unable to follow the mathematical operations by which this equation was derived or by which it is solved; nevertheless he should see it and look upon it with great respect, for it contains in its mysterious recesses more of information concerning atoms and electrons than can be found in any other equation. Since its presentation, volumes have been written in its elucidation. In its simplest form it may

[1] Erwin Schroedinger, born in Austria, 1888; student in the University of Vienna; professor of Mathematical Physics, Zürich, 1921; professor of Theoretical Physics, Berlin, 1926; now in Oxford. With Dirac of Cambridge he shared the Nobel prize in 1933.

be written

$$\frac{d^2\psi}{dx^2} + \frac{8\,\pi^3 m}{h^2}(E - V)\psi = 0. \tag{1}$$

Here we are concerned with a particle of mass m, moving along x, having a total energy E and a potential energy V due to certain forces. If we are thinking of an electron moving freely, $E - V = \frac{1}{2}\,mv^2$ and $E = mc^2$. Then the equation can be solved at once and we obtain $\lambda = h/mv$. This is the de Broglie wave length! We may regard it as marvelous that we have obtained this result. But in the way in which Schroedinger derived the equation, it was put in a disguised form. In the very unorthodox method used by the author in the appendix (Appendix 15–1) it enters directly. It would be very disturbing if we did not get out of a mathematical equation everything that was put in. But to get out a great number of details the existence of which was unsuspected by the designer of the equation when he merely constructed it to represent some general physical principle—that is the supreme function of mathematics.

A few very simple illustrations of this point may here be given. A pendulum bob is pulled away from its equilibrium position and then released. We construct an equation expressing the fact that a force is acting on it proportional to its displacement from the equilibrium position. We solve the equation and find that the motion is periodic and that the period is proportional to the square root of the length of the pendulum. We suspect damping and we put in a term stating that the damping force is proportional to the velocity, and we find that the equation announces that the period is slightly altered, also that the successive decreasing amplitudes bear a constant ratio to one another. All of this is experimentally confirmed. Thus when we construct the above equation we have summarized the operations of a vast number of ordinary phenomena.

Similarly when a stretched string is plucked, we form an equation giving its subsequent motion. But in solving it we must take account of the fact that it is fixed at two ends and that initially it had a certain form. A surprising amount of information boils out of the solution: that the subsequent motion contains a number of frequencies of limited amplitudes; but of all the infinite number of solutions that are possible, only those are acceptable which satisfy the imposed conditions. The fre-

quencies, for example, are limited to whole-number ratios, 1, 2, 3, $\cdots$, for a string, or 1, 3, 5, $\cdots$, etc., for a rod clamped at the center or at one end. In other forms of vibrating bodies, a membrane, a bell, the frequencies are not necessarily restricted to these simple values. These allowed forms are called Stationary States and the frequencies are known as Proper Frequencies.

Now the Schroedinger equation resembles that giving the motion of a stretched membrane, or of a steel shell filled with fluid. Why then is it such an extraordinary equation? The interpretation that it represents a family of surfaces having a velocity u at any point equal to $E/\sqrt{2m(E - V)}$ and the further view that E is equal to hf, these added adornments apparently lift it into a new realm.

We have defined all the quantities except ψ. In the case of the stretched membrane or steel shell, ψ represents the amplitude of vibration; its square is a measure of the energy at any point. But in this equation, "*was bedeutet* ψ*?*" In 1928 Schroedinger gave four lectures before the Royal Institution in London and, although he gave tentative interpretations to ψ at various times, at the end of his fourth lecture he turned to the question, How shall we interpret ψ? Hesitatingly he gave the answer which is now invariably given: the square of ψ is the *probability* that the particle m shall be at a certain point. In the case of electrons it would be a measure of the number or of the electric charge. And now we see that, instead of speaking of the *amplitude* of a vibration of a string or membrane, we may speak of *the probability* of the displacement being a certain value.

The difficulty of interpretation of ψ recalls a similar conceptual difficulty in Maxwell's equations. It will be recalled that Maxwell made use of two well-known experimental results, that of Oersted which gave the intensity of the magnetic field around a wire carrying a current, and that of Faraday which gave the electromotive force in a circuit due to a changing field. But he extended the idea of an electric current to include what he called *the displacement current.* It was, for its time, a difficult concept. But when he put all this in mathematical form in his immortal six equations and gave them the proper mathematical treatment, the result was beautiful—he showed that an electric disturbance generated at one point would pass through a material like glass with a velocity equal to that of the velocity of light divided by

the index of refraction. He prophesied the existence of electric waves, and he brought the phenomena of light and electricity together. The mathematical operations were perfectly clear.[1] But that displacement current? It is stated that one of the foremost French physicists said that he understood completely Maxwell's electromagnetic theory—only he did not know what he (Maxwell) meant by electricity! New concepts are difficult of acclimatization.

We come to discuss two cases in which the Schroedinger equation has led to correct (experimental) results. The case of (1), the spinning or rotating molecule with free axis; and (2), the hydrogen atom.

1. The Spinning or Rotating Molecule.

We picture a simple "dumb bell" molecule spinning about an axis at right angles to the line joining the two components of the molecule. It is necessary to transform the equation to r, θ, ϕ, coordinates but with r constant, and to give to the energy the ordinary kinetic energy of a rotating body. We find that the energy can then have only the following values,

$$\frac{l(l+1)h^2}{8\,\pi^2 I}$$

where $l = 0, 1, 2, \cdots$, and I is the moment of inertia about the axis. It is seen that since $l(l+1) = (l+\frac{1}{2})^2 - \frac{1}{4}$, energy levels $l(l+1)$ and $(l+\frac{1}{2})^2$ are equivalent, as only *differences* in energy levels are desired.

Hence these values of the energy are those which are necessary in explaining band spectra and the Raman effect (Chapter 10). Here then we have the proper quantizing of rotational energy levels.

[1] To students who do not think in mathematical symbols and who are not thoroughly at home with differential equations, they are not clear. But for certain physicists they are enshrined in song and story. The author was a member of the Cavendish Society (the physics research group of the University of Cambridge) and was at the annual banquet when these equations first appeared in musical form. We sang them (!) to the tune of "The Interfering Parrot." The "song" can be found in Appendix, and by way of contrast, showing a complete change of the physicist's point of view, a more recent song of the Cavendish Society glorifying "$h\nu$."

Physicists are not the only class to quote poetry to suit their purpose. Mathematicians have been guilty of this frailty. Witness (Appendix) the attempt to make a poem out of "π to thirty places"! But they have never sung it!

2. The Hydrogen Atom.

We start with the Rutherford picture, a nucleus and an electron. We take for granted that the old law of attraction holds, $F = e^2/r^2$; therefore that the potential energy V is $- e^2/r$. We transform the equation to r, θ, ϕ coordinates. As that equation so transformed would be rather terrifying to a non-mathematical reader, it is discretely placed in the appendix. The equation is then subjected to ordinary mathematical operations. Then various results appear. First the permitted values of the energy (leaving out m_0c^2) are: $2\,\pi^2me^4/h^2n^2$ where $n = 1, 2, 3, \cdots$. These are the Bohr values! There has been no assumption regarding orbits either circular or elliptical. Of course we are led at once to the frequencies of the radiated photons—they give the Balmer series and the other lines of hydrogen (see, however, our discussion of the de Broglie atom). But to continue, regarding results. When we break up the transformed equation into its three parts, we have not only the constant n which limits the energy states, but we have two others, l, m (m is not the mass). The equivalent of the old angular momentum comes out equal to $l(l + 1)h/2\,\pi$ where $l = 0, 1, 2, \cdots$, and the constant m corresponds to the old magnetic quantum number. It may have values l, $l - 1$, $\cdots (- l + 1)$, $- l$. All of this is purely mathematical, without any pictorial equivalent except as we connect it with the Bohr-Sommerfield model.

There are refinements. For example, the mass, m, may be replaced by the effective mass μ as was done by Bohr, then

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$$

(see Chapter 6, page 91). This justifies the variation of the Rydberg constant as the mass of the atom is changed. The spinning electron may be inserted. This operates as it did in the old theory. There are other refinements which we cannot here consider.

But there is an outstanding difference between the Schroedinger and Bohr atoms. The electrons now need not move in definite orbits. The electric density, the probability that the electron will be found in a definite region, is a very complicated function of r, θ, ϕ. When the computations have been carried out, the *electron cloud*, or the probability in a visible form, is shown by

Fig. 15–6, which we owe to Professor H. E. White. You are asked to draw a vertical line through the center of each figure (from bottom to top of the page) and rotate the figure about that line as axis. The resulting space figure represents the density of the electrons for a great number of atoms in their various energy states.

The atom now is a strange combination of Bohr terms regarding orbits which need not exist, of azimuthal and other quantum numbers; of de Broglie wave lengths and frequencies which cannot be measured; of Schroedinger densities which may possibly be subjected to experimental test. The author regrets that he cannot smooth out the irregularities in this picture. But he recalls the injunction given by Oliver Cromwell to the artist painting his portrait: "Paint me as I am. If you leave out a scar or a wrinkle I'll not pay you a shilling." So be it.

A New Aspect of Probability

We come to discuss an important new field opened up by the Schroedinger equation concerning the motions of particles. We consider the simple experiment of throwing a stone into the air. As we ordinarily state the case, the stone loses kinetic energy but gains potential. Finally when all the energy is potential, the stone stops, then descends. In equation (1) $E - V$ is then zero. Before this it had the value [1] $\frac{1}{2} mv^2$ and equation (1) then took the form

$$\frac{d^2\psi}{dx^2} + \frac{4\,\pi^2 m^2 v^2}{h^2}\psi = 0.$$

All students of physics recognize this as the most common of all equations in physics, the simple harmonic motion equation (on the assumption that v is constant). The solution represents $\psi(x)$ as a wave motion, $\psi(x) = A \cos (2\,\pi/\lambda)x$, where $\lambda = h/mv$. As the particle goes up, v decreases and λ increases. What would happen if V would become greater than E? Then the solution takes an exponential form, $\psi(x) = Be^{-kx}$ where $k = (2\,\pi/h)\sqrt{m(V - E)}$. The probability that the particle would go a distance x is the square of this quantity $\psi(x)$ or $e^{-2\,kx}$ (B becomes unity). Now k depends on m and $V - E$. Let us think of an electron trying to escape from a region in which it

[1] We are neglecting the energy equivalent of the rest mass.

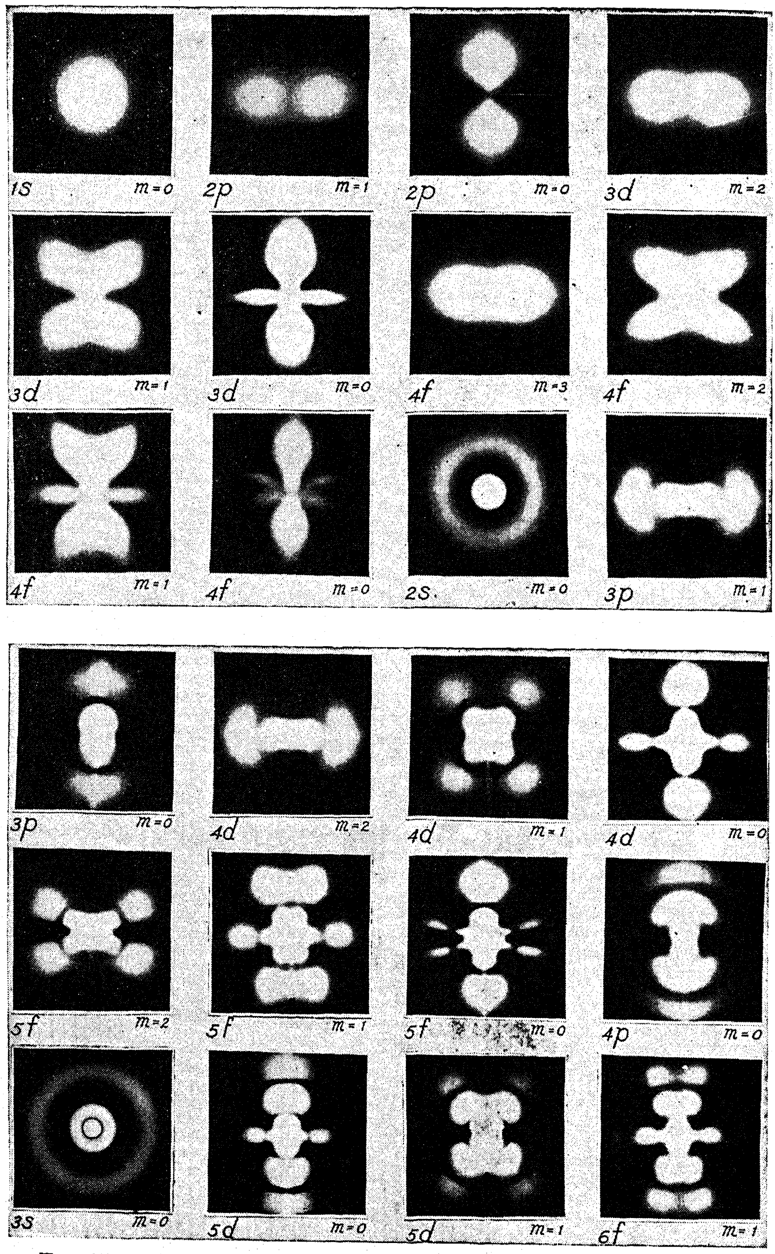


FIG. 15-6. The "latest picture" of the hydrogen atom. A cross-section of the electron cloud in the various "states" made by H. E. White. (From *Introduction of Atomic Spectra* by H. E. White, McGraw-Hill Co.)

moves back and forth. We ask the question, what is the probability of its escaping a distance of 3.2 angstrom units beyond the boundary requiring an electric potential of 1 volt. We substitute the values of $h = 6.5 \times 10^{-27}$, $m = 9 \times 10^{-28}$, $V - E = 1$ e.v. $= 1.6 \times 10^{-12}$, $x = 3.2 \times 10^{-8}$. Then the probability is, $e^{-2.3} = 0.1$. Had we been dealing with a proton, $m = 1.65 \times 10^{-24}$ and $e^{-100} = 10^{-43}$. Thus we see that an electron may escape from a region in which it is supposed to be confined, but it is highly improbable that a proton may escape.

There are many illustrations of the above conclusions. We may deposit an exceedingly thin film of gold on glass. At ordinary room temperature the molecules of gold are bouncing back and forth with an energy which may be expressed as a fraction of 1 e.v. Yet many years must pass before any diminution of the film thickness can possibly be noticed. On the other hand, electrons in metals move under the action of minute potentials. But the atom nuclei remain fixed.

Thus Schroedinger's equation reaches out to include problems concerning the motions of atoms and molecules, problems similar to those discussed in our first chapter.

CHAPTER 16

LIMITATIONS IMPOSED BY NATURE—THE UNCERTAINTY PRINCIPLE

Limitation in the Sharpness of an Image Due to the Wave Nature of Light.

Let us suppose that a strong beam of light is directed towards a small hole in a screen and that the emerging light is focussed by a lens on another screen. When the image is carefully examined (Fig. 16–1) it will be found that it is not a fine point like the hole from which the light emerged. There will be a central mass of light surrounded by colored rings. It might be thought that the fuzziness of the image was due to imperfection in the lens or the fact that the light was not monochromatic. But when the light is made as "monochromatic" as can be done and when the lens is figured and worked to the greatest precision, the image still appears as a central fuzzy spot with faint rings. Again it might be thought that if the light beam were squeezed together as it passes through the lens, that might bring all the

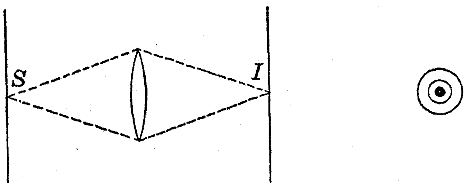


FIG. 16–1. The image of a very small object consists of a diffuse central part surrounded by rings of maxima and minima intensity.

FIG. 16–2. a, Light from a fine slit passes a (parallel) straight edge; the shadow is not sharp; bands of maximum and minimum intensity are seen near the edge; b, the shadow of a wire; the faintly luminous line in the center of the shadow cannot be seen; c, the shadow of a finer wire showing the luminous center; d, the shadow of a small triangle due to light from a small circular opening; e, shadow of a hexagon. (d and e due to Scheiner and Hirayama.)

The O's of the lower figures are due to light from a small circular opening passing through circular openings of different sizes. When the path difference between the central and outer rays is 2, 4, 6, 8 half-wave lengths, there is destructive interference in the center of the shadow: there is a black spot. When the path difference is 3, 7, 9, there is a bright spot. The P's are shadows of circular discs of the same size as the corresponding opening. (Arkadiew.) Compare with Figs. 15–3 and 15–4.

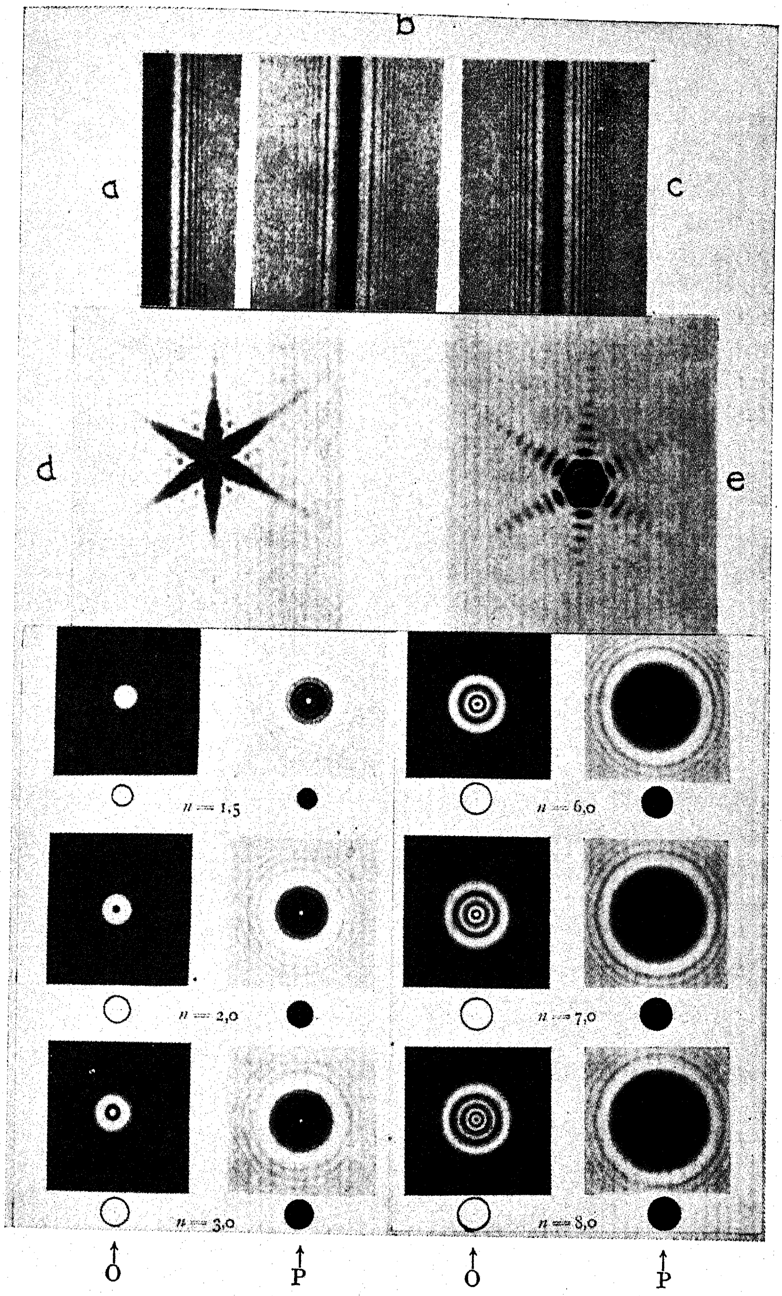


Fig. 16–2.

light together at a focal spot. But this makes matters worse. The *spot* and rings are larger and fuzzier than before. The smaller the lens, the larger but fainter the rings.

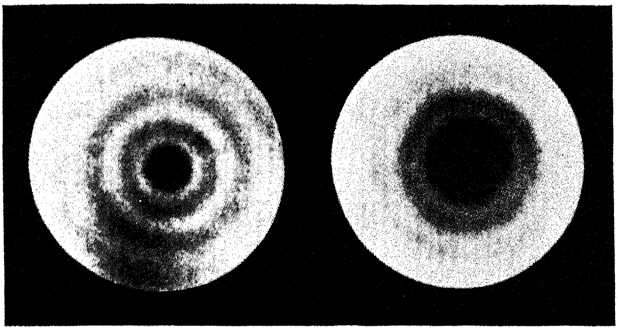
The lens has been constructed in the belief that the law of refraction holds, that a *ray* of light from a point S will follow a precise path to I and that this will be true for all rays from S passing through the lens. But it is found that this is not quite true. *Of the photons emerging from S and passing through the lens, not all of them can ever again be brought together to a point similar to that from which they emerged.* The wave character of the photons asserts itself and guides some here, some there. But the probability is that most of them will be found near the central point.

It results therefore that an exact image of a very small object can never be obtained. There is a limitation imposed by the wave character of the photons. By decreasing the wave length, it is true that the indefiniteness can be decreased. The ultraviolet microscope is designed for that purpose. But finally the absorption of matter for short waves and other considerations limit the possibilities of obtaining an exact image of a small object.

A result of the same nature is obtained when we attempt to cast a shadow of a fine wire by means of light from a fine parallel slit. Effects like those of Fig. 16–2 are obtained. The central part of the so-called geometrical shadow may contain a number of bright lines, while outside the shadow there are large and undulating variations of intensity. All of this proclaims, and is in accord with, the wave character of light.

Electrons Scattered from Nuclei Show the Same Phenomena.

When we greatly decrease the wave length, by the order of 1/100,000, and use *electrons* as our source and *atoms* as our *opaque* or scattering bodies, we obtain effects shown in Fig. 16–3 a and b. The scheme for the experimental setup is shown in Fig. 16–4. Electrons driven by a high voltage pass through a small opening in a metal tube, then through a thin stream of carbon tetrachloride vapor to a photographic plate. The vapor is condensed, on the metal or glass surface above and facing the jet cooled by liquid air. The wave length of the electrons decreases as the accelerat-



a b

FIG. 16–3. Diffraction of electrons by carbon tetrachloride (CCl_4) vapor; a, 40 kilovolts; b, 54 kilovolts. (Mark and Wierl, *Z. für Phys.*, 1930.) Exposure time between 0.1 and 1 second!

ing voltage increases ($\lambda = \sqrt{150/V}$ Å). The greater the voltage, the smaller the rings. This is demonstrated by the photographs. Observe that the exposure time for the photograph may be less than a second.

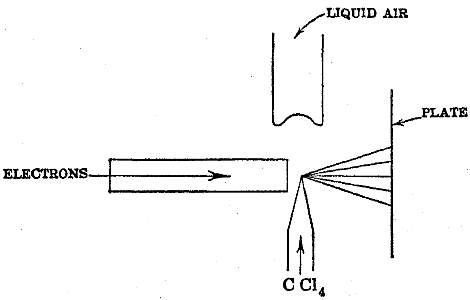


FIG. 16–4. Electrons streaming through CCl_4 vapor are scattered to produce the rings of Fig. 16–3.

Scattered X-Rays Also.

Photographs similar to those of Fig. 16–3 may be obtained by X-rays as a source, but there is this difference. X-rays, consisting of what may be called electromagnetic photons, are scattered chiefly by *electrons* around the atoms. Electrons are scattered chiefly by the *nuclei*. It results that by means of the radii of the rings in Fig. 16–3, and the wave length associated with the electrons, we are able to measure the atomic distance in a molecule like carbon tetrachloride (CCl_4). And when we replace carbon by heavier atoms, as in the molecules below, we obtain these distances, in angstroms: Carbon, CCl_4, 2.96; Silicon, $SiCl_4$, 3.28; Titanium, $TiCl_4$, 3.60; Tin, $SnCl_4$, 3.90. The measurements by X-rays give nearly the same values.

We can go even further than this in the direction of smallness. By measuring the intensity of X-rays scattered from gases, we can get an estimate of the density of the electrons around one atom. Wollan and Compton have very recently done this. Basing their computations on a rather involved theory, they are able to give us a "picture" of an atom. It is interesting to note that the "pictures" of helium, neon, argon, do not differ greatly from the 1 *s* "picture" of hydrogen (with a slight fuzziness suggestive of the 2 *s*) as computed by White (Fig. 15–6) by the use of the Schroedinger equation.

But it will be recalled that Rutherford, by measuring the scattering of alpha particles from gold foil (Chapter 3), computed the size of the nucleus of an atom and found it to be of the order of 10^{-12} cm. Is it likely that we can go to much smaller quantities? A new point of view enters bearing upon this question. It is called the Uncertainty Principle or the Principle of Indetermination.

The Uncertainty Principle.

Before considering this principle let us ask this question—Is there any limit to the smallness of an electric charge? The vast number of experiments which have been performed dealing with an electron since its discovery in 1897 all lead to one answer. The charge on, or of, the electron is the unit of electricity. We do not have fractions of an electron. It is a unit of nature. It is the absolute limit to which we can measure an electric charge. This at least is our present view.

Similarly Planck's constant h is a unit of nature. It is the basis of the vast body of experimental fact which established the quantum idea. It has frequently been pointed out that h is the unit of Action; it is equivalent to energy $\times$ time, or momentum $\times$ distance. Hence it follows that an inferior limit is placed upon these products. We derived the circumference of the first Bohr orbit by simply making circumference $\times$ momentum $= h$. This is a special case of the Principle of Least Action.

From the above it follows that we cannot measure both the position of a particle and its velocity to any degree of fineness that we please. If one is measured to high accuracy e_1, the other is indeterminate to the extent e_2, so that the product e_1e_2 must be of the order of h/m where m is the mass of the particle.

This must have been evident to many physicists soon after Bohr set forth his theory, but its recent statement by Heisenberg of Leipzig has made a very great stir, chiefly among philosophers. Had the constant h been established before his time, Hamilton would surely have pointed out this principle one hundred years ago.

This principle is of great philosophical importance. For example, we ask a simple question—What will be the condition of the universe n years hence? That is a simple question for a philosopher. It is reported that Laplace was asked this question. He was at the height (1780–1790) of his great mathematical power. He had just proved, mathematically, that the solar system was stable, thereby reassuring a timid world. His answer to the question left no doubt regarding his estimate of his mathematical power: "If you tell me the present positions and velocities of all the particles in the universe and the forces acting on these particles, I shall tell you all future history." It was a safe bet, of course. However, we now see that the phrase appearing everywhere in mechanics—the *velocity* at a *point*—is an over-specification. If the *point* is exact, the *velocity* is not.

Would it have disturbed Laplace's serene confidence in his mathematical power if he had suddenly been shown the Brownian movement? We wonder. Yet out of the apparently haphazard, chaotic, movements of those particles there was worked out the first close estimate of Avogadro's constant.

While the motions of individuals are in doubt, the average motion is known with high accuracy. The laws regarding precise individual motions give way to the laws of chance, to the rules of statistics.[1]

The Schroedinger equation which shows the fuzziness of the hydrogen atom is a special illustration of the law of probability.

The statement of the Uncertainty Principle, while it gave to philosophers an almost boundless theme for discussion, produced in the ranks of experimental physicists very little excitement. For it was known that the limitations imposed by that principle are exceeded by more serious experimental limitations. It was also known that an atom or electron could not send a signal to an

[1] This recalls Tennyson's lines concerning the mode of operation of Nature:

"So careful of the type she seems,
So careless of the single life."

observer without being disturbed in its motion by the effort of sending the signal. Let us consider the case of electrons passing through vapor, as in Fig. 16–3. The electrons are scattered from the nuclei; the latter must rebound in another direction. And what particular nucleus was it that caused an electron to bounce in a certain direction? Or what particular electron of all those passing through the vapor would strike the photographic plate on a certain point?

And still another illustration of the reaction of the signal may be given. In observing a particle in a microscope, we must throw light upon it. The scattered light enables us to "see" the particle but with the limitations already given. However, when the photon is bounced off the particle, its energy is lowered, as we saw in the Compton effect. The particle rebounds with a momentum equal to the difference (geometrical) between incident and scattered momentum. The position and velocity of the particle immediately after starting the signal (photon) to us are different from what they were before. The theoretical limitations imposed on the accuracy of determining position and velocity are determined by the constant h.

Let us apply this Uncertainty Principle to a photon. The momentum of a photon is hf/c. Hence the uncertainty in its position is given by

$$\frac{h/hf}{c} = \frac{c}{f} = \lambda.$$

That is, the position of the photon is uncertain to an amount equal to its wave length. This is a suggestive result, perhaps even a sensible one. The wave length of a photon might be pictured as a fuzziness around its center. Some place within that fuzziness the photon is located. For red light this uncertainty is of the order of 0.0006 mm., for a very short X-ray, of the order of 10^{-12} cm. This is about the diameter of a nucleus. Thus we approach limitations in smallness.

Let us go in the other direction. Is there any limitation in *largeness* of any physical quantity? We exclude questions regarding nebulae, galaxies, the universe. What about ordinary physical quantities? The answer is, yes. There seems to be a superior limit to *velocity*. It looks as though the velocity of light (a signal, not a wave motion) is the upper limit.

The argument is rather long; we start with first principles. On the bank of a stream we picture two men A and B, each of whom can row 5 miles per hour in still water (Fig. 16–5.) The stream is flowing at the rate of 3 miles per hour. A is to row 10 miles directly across stream, to a point C, and back. B is to row 10 miles down and back. Which wins? A must point his boat upstream so that the component of his speed against the current will just balance the speed of the stream. It is seen that he must point his boat so that the triangle 5, 3, 4, is formed. Then he will go straight across with a speed of 4 miles per hour and it will take him 5 hours, across and back. B's speed downstream is 8, and back 2, miles per hour. It will take him $6\frac{1}{4}$ hours, down and back. A wins.

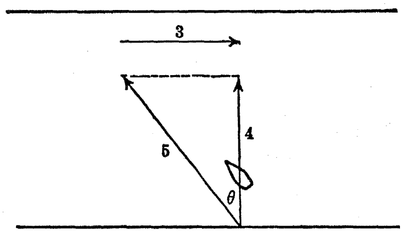


FIG. 16–5. A boatman desirous of crossing directly a rapid river must point his boat upstream.

We have the same kind of problem in light. The earth is rotating on its axis. On this account a person at the equator has a speed of about 1/3 mile per second. Due to the rotation of the earth about the sun, we have a speed of about 18 miles per second. At midnight these two speeds would be tangential to the earth, (nearly) parallel with the equator, and would be about 18.3. At midday the speed would again be tangential to the earth and would be about 17.7. Away from the equator these speeds approach one another. We may take the speed then as 18 miles per second.

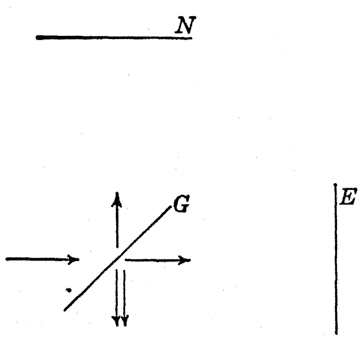


FIG. 16–6. A thinly silvered surface G breaks a beam of light into two parts. These may come together to produce interference fringes as shown in Fig. 16–8.

The Michelson-Morley Experiment.

A beam of light from west to east enters an interferometer. It divides at the separating surface G (Fig. 16–6); half continues east, half is reflected north.

Assume that the earth is cutting through the ether with a speed v. Then the speed of light east is $c - v$ and back, $c + v$. Similarly the speed north and back is $\sqrt{c^2 - v^2}$ (we have not shown the small angle of departure, θ, from the right angle). If the two interferometer arms are equal, the north wins. Straight line interference fringes are seen in the observing telescope. If now the massive support of the interferometer is gradually rotated about a vertical axis, the east-west and north-south arms change places and there should be a shift of the fringes. But it is very small. The velocity of the earth is about 1/10,000 of the velocity of light. The maximum fringe shift would represent only $2/10^8$ of the length of an arm. That is, if conditions were as those given in the stream problem. In one of the early experiments the arm l was 10 meters = 10^4 mm. = 2×10^7 wave lengths of light (nearly). Then the fringe shift should have been 0.4 fringe. The amount that was found was less than one-twentieth of this. Hence the conclusion was—*that the earth was not cutting through the ether*. The velocity of light is not increased when the earth approaches a source.

This is the famous Michelson-Morley experiment which was performed in 1887. Why is it discussed in a text dealing with Modern Physics? Because it is probable that it will for ages be a part of the modern physics of that time. *Explanations* have been given of this result but, in the author's opinion, not one of them is satisfactory.

Miller's Modification.

The experiment has been performed repeatedly and in various ways since 1887. The great preponderance of evidence is in favor of a null effect, or no shift. But the most persistent worker in this field, D. C. Miller, who has been investigating this problem for about 35 years (since 1900) claims that there is a definite shift, although it is small compared with the amount to be expected from the view that the earth is cutting through the ether. Moreover, the direction of cutting the ether is not that to be expected from the argument which has been presented above.

Miller's apparatus (Fig. 16–7 a) was a modification and enlargement of the original. By reflecting the light back and forth, the total length of an arm (Fig. 16–7 b) was 64 meters or 112,000,000 wave lengths. The shift to be expected on the above

theory was 2.25 fringe and this maximum should be obtained at midnight or midday. But Miller abandoned this simple view. He made observations of the fringe position at regular intervals through the day and night and *plotted the shift against sidereal,*

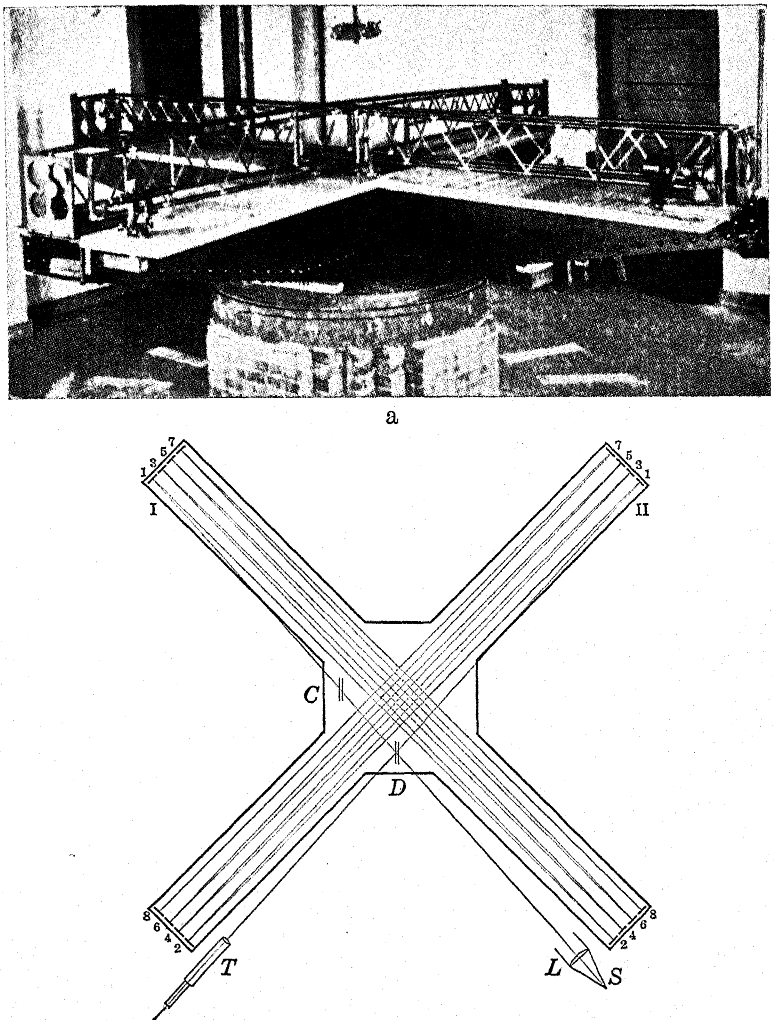


FIG. 16–7. a, The huge interferometer for measuring ether drift is floated on mercury so that it can be easily rotated; b, the light path is increased by many reflections.

not solar, time. The fringe system as seen by Miller is shown in Fig. 16–8. It is clear that the mirrors were of unusual excellence

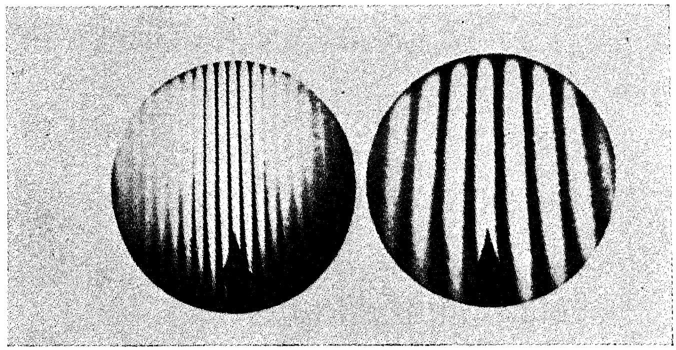


FIG. 16–8. The excellent fringes as seen in Miller's apparatus.

and that the technique was the best. The result was that he obtained a correlation between these quantities which led him to the conclusion that the solar system has a *cosmic* motion of 208 kilometers per second and is directed towards the constellation Dorado, the Swordfish. This apex is about 7° from the pole of the ecliptic and is *south*. Now the solar system is supposed to be moving, with a speed of 19 kilometers per second, towards the constellation of Hercules which is *north* of the ecliptic almost opposite to the apex of the motion found by Miller. This suggests that the constellation of Hercules is moving *south* with a speed of 227 kilometers per second. Now an ether drift of 200 kilometers per second would have produced a shift of 25 fringes, whereas Miller never obtained more than a fraction of a fringe. How then did he arrive at this estimate of speed? By finding the small variations due, as he thought, to the earth's speed in its orbit and then using the same ratio for the maximum displacement.

Miller believed that he had eliminated all effects due to temperature change, to flexure of the interferometer arms, to air currents. The final experiments were performed on the top of Mount Wilson under the most favorable conditions. He plotted 25,000 observations and his plotted data agree well with the smooth curves computed on the basis of his assumptions. Yet his results are not convincing—neither can they be ignored. The question is still open. Is there any change whatever in a light

path as it is rotated on the earth's surface? Is there any change whatever in the velocity of light on the earth's surface, due to the earth's motion?

Explanation of the Michelson-Morley Experiment.

There are various explanations of the Michelson-Morley null effect. The first was the Fitzgerald-Lorentz shortening—that any rod was shortened in the direction of its motion in the ratio of 1 to $\sqrt{1 - v^2/c^2}$. This merely amounted to equating the two times. Let us illustrate by the stream problem. Can we come to the aid of *B*? Poor fellow, he will have a hard time rowing back against that stream. That is very easy; we will shorten his distance. Let him row 8 miles down, 8 back, and his time will equal *A*'s. Now 8 is $10 \times \sqrt{1 - (3/5)^2}$. But Fitzgerald and Lorentz would still call it 10, not 8 miles. They would say that if you get on a boat and try to measure that distance, the meter stick or measuring rod would be shortened in the same ratio—so it would still be 10 miles. They merely computed the ratio of shortening in order to bring the two times equal—then said that is what happens in nature!

The other method is to assert that the velocity of light is constant, however observed. This may be taken as the result of the Michelson-Morley experiment. It is the basis of the theory of relativity. But we have to do a lot of *explaining* before we go very far in this theory. (We don't propose to go very far!)

We start with the simple idea that all motion is relative. If two automobiles start from a point in opposite directions, one with a speed of 30 and the other with a speed of 20 miles per hour, their relative motion as judged by anyone in the cars or out of them is 50 miles per hour. But if a person in a car is approaching a light source with a speed of 100 *miles per second,* what is the velocity of light as measured by him? The answer is, it is still c, the unchangeable velocity, 3×10^{10} cm./sec. How does this come about?

What is velocity? It is *distance* divided by *time.* Let us consider one of these, *time.* Suppose *time* as observed by a person in motion depends on his velocity. What strange results might be obtained. As a playful illustration, suppose a person in an automobile has a clock on the dashboard geared in some way to the machine. We might make it run fast when the car velocity

is large, in which case it might clock up 100 hours in going from Boston to New York. Or we might make it slow down as the car speeds up, or indeed run backwards so that the driver finds that he reached New York one hour before he started from Boston! His average velocity from Boston would then be a negative quantity. A very absurd result, you will say. But it must be conceded that if velocity depends on time, and time depends on velocity, and velocity . . . , we may get tied up in a knot.

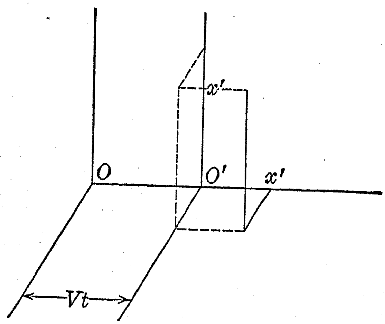


FIG. 16–9. A body may be referred to fixed or moving axes.

Now let us consider this matter more seriously. The position of a body must be determined by measuring its distances to three planes of reference (Fig. 16–9). Let OX, OY, OZ be frame 1, and $O'X'$, $O'Y'$, $O'Z'$ be frame 2. And let frame 2 be moving to the right with speed v so that $OO' = vt$.

Then $x = x' + vt$, $y = y'$, $z = z'$. In all ordinary cases *time* in frame 2 is the same as in frame 1. But Lorentz found those relations inadequate for the complete statement of Maxwell's equations. He was led to these:

$$x = k(x' + vt'), \qquad y = y', \qquad z = z',$$

$$t = k\left(t' + \frac{vx'}{c^2}\right) \qquad \text{when} \qquad k = 1\bigg/\left(\sqrt{1 - \frac{v^2}{c^2}}\right).$$

Time in a moving frame differs from time in another frame.

Frame 2 is moving to the right relative to frame 1.

Similarly frame 1 is moving to the left relative to frame 2. If we eliminate t', then x, from the above relations, we get

$$x' = k(x - vt), \qquad y' = y, \qquad z' = z, \qquad t' = k\left(t - \frac{vx}{c^2}\right).$$

In other words, these relations are perfectly symmetrical. We have changed the sign of v, as we should do, and we have interchanged x and x', t and t'.

These are the celebrated Lorentz transformation equations. They became a part of the Theory of Relativity.

From the above equations we can easily derive the following results. The sum of two small velocities, u and u' (small compared with c) is equal to $u + u'$. But the sum of c and u is not equal to $c + u$ but to c. Similarly the difference of c and u is still equal to c. For it can be shown that the sum of two velocities is

$$\text{Sum} = \frac{u + u'}{1 + \frac{uu'}{c^2}}.$$

The rest follows. Thus is the negative result of the Michelson-Morley experiment explained. The velocity down stream was not $c + u$ but c; up stream also c; across, c. Of course there is no shift in the fringes as the instrument is rotated.

It is not our intention further to follow these algebraical operations, nor to discuss the perplexing performance of clocks in different frames of reference. We come to an appraisal and criticism of some of the results.

We Discuss Some Conclusions of the Theory of Relativity.

Frequently we have in this text used the relation $m = m_0/\sqrt{1 - \beta^2}$ where $\beta = v/c$. This is equivalent to saying that the mass of a body increases with its speed and that the latter cannot become as great as the velocity of light. This develops from the theory of relativity and is often spoken of as the relativity mass. But Lorentz derived this relation from the classical electromagnetic theory. Moreover, it was experimentally established by Bucherer, Neumann, Tricker, and others. We do not need the relativity theory for that relation. We prefer to call the above relation the Lorentz relation.

Again, we have used the relation $E = mc^2$ for the equivalent of the energy associated with the mass m. But using the Lorentz relation we can derive this result (Appendix 7–1). Here, however, we feel reassured when we see that is part of a substantial theory.

There is another result generally thought to be uniquely due to the general relativity theory but which follows from simple considerations. We have used these in the discussion of the de Broglie [1] atom. The frequency of an electron depends on its

[1] It is true that de Broglie used the Lorentz equations in deriving some of his relations.

energy, $f = E/h = mc^2/h$. If the potential energy changes, the frequency changes. Let f_0 be the frequency of an atom on the earth, and f on the surface of a great star. The potential energy V is Gm_0M/r where G is the Newtonian constant of gravitation, M the mass, and r the radius of the star. Then we have

$$f = f_0 + \frac{V}{h} = f_0 - \frac{Gm_0M}{rh} = f_0 - \frac{Gf_0M}{rc^2} = f_0\left(1 - \frac{GM}{rc^2}\right).$$

This asserts that the phase or wave frequency of an electron in any definite atom is smaller on the surface of a great star than it is on the earth. As all frequencies would be altered in the same ratio, it follows that a radiated photon would have a correspondingly smaller frequency—that a red hydrogen line would be redder; that all lines would be shifted slightly towards the red if the source is a great star. For the sun, this amounts to only two-millionths of the wave length. For orange light, $\lambda = 6000$ Å, the shift to be expected is then 0.012 Å. But for a very dense star, as in the case of the companion of Sirius, M is large and r is small. The shift to be expected is thirty times that on the sun. This result has been confirmed.

A Digression regarding the Shift of Spectral Lines.

While we are on the topic of the shift of spectral lines, we digress in order to record that there are, for this effect, three causes other than that just discussed: the shift due to pressure, the Doppler effect, and the shift due to the light source being at a vast distance, which may or may not be a special case of the Doppler effect. The pressure shift is small towards the red. We shall not consider it further here.

We may have a shift towards either lower or higher frequencies due to the Doppler effect. When an observer and source are approaching each other, the frequency increases, the line shifts towards short waves; when they are receding from each other, the shift is in the opposite direction. The fractional change in frequency is equal to v/c where v is the velocity of observer and source towards or away from one another. The velocities of ordinary bodies on the earth's surface are so small that generally the Doppler effect (for light) is not in evidence. But here are two possible cases. If two airplanes were flying away from each other, each with a speed of 300 miles per hour relative to the earth,

the displacement of a line of 6000 Å coming from one ship to the other would amount only to 0.006 Å, which could easily be measured by a good instrument. If a disc with a white surface be illuminated by a strong monochromatic source and be set spinning with high speed, the top and bottom of the disc viewed by a spectrometer in the plane of the disc will show a displacement between the two lines. If mirrors are mounted on the rims of two coaxial cylinders rotating in opposite directions and so oriented that light may be reflected from them into a spectroscope, we may have a measurable doubling of a spectral line, as was observed by Bielopolsky in 1901.

In Astronomy we have a great number of applications of the Doppler effect; the rotation of the sun, of the rings of Saturn, of the components of a double star, of stars receding from or approaching the earth. But when we come to deal with bodies at vast distances, the so-called extra-galactic nebulae, we find a shift in spectral lines which, interpreted on the basis of the Doppler effect, leads to an astonishing result—those nebulae are receding from us at speeds depending on their distances. For a distance of 25,000,000 light-years, the velocity is about 3000 miles per second; for 50,000,000 light-years, 4500 miles per second; for 100,000,000 light-years, 10,000 miles per second (a shift of 300 or 400 angstroms!). This has led astronomers to the view that our universe is expanding—and rapidly. But there may be other explanations.

The General Theory of Relativity Makes Two Successful Predictions.

But we come to a place where the Einstein (general) theory gets ahead of us. Let us come to the topic of *mass*. That suggests that we are starting elementary physics!

There are two aspects of mass: inertial and gravitational. Let us illustrate.

Let us think of an electron shot out from a point in a certain direction, then coming under the action of a magnetic field at right angles to its motion. The electron will describe a circle of radius r given by $Hev = mv^2/r$. A proton having a large mass will describe a circle of large radius. Here the force on the electron is the same as on the proton, but the great *inertial* mass of the latter prevents its quick departure from a straight line. Now if

the velocity of the electron is increased, its inertial mass is increased, the radius increased accordingly. It is the increase in this inertial mass due to increase in velocity that causes the orbit of the electron in the Bohr-Sommerfeld theory to precess—thus introducing a variation in the energy of that orbit.

Of a different nature but with the same result is the interaction of the sun and the planet Mercury. As the planet approaches the sun, its mass increases due to increased speed. But the relativity theory states that the *gravitational* mass increases with the *inertial* mass. When we say that the gravitational mass increases, we mean that the attractive force due to gravitation increases. Hence, since mass and force have increased in the same way, the *acceleration* would not change. But now the theory asserts that *space* near a gravitational body is curved. On this account there is an advance in the perihelion of the orbit of Mercury. However, there is another agency which contributes to this advance; it is the perturbing effect of all the other planets on the motion of Mercury. This accounts for an advance of 532 seconds of arc per century. But the observed amount is 574, leaving 42 seconds unaccounted for. The relativity theory (general) supplies exactly this discrepancy. In no other way known to the author can this result be derived.

Similarly for light passing near the edge of the sun. Newton held the view that light was corpuscular. He had stated the law of gravitation—that every particle of matter attracted every other particle. . . . Consequently it was reasonable for him to propose this query (in *Opticks*). "Do not bodies act upon light at a distance and by their action bend its rays, and is not this action strongest at the least distance?" But he does not seem to have worked out the problem of light passing near the sun. A young German astronomer, Soldner, of Munich, about the time (1805) that the corpuscular theory was (supposedly) passing into the limbo, worked out the problem and arrived at this result—light passing near the edge of the sun will be bent through one second of arc. At that time astronomical instruments were entirely incapable of being used to that accuracy. The result was forgotten. Einstein revived it in 1911 when his theory (of that time) led to the same result. But his general theory (1919) led to a deflection of 1.7 seconds of arc. This was confirmed by photographs taken by British astronomers during the eclipse of

the sun in 1919 and very specially by American astronomers in 1922. The result may be qualitatively stated thus: two stars seen on opposite sides of the sun during an eclipse are 3.4 seconds of arc farther apart than they are when the sun is in another part of the heavens. Here again not only is there gravitational mass in the photons but there is curvature of space in the gravitational field of the sun. It is this latter feature that was not included in Einstein's first computation.

The angular displacement in radians for photons which have passed near the edge of the sun, according to theory, is $4\ GM/c^2d$ where G, M, d signify the gravitational constant, the mass of the sun, the distance of nearest approach of a photon to the center of the sun. This may be changed to the value $1.70 \times r/d$ seconds where r = radius of the sun. Hence light from a star which just grazes the sun on its way to us would be bent inwards; therefore the star would be displaced outwards, 1.70 seconds. Obviously it would be practically impossible, even during an eclipse, to measure accurately the position of a star, the light from which grazed the sun. All that can be done is to photograph all the stars in the neighborhood of the sun during an eclipse and to measure the displacements as compared with their ordinary positions. The measurements made in Australia by Campbell and Trümpler of the Lick Observatory during the 1922 eclipse are shown in Fig. 16–10. The sun, the inner and outer coronae, and the general positions of the stars are drawn to one scale (1 cm. per degree) and the displacement of the stars as shown by the lines drawn out from the dots (the usual positions) has been magnified about 2350 times as compared with the solar scale. It is seen that the near stars are displaced the most—and outwards from the sun. (But there is one conspicuous star on the left of the picture that is displaced a large amount in the wrong direction.) For the nearest star $d = 2\,r$, and the displacement should be $0.85''$; the measured value is $0.7''$. When the radial displacements of all stars are plotted against d, they are seen to fall reasonably well on the theoretical curve; this at least is true for the near stars.

Moreover, if the displacement of the light grazing the sun be computed from the mean curve, it comes out $1.72'' \pm 0.11$ in place of the theoretical value 1.70. (This close agreement is obviously due in part to chance.)

Thus there are two experimental results which support the theory of relativity: the advance of the perihelion of Mercury, and the bending of light passing near the sun. The other results, $m = m_0/\sqrt{1 - \beta^2}$ and $E = mc^2$ generally attributed to the theory, may be based on other considerations. But there is one characteristic of the theory which makes it play only a small part in the physics of atoms, electrons, photons. *It does not include the*

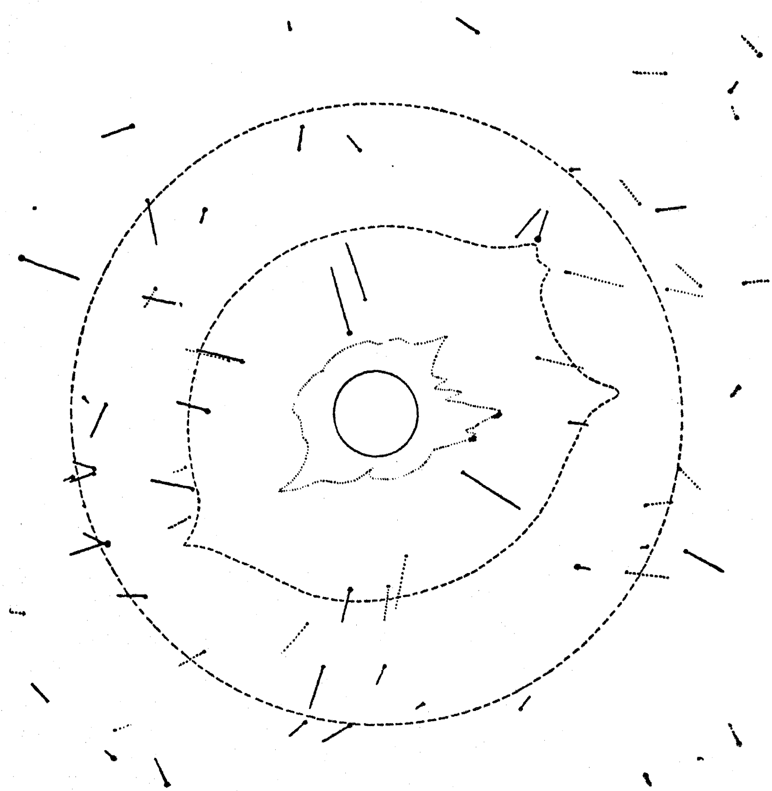


FIG. 16–10. At the time of a solar eclipse stars near the sun appear shifted (generally) outwards from their usual positions indicated by the dots. Those nearest the sun are shifted the most. The shifts, represented by the lengths of the lines, are magnified 2350 times in comparison with the solar scale.

quantum idea. And we have seen that this idea is of outstanding importance in the whole realm of modern physics. On the other hand, the quantum idea plays but a small part in large-scale

phenomena. There the relativity theory, as an extension of Newtonian ideas, has scored its success.

We regard as beyond the domain of this text a discussion of the philosophical importance of the theory of relativity, of the importance to be attached to an analysis of the concepts of space, time, simultaneity, to an analysis of mass, energy, gravitation, electrical action. Heroic methods must be used to bring new ideas out of these concepts and these phenomena.

Features of a Proposed Relativity Theory.

But the relativity theory is undergoing revision. Professor Einstein and Dr. Rosen have recently given out the prospectus of the new theory: "The writers investigate the possibility of an atomistic theory of matter and electricity which, while excluding singularities of the field, makes use of no other variables than the gravitational field variable of the general relativity theory and the electromagnetic field variable of the Maxwell theory. By the consideration of a simple example they are led to modify slightly the gravitational equations which then admit regular solutions for the static spherically symmetric case. These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a 'bridge' connecting these sheets. One is able to understand why no neutral particles of negative mass are to be found. The combined system of gravitational and electromagnetic equations are treated similarly and lead to a similar interpretation. The most natural elementary charged particle is found to be one of zero mass. The many-particle system is expected to be represented by a regular solution of the field equations corresponding to a space of two identical sheets joined by many bridges. In this case, because of the absence of singularities, the field equations determine both the field and the motion of the particles. The many-particle problem, which would decide the value of the theory, has not yet been treated."

A particle is a bridge between two sheets! And if there are 10^{24} particles in a small vessel, are there 2×10^{24} sheets? And if the particles move with a speed of one mile per second in all manner of directions, do the sheets move with them? The authors take

satisfaction in noting that now we can understand why *no neutral particles of negative mass* are to be found! But they had a narrow escape in missing that result, for they find that the most elementary *charged* particle is one of *zero* mass. What would happen to it in a magnetic field? And may we add to it de Broglie waves? And may we then put it into the Schroedinger equation?

It would appear that if this theory enters physics of the future, the subject then would be dealing with disembodied spirits.

CHAPTER 17

MODERN APPLICATIONS OF PHYSICS

It is an open question as to whether the degree of *civilization* in a community may be measured by the application of science to industry, to the arts of peace and of war. But at least we are apt to measure *progress* by the extent of such application. Industry today depends almost entirely upon the use of the sciences, physics, chemistry, etc. It will be possible in this chapter merely to give a partial catalogue of such applications.

Electron Tubes.

In an earlier chapter attention was called to the extraordinary development of the electron tube. If we include under this heading *photoelectric cells* and the *cathode ray oscillograph* we can group together a great number of industries. Perhaps some data regarding the money value of industries depending upon these devices will tell part of the story. Here are the estimates for the year 1934 in the United States (the unit is a million dollars): sound pictures, 750; long-distance telephones, 250; broadcast receivers, 200; radio and electronic tubes, 75; broadcasting stations, 73; medical and industrial, 20; radio communication, 8; recordings, 4; altogether about 1,400,000,000 dollars!

In all of these industries the *electron tube* is an essential component. They have all grown up in about fifteen years.

We might list some special modes in which the electron tube serves us.

The All-Metal Radio Tube. The man in the street who thinks of an electron tube as a glass bulb with a few wires inside it will be apt to change his view if he glances at Fig. 17–1. The figure shows that this tube is a compact, complex instrument of precision as well as an extraordinary scientific device. Its manufacture has in a few years grown up to be a great industry. It is interesting to note that in its manufacture the electron tube itself is called upon to control voltages and automatically to adjust the time of flow of a 75,000 ampere welding current!

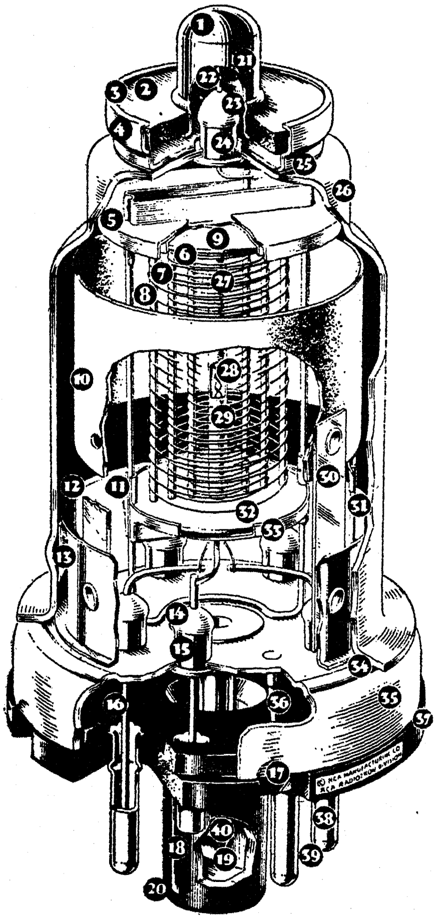


FIG. 17–1. Internal structure of an all-metal radio tube. (From R.C.A. Technical Bulletin.)

The Telephone.

Let us consider a most ordinary device, the telephone. We are so accustomed to picking up a telephone and speaking to a person whose voice we clearly recognize, even though he may be a hundred or a thousand miles away, that we regard the performance as very ordinary. But with wires in sheathed cables (now in the experimental stage) and for a carrier wave of a million cycles, it is necessary to have *electron tube relay amplifiers*

at intervals of about ten miles, each relay giving an amplification of about a million. In a call from Maine to Texas (the sheathed cable line is proposed), a distance of 3000 miles, there would be 300 relays with a total amplification [1] of 10^{1800}. It is impossible to form a picture of this quantity. Sometimes we try to do it by comparing a quantity inconceivably great, like the universe, with another quantity inconceivably small, the electron. Does it help us to conceive a magnitude by comparing two inconceivable quantities? Let us show that we are superior to such considerations and try! The miserably small universe [2] pictured for us by Jeans in *The Universe around Us* has a diameter of only one million light-years or 6×10^{18} miles or 10^{24} cm. We may take the diameter of an electron (or atom) [3] to be 10^{-8} cm. Hence the ratio of the diameter of Jean's little universe to that of an electron is of the order of 10^{32}. We would have to multiply this ratio by 10^{1765} in order to get our amplification ratio. Then we would have to start all over again to picture 10^{1765}! Even if we compare the *volume* of the universe with that of an *electron*, we get a ratio of only 10^{100} or 10^{120} depending on the kind of universe we start with. The latter ratio would be for a super-astronomical universe.

Moreover, there must be precision in this amplification. We must not constantly amplify one quality or frequency in a voice to a greater extent than we amplify another quality or frequency, otherwise speech would be absolutely unintelligible. And perhaps the most marvellous part of this matter is that this amplification is made to correct itself automatically!

[1] The author is indebted to Mr. O. B. Blackwell, manager of Staff Departments, Bell Telephone Laboratories, for these data.

[2] Miserably small compared with the 10^{10} light-year universe we discussed in Chapter 13. This universe has a diameter of 10^{28} cm. or 10^4 times the diameter of Jean's little universe. Its volume therefore is about 10^{12} times the volume of Jean's universe.

[3] From the electromagnetic theory the diameter of an electron is $\frac{4}{3}\, e^2/m$ or 3.8×10^{-13} cm. But in the quantum theory the diameter may have various values, or perhaps we should say that no meaning can be attached to the term *diameter*. For in this theory there is a wave length λ associated with a mass m equal to h/mv. As we have shown in Chapter 15, the wave length in the lowest orbit is equal to the circumference of that orbit. Hence if we use this model we might say that the diameter of the electron is the diameter of the orbit. For the first hydrogen orbit this would make $d = 1.03 \times 10^{-8}$ cm. But here is a question—if we can attach no meaning to the term "diameter of an electron," can we attach any to "diameter of an atom"?

An astronomer may take liberties with the masses and distances of nebulae near the rim of our universe, but a physicist deals with super-astronomical numbers in a laboratory, here and now. He can be checked if he makes an error; [1] indeed, an error might mean the failure of a great structure, a great industry, or it might mean the loss of human life.

There is another aspect of this matter to which attention should be called. Let us suppose we have a carrier frequency in this coaxial cable of 1,000,000 cycles per second. This band is wide enough for 1 television signal or the simultaneous transmission of 200 different telephone conversations or 4000 telegraph messages. Every user of a telephone would receive only his own message and none other of the 200 messages passing through the cable at that time. Similarly every telegraph operator, receiving his own message, would not be disturbed by the other 3999 messages. The man on the street cannot comprehend the infinite care that must be taken in regard to every detail of the apparatus which must be used to bring about the above results.

Television.

One reason why television has been so long delayed is indicated by the data in the above paragraph. *The carrier wave will carry 1 television signal or 4000 telegraph messages!* Television, including the voice and synchronizing features, requires a broad band of frequencies. Hence it is expensive. But it will come.

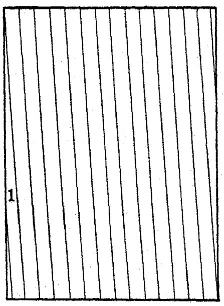


FIG. 17-2. In television the photoelectric cells receive light from the picture line after line.

It will be recalled that in television an intense spot of light is flashed across the scene (a person's face), first along a line 1 (Fig. 17-2), then in turn along adjacent lines until the entire scene has been searched out; this is repeated, over and over, the entire time for one picture being about 1/20 second. Large sensitive photoelectric cells catch the light reflected or scattered

[1] A physicist, like a surgeon, may be convicted of having made a mistake. For the latter, though he may claim to have done the better thing, may be correctly accused of inaccuracy if he cut off a person's head when he intended to cut off a foot. But a political orator, a philosopher, an economist, and an educationalist are never in error; they are merely advancing new ideas.

back from the picture; the electric impulses, greatly amplified, modulate the carrier frequency; the receiver at the distant station transmits these impulses to the appropriate elements of a cathode ray oscillograph. There the mechanism moving the light spot must be absolutely synchronized with that which moves the light spot across the picture at the transmitting station. Then there will flash out on the fluorescent screen of the oscillograph tube a variation of light and shade similar to that which was illuminated at the transmitting end. Such is television.

And when it arrives, will it become as ordinary as using the telephone? The author recalls the first time he sat in a darkened booth and unexpectedly saw the face of a friend, whom he knew to be miles away, flash out, animated, completely life-like; heard his friend's voice call him by name, speak to him naturally, the motions of the lips synchronizing with the words; then heard and saw his friend laugh at him, for the friend had seen by the author's face that he was startled by the apparition! Science and laughter!

Other Applications of the Electron Tube.

The *direction finder* on a ship may be used to locate radio beacons on lighthouses or on shore and thus may enable the officer of the ship to find his way through a fog. Airplanes are guided by radio beams sent out in definite directions from a transmitting station at an airport. An indicator on the panel shows the aviator whether he is out of, in the center of, or in the right or left edge of the beam.

An aviator may hold two-way communication with stations several hundred miles distant from the plane. This fact alone is of the greatest importance in commercial aviation.

The recording of the human voice, whether in song or in political oratory, so that it may be reproduced at some future date—this may not always be an unmixed blessing, but at least the sound and foam specimens may be destroyed.

By means of the *audiometer* sounds of definite frequency, generally in octave steps from 64 up to 8192 vibrations per second (7 octaves) and varying in intensity by known amounts, can be used in calibrating or testing the human ear. This is the only scientific method of testing hearing.

In the same way the quality of the human voice or of a musical instrument can be quantitatively determined.

By means of the *stethophone* the heart pulses may be amplified and all the various frequencies present in the heart motion may be identified. The progress or decay of a certain disease may be observed.

Electronic Effects Due to Light.

Photoelectric cells may be used for all manner of counting operations, automobiles passing over a bridge, or white, red, and blue balls rolling down a groove. Photocells may be made (in part) sensitive to different colors and may, within limits, identify an object by its color.

But one important application of the photocell is in the *light intensity meter* or *exposure meter*. A photocell is connected to a very sensitive (micro) ammeter. The current which is produced causes the ammeter pointer to traverse a scale which may be calibrated in photographic units so that a photographer knows precisely the time (or stop) required for an exposure in the light available. This is of very great importance not only to the amateur photographer but very especially to the motion picture man who thereby may be saved the cost of many rolls of waste film. The particular kind of cell used in this way is called, by one manufacturer, the *photronic cell.*

In the photoelectric cell previously described, the light beam, falling on the sensitive surface, liberates electrons but a battery (90 to 180 volts) is necessary to produce a continuous current. In the photronic cell, however, no battery is necessary; the light beam not only sets free the electrons but provides the necessary electromotive force. The current produced is very nearly proportional to the amount of light falling on the surface. It amounts to about 0.150 milliampere for a 100-watt lamp at a distance of 1 meter. Consequently the cell is sufficiently sensitive to operate a relay for rather moderate variations of light intensity. By means of successive relays, if necessary, a current of any desired intensity may be built up.

One rather important feature of the photronic cell is that it has very nearly the same "visibility" curve as has the human eye; in other words, it has the same color sense.

Different manufacturers use different sensitive surfaces, but the one chiefly used is probably a film or deposit of cuprous oxide on copper. A relay is electrically connected to these surfaces. No vacuum is necessary. See also Chapter 5, page 85.

Electronic Effect Due to Pressure.

Connected with the operation of electron tubes as transmitters of electric waves, there is a phenomenon that is having a large application. It is called the *piezo-electric* (*piezo = pressure*) *effect.* Various crystals, quartz, rochelle salt, tourmaline, when compressed in definite directions become electrified or vice versa. While rochelle salt is by far the most sensitive of these crystals, on account of the fact that it is fragile, it deliquesces, and its elasticity is low, it is not used for the purpose to be described. Quartz is the crystal used.

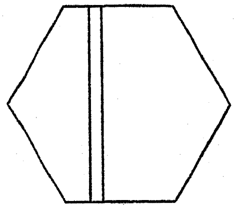


FIG. 17–3. The axis of a block of quartz is parallel to the hexagon faces (perpendicular to the paper). A plate cut parallel to the axis may control the frequency of a high power transmitting station.

After a block of quartz has been cut from a good crystal perpendicular to the crystallographic axis, it is again cut into plates parallel to the axis and either perpendicular (Fig. 17–3) or parallel to one of the crystal sides. (There are also other modes of cutting.) A plate with its faces reasonably parallel is placed in a circuit as shown (Fig. 17–4). With proper values of inductance, capacity, etc., the circuit will oscillate.

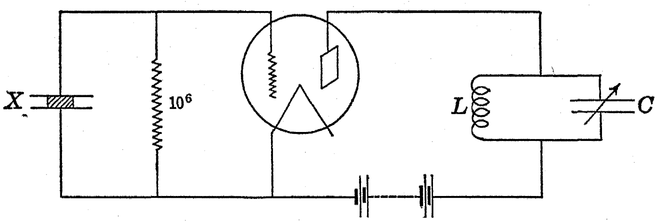


FIG. 17–4. A simple circuit for producing oscillations of a quartz plate. The frequency of these oscillations then controls the frequency of the oscillations in LC.

But the surprising thing is that the frequency of oscillation is not necessarily $1/(2\pi\sqrt{LC})$ which is the ordinary electric value,

but it is determined, within limits, by the quartz crystal itself. In fact, it is equal to the *mechanical frequency*[1] of oscillation of the plate. Now it is true that a plate may have various frequencies depending on its mode of vibration. But with proper choice of the electrical quantities it may be made to vibrate as if the two condenser plates were approaching, then receding, etc. In other words, the *thickness* of *the plate* is a *half-wave length* of the mechanical *vibration* of the *quartz plate.* This mechanical vibration of the quartz then controls the electrical vibration in L and C, and an electrical wave may be sent out with that frequency. Generally, however, this electrical variation is amplified so as to *control the frequency of a high-power station.* Now quartz is almost perfectly elastic, its temperature coefficients are small, it is not affected by atmospheric conditions. Moreover, as the crystal need be only an inch square, it can easily be mounted in a constant temperature box so that the frequency will not change by more than a few parts in a million for months or years. *All high-power stations now are crystal controlled.*

Another application of the piezo-electric effect is in the *measurement of pressure*, whether due to the ignition of gasoline in a cylinder or to the firing of powder in a gun. It is used chiefly for the latter purpose. A number of thin quartz plates are placed in a proper housing in the breech block of a gun. When the gun is fired, a plunger is driven against the plates. The electric potential generated is amplified by a resistance-coupled amplifier and the whole story of the rise and fall of the pressure of the exploding gases is written on a film. Timing marks, one-thousandth of a second apart, made by a tuning fork operated and controlled by an *electron tube*, are also written on the film. The total duration of the explosion may cover ten of these marks, or 1/100 second. As quartz can stand very high pressures (50,000 lbs./inch^2), this method has become standard for testing powders, not only as to the total pressure produced but whether slow or fast burning. During the war the author devised this kind of pressure gauge for the above purpose at the Aberdeen proving ground. Since then it has been improved and standardized under the able direction of Mr. R. H. Kent, head of the Instrument Section of the proving ground.

[1] For the $\times$ cut shown in Fig. 17–3 the length of the electric wave is nearly 105 meters for a crystal of 1 mm. thickness; or $f = 2.85 \times 10^6$ cycles/sec.

Acoustics.

Now that we are able by means of the electron tube to produce sounds continuously of any desired pitch and intensity and also to measure accurately the intensity of the sound at any point, the old branch of physics, acoustics, has had a new lease on life.

To begin with, we can produce sounds of one frequency—pure tones. For we can filter out of a complex note undesired harmonics. We can then measure the "resistance" to sound for all ordinary materials, as well as the reflection coefficient of a surface. We are then able to "sound out" a lecture room or auditorium for dead spots or for points of large intensity or for the reverberation time and are able to prescribe the proper amount of deadening of the walls here or there in order to render the hall acoustically satisfactory.

Similarly we may study the performance of a microphone, telephone receiver, or loud speaker. The great industries concerned with the telephone and sound film are dependent upon these devices. Moreover, an entirely new field has been opened up, that of supersonics, for an electron tube can generate a frequency of 10^4 or 10^5 as easily as one of 10^2.

We may measure noise and observe its effect on the efficiency of workers. Altogether we may say that physiological acoustics is taking on aspects of an exact science.

The application of these acoustical devices to war problems cannot be presented here.

Recent texts dealing with these matters are *Applied Acoustics* by Olson and Massa, *Acoustics of Buildings* by F. R. Watson, and *Acoustics* by Stewart and Lindsay.

Gravity Apparatus.

If the earth were a perfect sphere of uniform material, all bodies on the surface would have the same acceleration towards the earth if the effect due to the earth's rotation is allowed for. Then g would be a constant quantity over the earth's surface. But the earth is an oblate spheroid and g at the pole is 201/200 of its value at the equator. But variations of density in or near the earth's surface are sufficient to cause minor variations of g.

The development of radio has made it possible to carry out determinations of these variations in density with great ease and

with great accuracy as compared with a few years ago. For, in general, the measurement of the period of some kind of pendulum is necessary. We have the old formula $P = 2\pi\sqrt{l/g}$ for the period P where l is an equivalent length of the pendulum. If the length of the pendulum when carried from place to place does not change, then P^2 varies inversely as g. Then we must measure merely the period in order to find the variations of g. An accurate time scale is wanted. Radio supplies it. A clock in a constant temperature room in Washington, by shadowing a photoelectric cell once every half-vibration, transmits seconds through a circuit. The signal is carried by wires to the transmitting station at Arlington, there sent out, picked up by a geophysical party perhaps out on a desert, recorded on a drum on which is also recorded the period of a pendulum at that station. Electron tubes are necessary for amplification of the signals.

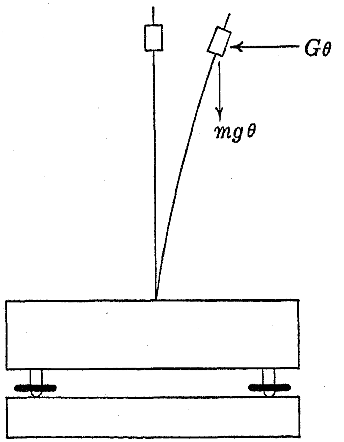


FIG. 17–5. A small variation in g may be indicated by a variation in the period of an elinvar rod; a gravimeter.

The pendulum may be of quartz or elinvar, for both of which the coefficient of expansion due to temperature change is very small. It is mounted in a case which can be evacuated. During transportation it is supported otherwise than by its knife edges. Photocells and all electrical equipment are permanently mounted in a truck. Thus the whole time spent in making an observation is very small.

But another kind of pendulum is far more sensitive. Figure 17–5 shows a thin strip of elinvar with a small mass attached at the end (known as the Lejay-Holwick gravimeter). When this strip is displaced from the vertical, two forces act, proportional to mg vertically down, G (equivalent) horizontally, where G is the constant of the spring. Hence the period P is

$$2\pi\sqrt{\frac{ml}{G - mg}}$$

(some constants omitted both here and in the figure), and we see that P depends on the difference between the constant G and the variable mg (assuming that g is variable).

It ought to be noted, however, that though we have spoken of g as being the only variable in the above relation for P, both G and l may vary due to temperature change. This means that the instrument should be calibrated in some standard place for variations of temperature. Moreover, there may be magnetic effects. These also must be inquired into during calibration. This, like any other instrument of precision, requires expert use.

We arrange it so that G is just greater than mg is likely to be. Then P is large and a small change in g makes a large change in P. Therefore small variations in g can be measured. This arrangement is about 100 times as accurate as the ordinary pendulum method.

These small variations in g are indications of low density due to oil domes or water accumulations, or they may indicate high density due to heavy rock possibly containing ore.

But the gravity device which surpasses all others is the *Eötvös torsion balance* (or variations of that instrument).

Let us consider this matter quantitatively. The ordinary value of g is taken as 980 dynes per gram or 980 cm./sec.^{2}. The highest accuracy in measuring g that may be obtained by the pendulum method is one part in a million. The gravimeter described above may show a difference of one part in one hundred million. The Eötvös instrument may show *one part in one million million.*

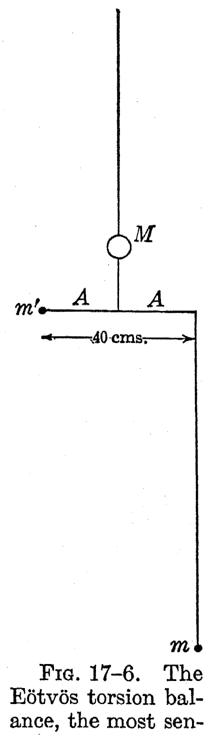


FIG. 17–6. The Eötvös torsion balance, the most sensitive apparatus for showing variations of gravity.

In Fig. 17–6, AA is a light aluminium rod suspended by a long fine platinum-iridium wire; on one end is a gold or platinum cylinder m' (20 gms.); suspended by a fiber from the other end is a nearly equal mass m. All of this is mounted in a heavy metal case which can rotate about a vertical axis. Let us suppose that we are out on a uniform plain, and that the position of the light

spot reflected from the mirror M has been noted. The case carrying the light source and telescope is then turned around the vertical axis. The reflected light spot will be unchanged in the telescope. But if we bring up towards but below the instrument a large stone or heavy object, the mass m will tend to turn towards it. If the Eötvös instrument were moved to different positions around this heavy object, the mass m would always point slightly towards it. (Only the torsion of the wire prevents a direct pointing.) If the heavy object were below the surface of the earth, this would still be true. Consequently in order to locate a heavy body, it is necessary to map a whole region, finding at every station the direction of the heavy object or dense portion of the earth.

The period of this torsion balance may be of the order of 30 minutes. This would make visual observation with a telescope and scale tedious. The better way is to mount a light source and photographic plate near the top of the instrument and by mirrors reflect back the light to the photographic plate. The whole apparatus then can be made automatic, the plate exposed a certain number of times, the balance arm rotated, at the end of a few hours, the plate again exposed, etc.

The densities of material which may be found near the earth's surface run as follows: oil, 0.9; loose sand, 1.3; clay, 1.8; rock salt, 2.3; most stone, 2.7; magnetite, 5.2. Professors Eve and Keys of McGill University in their text *Applied Geophysics* compute that if there is a fault, as shown in Fig. 17–7, *18 miles below the earth's surface* it can be detected by the Eötvös instrument. Figure 17–8, also taken from the above text, shows the indications of the instrument above and near a salt dome. In the upper part of the figure the arrows are definitely pointing to a region of large density. On the assumption that this region was a salt dome, the survey showed that the top of the dome would be about 700 feet and the lower rim about 4000 feet below the earth's surface. Borings showed that the torsion balance survey was correct.

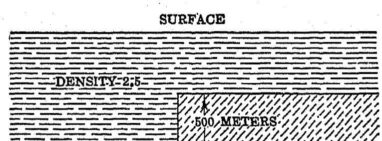


FIG. 17–7. An Eötvös torsion balance can detect a fault of the above dimensions at a depth of eighteen miles below the earth's surface.

Oil or sulfur may be associated with salt domes. According to Eve and Keys, the sulfur on one underground dome melted by superheated steam and then pumped to the surface exceeded

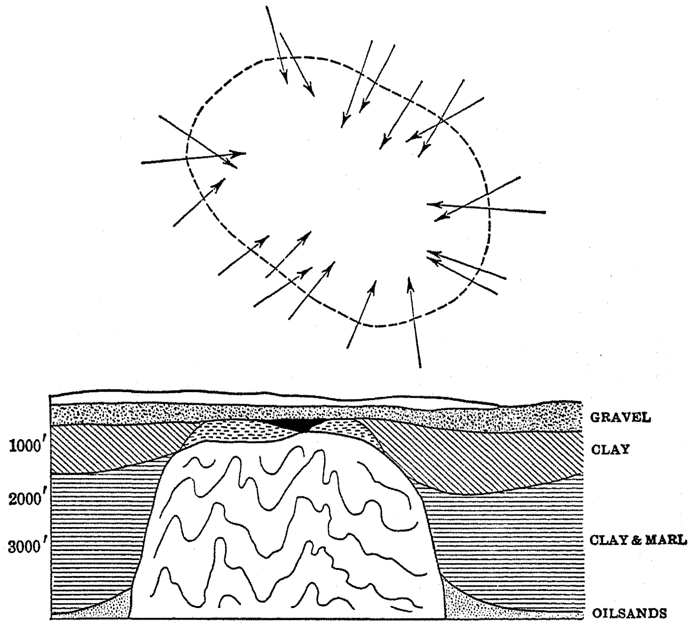


FIG. 17–8. Above are shown the gradients of gravity near a salt dome as indicated by the Eötvös instrument. In the lower figure is shown the dome in section.

150 million dollars in value. Obviously the discovery of one dome may pay for all the geophysical prospecting which may be done in all parts of the world for many years.

An instrument as sensitive as the Eötvös may be influenced by many local conditions, surface rocks, hills, streams; sunlight falling on one part of the instrument may set up air currents. Obviously it can be used only in very uniform terrain and must be handled by expert technicians. We come now to deal with a very modern method of prospecting known as the seismic method.

Seismic Method.

If a boring is made in the earth and a charge of dynamite in the bottom of the boring is fired, the disturbance travels outward as a

compressional wave. If the medium all around the boring is uniform, the speed is the same in all directions. But these speeds vary; for water, 4800 ft./sec.; for limestone, 12,500; for granite, 23,000; etc. If we measure the time required for the disturbance to arrive at certain points at known distances from the source, we can determine the kind of material through which it passed—this on the supposition that it has not been reflected or refracted.

The electron tube again is called upon to aid us in the measurement of the times of arrival of the disturbance. We suspect that there is a salt dome, A, in a certain region. We plant our dynamite charge at S (Fig. 17–9) and detecting microphones four or five miles distant at 1, 2, 3, etc. The mechanism that fires the charge releases a radio wave that is instantly recorded at all the stations. This signal together with those due to the earth disturbances are all recorded on a revolving drum upon which time marks are made by a tuning fork driven by an electron tube. The distances S 1, S 2, etc., all being known, the velocities then can be found. The interpretation of the records is a matter for experts in geophysics, for there are many variables.

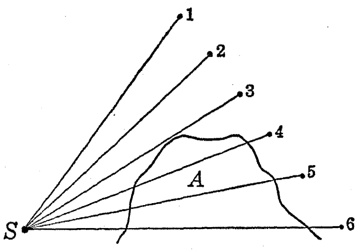


FIG. 17–9. Shock waves due to a dynamite discharge at S are picked up by microphones at 1, 2, . . . 6. The nature of the material in region A may be determined.

Again we may use this method for locating the depth of reflecting layers. We place a dynamite charge, D, in a boring, SD, 100 or 200 feet deep, and six specially constructed microphones in a line through S. After the charge is fired, direct signals go to the microphones but these are easily recognized. Indeed, the velocity of the signal through the surface layers can be determined and the correct time of firing can be computed. There will be reflections perhaps from various layers of rocks which may be identified (Fig. 17–10).

Obviously a variation of the above method may be used for measuring the thickness of a glacier or the depth of the ocean.

This method may be combined with one of the methods described below.

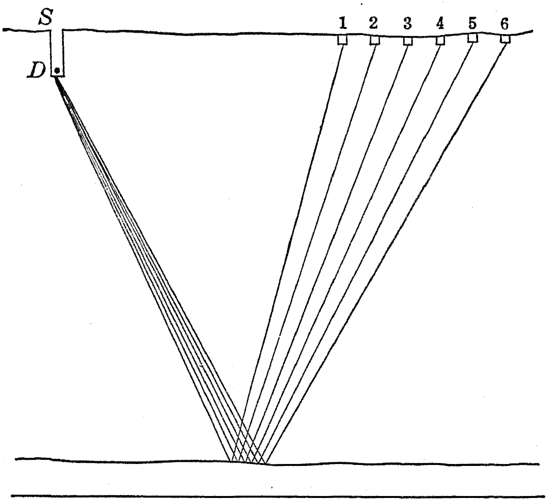


FIG. 17–10. As in Fig. 17–9, the depth of a reflecting surface may be found.

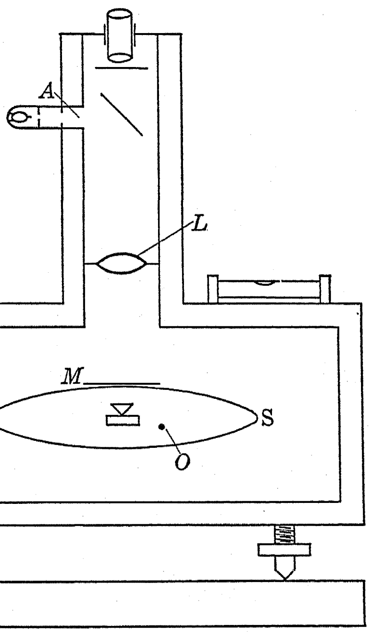


FIG. 17–11. A sensitive magnetometer for finding variations in the vertical component of the earth's magnetic field.

(1) *Magnetic.* The horizontal or vertical component of the earth's magnetic field, generally the latter, is nearly compensated in a standard magnetometer at some standard station; then *variations* of the field assume large values. Bodies of iron ore can be located in this way.

There are various ways of bringing about this compensation. For example, in Fig. 17–11, NS represents a magnet in the form of an elongated ellipse. It can swing on quartz knife edges which are supported by quartz rods. It has been loaded at a point O so that the magnet is horizontal at a standard station. But there is one point which requires the closest attention, the matter of level. The instrument can be leveled by an optical arrangement or by ordinary sensitive spirit levels on the case. Light from the fine slit A is reflected down to the mirror M and back to the scale which is viewed by the eyepiece. Then the departure of NS from the horizontal is shown by this reflected light. The needle can be brought back to the horizontal by a definite field produced by an electric current.

(2) *Electrical.* A region is staked out with metal posts at definite distances and with two lines of posts on opposite sides of a rectangle. The electrical resistance between opposite posts, or between one post and any other, is measured. If two posts happened to be above an ore vein, the resistance between them would be small. Or we send a current between two posts and the manner of the spreading out of the current is measured. The current will tend to flow through good conductors, presumably ore bodies.

(3) *Electromagnetic.* An electromagnetic wave is sent out from a loop of wire either embedded in or lying on the ground. A detecting loop in the form of a rectangular or circular coil is mounted so that it may be oriented in any direction. The current that it picks up is amplified by electron tubes and the relative intensity of the current as the coil is turned in various directions is noted by ear phones. From the chart giving the directions of silence (if any) for the loop at various stations, the location of a mass of conducting material may be determined.

There are several different variations of the above method. The transmitting loop may be large or small, the electrical frequency may be a few hundred or a million; the locating of the ore body may depend on reflection, or upon the degree of pene-

tration. Altogether this is a large technical topic and cannot be further discussed here.

X-Rays.

As everyone in a civilized community knows, an X-ray machine is an essential device in every hospital and in every dentist's office. By means of X-rays we can detect abnormal conditions in almost every organ in the human body. Ordinarily only one tube is used and a shadow photograph is taken of that part of the body which is to be examined. This gives a projection of a body in space upon a plane surface. In order to locate the various features in their correct space or their dimensional relations, it is necessary to take another photograph from a different angle. Then the two pictures may be arranged to give a stereoscopic view of the body. But very recently a new method has come into use whereby two tubes close together are operated intermittently 60 times a second and shutters through which an observer looks are operated in synchronism, with the result that the eyes see the body in space. A foreign object in the body or the special position of a fracture of a bone may be located with considerable accuracy. It is a rather extraordinary sensation to see the whole interior of a human body, as if almost every part of it were transparent, yet knowing that it is all opaque, judged by visual rays.

In addition to their use for making visible the interior of any part of the human body, X-rays are used in hospitals for therapeutic purposes. Tumors and abnormal growths are "dosed" with X-rays. Incipient cancers are similarly treated. But this work must be done with great caution, since more harm than good may result.

In this connection—the influence of X-rays on organisms—there should be noted one outstanding application. *Mutations* may be brought about in certain insects (flies) by means of X-rays. We quote one of the foremost workers in the field of genetics, Professor H. J. Muller, "X-rays and their relatives remain the only prime cause of mutations yet known. . . . For the first time the biologist can now produce mutations. . . . The effect in producing mutations is 1000 times the normal effect."

Another illustration of the power of X-rays to produce mutations has just been made public. In the Research Laboratory of the General Electric Company, Mr. Chester Moore "dosed" with X-rays 75 bulbs of regal lilies. From the "freaks" of the second generation there has been developed a true-breeding strain of lily not only possessing beauty but having an attractive characteristic—it does not shed its sticky pollen as its parent did.

What do X-rays do to the flies in the one case or to the bulbs in the other?

This is a matter of the greatest importance. To be able rapidly to change the form, dimensions, or characteristics of living creatures or of growing forms at will and under control is a new power never before possessed by man. Whether it can be extended to larger life-forms remains to be seen.

High Frequency Electric Waves.

Differing in nature from the therapeutic effect of X-rays are those of high frequency electric waves. Here waves of a frequency of about 20 million per second (15 meters wave length) are generated by a high-power tube. Two large condenser plates are connected to the oscillating circuit and a patient's arm, for example, is placed between the plates. High frequency currents are induced throughout the whole arm. There is no surface heat. The entire arm is warmed several degrees. Sometimes a wire from the oscillating circuit is placed inside a bottle of water. This feels cold to the touch but presently the part of the body in contact with the bottle will be definitely warmed. Arthritis is now being treated in this way.

Electron Optics.

There has grown up in recent years a branch of our subject under the above title. It might be thought that the de Broglie wave characteristic of the electron ($\lambda = h/mv$) would enter in a discussion of this topic, and so it might, but we are to treat it qualitatively and merely from the point of view of the influence of electric or magnetic fields upon the motion of an electron.

Let us picture an electron source at S in Fig. 17–12 a, and suppose that the electrons, going out from S, are accelerated by an electric field. Then let the diverging electron "rays" pass

through a "lens" L (Fig. 17–12 b). Obviously the rays will be bent inwards towards the "optic axis" of the system. By varying the voltage on the central rod or by changing the length of that rod as compared with the radius of the lens, we may cause

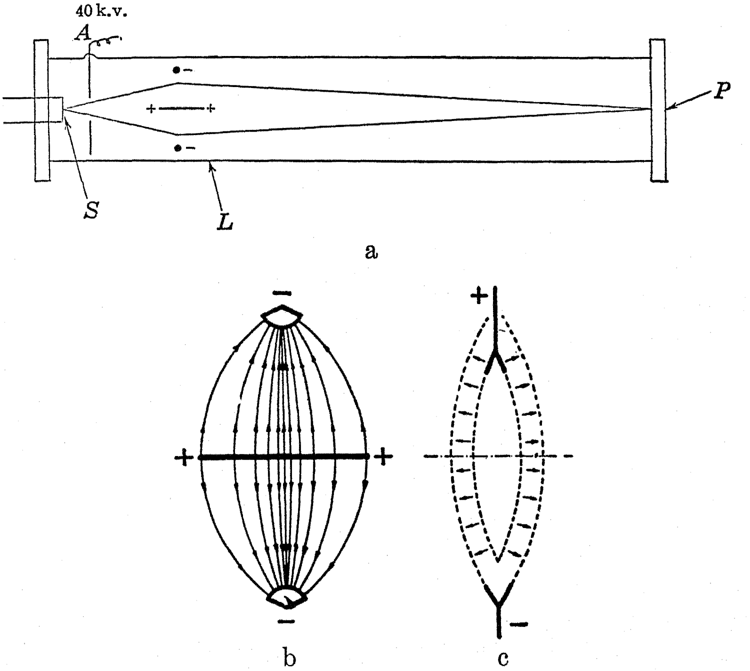


FIG. 17–12. a, An electron microscope; b, c, electron lenses. (Bruche and Johannson, 1934.)

all electrons going out from a point of S to be focussed on the photographic plate P. Similarly another point of S would also have its image in P. We would have then, with dimensions as indicated in the figure, an enlarged image of S on the photographic plate.

Another lens which will accomplish the same purpose is shown in Fig. 17–12 c. A lens-shaped mesh of fine wires is connected to the plus pole and a similar larger mesh to the negative pole of the battery. We can alter the focal length by changing the voltage. Or we can change the lens from a converging to a diverging by reversing the voltage. The paths of electron rays (made lumi-

nous by a slight amount of gas in the tube) passing through a converging lens (Fig. 17–12 b) are seen in Fig. 17–13.

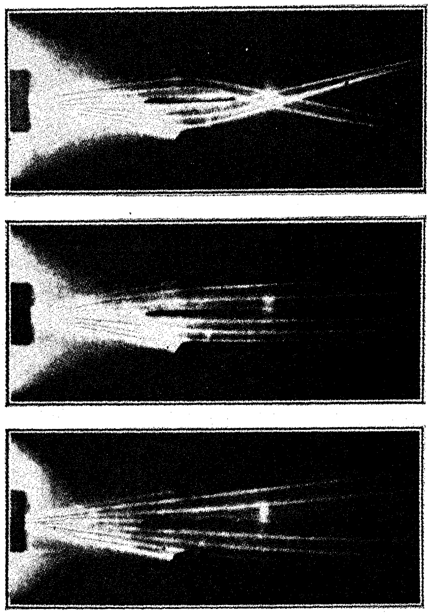


FIG. 17–13. Electron streams are made to converge by an electron lens (b of Fig. 17–12). The paths of the electrons can be seen, since the residual gas is made luminous by the high velocity electrons.

Let us now consider the motion of electrons which, going out from a radioactive source S (Fig. 17–14), pass through an opening in the anode in the form of a ring and let the tube be surrounded by a solenoid. Then we would have a uniform magnetic field H inside the solenoid parallel to the axis of the tube. An electron

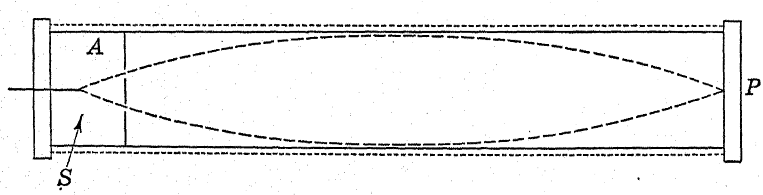


FIG. 17–14. Electrons sent out by radium at S are made to spiral and concentrate at P by means of the magnetic field in the solenoid.

going through the ring opening in A would start out at an angle θ to the tube axis. The component, $v \sin \theta$, of its velocity at right angles to the axis would not change. Consequently it would tend to describe a circle around the magnetic lines of radius given by

$$Hev \sin \theta = \frac{mv^2 \sin^2 \theta}{r}.$$

However, its forward speed $v \cos \theta$ would cause it to describe a helix and to bring it back to the axis at a distance d $(= tv \cos \theta)$ where t $(= 2\pi/He)$ is the period around the circle. All the electrons going out from S and passing through the opening would be brought back to the axis at that point. Similarly electrons from a point near S would be focussed at a corresponding point on P. Thus there would be an image on P of electron sources on a filament or plate at S. As $\cos \theta$ differs only slightly from unity for quite appreciable values of θ ($\cos 10° = 0.985$), we are able to collect practically all the electrons of definite velocity emerging from S and going through a hole in A at a distant point. This is an important point and will be referred to below.

Instead of a radioactive source, we picture a photoelectric

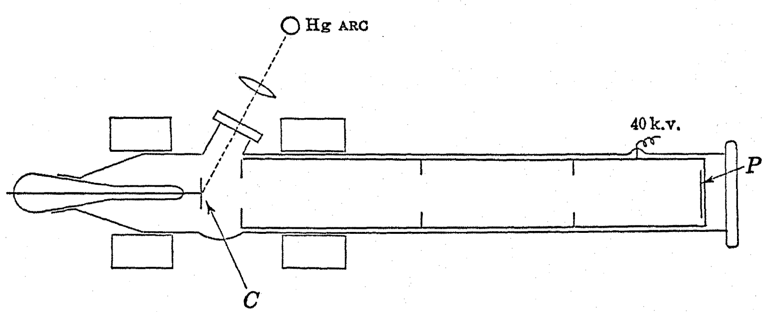


FIG. 17–15. Photoelectrons emitted by the plate C are focussed by magnetic and electric fields at P. The a photographs of Fig. 17–16 result.

source as C in Fig. 17–15. Here we accelerate the electrons by a very high potential (total 40,000 to 50,000 volts) and use a magnetic field due to coils as shown in the figure. By adjustment of the accelerating potential and the magnetic field, we are able to focus electrons from a point of C to a point on P. The magnification depends on the quantities above and on the geometrical relations. That an image is formed may be seen by inspection of

Fig. 17–16. The electron image *a* is to be compared with the photographic image *b* for the same magnification. It is seen that there is an accurate focussing. But the influence of oxide or oil

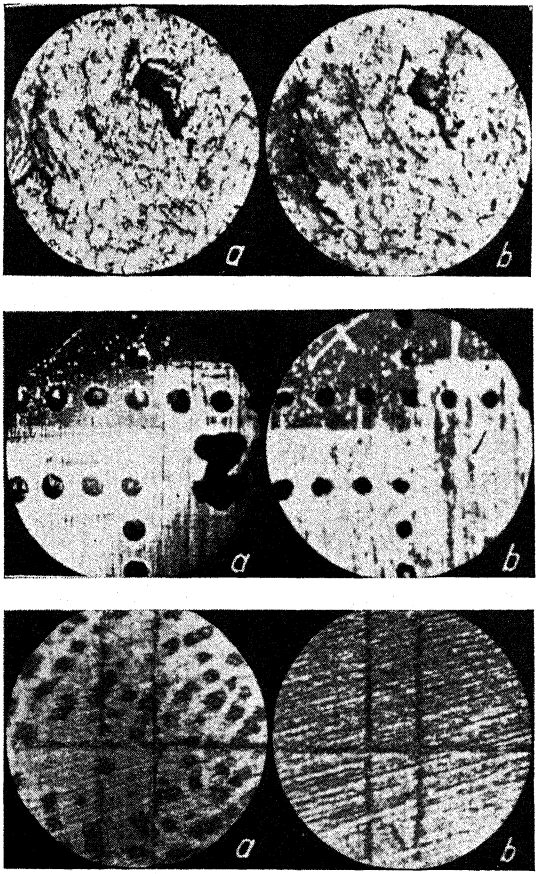


FIG. 17–16. The *a* photographs are due to electrons which, coming out from the surface, are focussed by magnetic and electric fields. The *b*'s are microphotographs taken by light reflected from the surface. The influence of invisible films on the emission of electrons is made evident.

films upon the photoelectric emission of parts of the surface is clearly in evidence. Thus we may test the uniformity of emission of electrons from a heated or photoelectric surface.

Of another character is the effect now to be presented. Sup-

pose we have two cylindrical magnetic pole pieces of iron M (Fig. 17–17) facing one another with a small air gap between them. There would be a strong magnetic field between the pole pieces and a weaker field as one goes out radially (as suggested by the dots at F). We might, if we wished, fill the space between the pole pieces with a sheet of hard rubber without altering the field appreciably. An electron starting out from S along the arrow would be bent inwards; coming into the stronger field near the magnet, it would change its direction rapidly. The consequence would be that it would describe a series of epicycloids(?) around the hard rubber between the pole pieces. This would be true for all electrons going out from S within quite a large angle. Positrons going out from S would go around the other way. Or, if we reverse the magnetic field, we may collect positrons instead of electrons in I (a Geiger counter or ionization chamber).

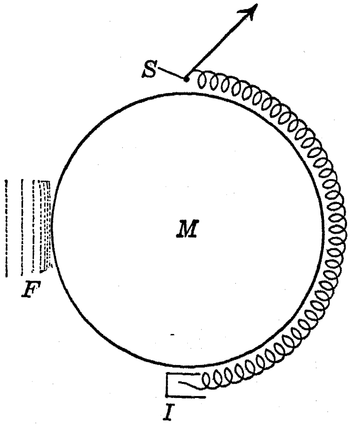


FIG. 17–17. Electrons starting out from S along the arrow describe epicycles(?) around M to I due to a magnetic field decreasing outwards from M as shown at F. Positrons would go around the other way.

This behavior of electrons in an unhomogeneous field was used by Villard (1908) in his attempts to account for the appearance of the aurora borealis, but it has been recently applied by Thibaud to compare the number of positrons and electrons coming from a feebly radioactive source. It is of very great importance in problems connected with artificial radioactivity.

The Top of the World.

The quest for the secret of the nature of cosmic rays has given an extraordinary impetus to "going up." Here is a partial list of journeys into the upper air, all, with the exception of the airplane flight, concerned with the measurement of cosmic rays. It is true that in the case of sounding balloons, instruments were sent up; observers did not accompany them.

An approximate indication of the pressure of the air and the height reached is given in the last column.

Observer	Device	Year	Height	p/p_0
Glockel	Balloon	1910	14,000 ft.	.60
Hess	"	1911	13,000	
Kolhörster	"	1913	30,000	.28
Donati	Airplane	1934	47,000	.17
Millikan	Sounding balloon	1922	51,000	.14
Picard	Stratosphere Balloon	1932	53,000	.12
Stevens-Anderson	"	1934	60,600	.08
Settle-Fordney	"	1933	61,200	
3 U.S.S.R. scientists (killed in descent)	"	1934	72,200	.06
Stevens-Anderson	"	1935	72,395	
Regener	Sounding balloon	1932, 1933, 1934	94,000	.03
E. and V. H. Regener	"	1934	100,000	.02(?)

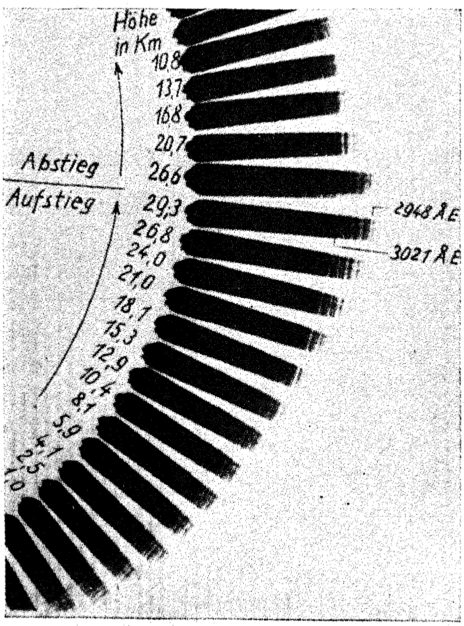


FIG. 17–18. The solar spectrum taken at heights up to eighteen miles. (E. and V. H. Regener, 1934.)

It is seen from this table that Regener's sounding balloons have greatly eclipsed all other soaring devices. Above his instruments there was only 2 per cent of the earth's atmosphere. (The Regeners state that p was only 9 mm.!) It does not seem probable that a stratosphere balloon will reach that height.

But in the case of the stratosphere flights, with observers on board, a variety of different kinds of measurements may be made; for cosmic rays, the number of particles

coming down vertically as compared with the east-west or west-east; the count for shielded, compared with unshielded, instruments; a rather accurate record of air conditions; samples of the air at different heights; the relative intensity of the lines of the solar spectrum.

However, concerning this last item, the Regeners sent up a small quartz spectrometer with a clockwork mechanism for taking photographs of the spectrum at various heights. Figure 17–18 tells the story. At 29.3 kilometers (98,000 feet) their instrument recorded the strong ultraviolet line, 2948 Å. One can see how the transmission of the air increases with height. It is estimated that though only 1.5 or 2 per cent of the atmosphere was above the instrument, 6 per cent of the ozone, the chief absorbing gas for the ultraviolet, was above.

The exposure in this case was ten minutes; the spectrometer very ordinary, as measured by light-gathering power. Swiss observers near the top of the Swiss Alps with excellent instruments and using the enormously sensitive Geiger counters have measured lines from the sun out to 2000 Å. Still, this does not detract from the performance of the Regeners.

APPENDIX

APPENDIX

There are many phenomena which are governed by the law of diminishing returns, the law of geometrical or exponential decrease.

Appendix 1–1

Light.

Let us consider a beam of (homogeneous) light passing through a uniform absorbing medium. Let the medium be divided into sheets of equal thickness normal to the direction of the light and let I_0 be the intensity entering the first sheet, I_1, leaving the first and entering the second, etc. Then for a very thin sheet the decrease in intensity dI would be proportional to I and to dx. Hence $dI/I = -\mu dx$ or $I = I_0 e^{-\mu x}$. Thus it is found that $I_0/I_1 = I_1/I_2 = etc = $ constant. When the sheets are made quite thin and the I's are plotted, we have the exponential curve $I = I_0 e^{-\mu x}$.

We might state this point another way. Of all the *photons* entering a sheet, a certain fraction will be stopped or absorbed in a distance x. This fraction is not dependent upon the number. We would then write the relation $n = n_0 e^{-\mu x}$ where n is the number of *photons* entering the x sheet.

A similar relation holds in Radioactivity. See Appendix 11–1.

Appendix 1–2

Atmospheres.

We divide a homogeneous atmosphere up into horizontal sheets. Then the pressure decreases as we go up; thus $p = p_0 e^{-kx}$ where k is a constant for a certain atmosphere and p_0 is the pressure for $x = 0$. Again this may be put in a form giving the number of molecules per unit volume $n = n_0 e^{-kx}$.

This may be put in a more detailed form. The decrease in pressure, $-dp$, as we go up a distance dx is $\rho g dx$, or $dp = -\rho g dx$, where ρ is the density of the gas. But from $pv = RT$ we have $\rho = pM/RT$ where $M =$ molecular weight and R is the gas constant for one mole. Also $M = Nm$ where $N =$ Avogadro's constant and $m =$ mass of one molecule. Hence

$$\frac{dp}{p} = -\frac{N}{RT} mgdx.$$

Or

$$p = p_0 d^{-kx} \qquad \text{where} \qquad k = \frac{Nmg}{RT}.$$

Thus the *relative* change in pressure per cm. height is proportional to the *mass* of the *molecule*.

This illustrates Perrin's method of finding N by the use of colloidal solutions.

Appendix 1–3

Energies.

From the law of velocities we have

$$_vN_{v+1} = \frac{N}{\alpha\sqrt{\pi}} e^{-v^2/\alpha^2}.$$

But $\frac{1}{2} mv^2 = E$, the kinetic energy of a particle, and since $\alpha^2 = \frac{2}{3} C^2$ we have $\frac{1}{2} m\alpha^2 = \frac{1}{3} mC^2 = kT$ where k is Boltzmann's constant. Hence

$$_vN_{v+1} = \frac{N}{\alpha\sqrt{\pi}} e^{-E/kT}.$$

This is Boltzmann's law of distribution of energies. Observe that it is a distribution of energies according to velocities.

Appendix 1–4

Electron Densities.

The potential energy of an electric charge e at a point of potential V is Ve. Hence the electron densities in two adjacent regions are given by

$$N = N_0 e^{-\frac{(V_0-V)e}{kT}} \qquad \text{(See Killian's data, Chapter 9).}$$

Appendix 1–5

Electron Flow from a Condenser.

If a condenser of capacity C, charged up to a potential V, is connected through a resistance R to a galvanometer, the current i at any instant is given by $i = V/R$. But $V = Q/C$ where Q is the charge on the condenser and $i = -dQ/dt$. Hence

$$\frac{dQ}{Q} = -\frac{1}{CR} dt \qquad \text{or} \qquad \log_e \frac{Q}{Q_0} = -\frac{t}{CR}$$

or

$$Q = Qe^{-t/CR}.$$

The time required for the charge, and therefore the potential, to drop to $1/e$ of the original value is given by $t = CR$. This is called the *time-constant* of the condenser-resistance circuit. It must be considered in resistance-coupled amplifiers and in sweep circuits (Fig. 8–25).

Appendix 1–6

Advanced students very frequently will want to know how to find the value of

$$\int_0^\infty e^{-x^2}dx.$$

We let it equal S. Then also

$$S = \int_0^\infty e^{-y^2}dy.$$

Hence

$$S^2 = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy.$$

We transform this to polar coordinates. Then

$$S^2 = \int\int e^{-r^2}rdrd\theta$$

since the element of area is now $dr \times rd\theta$. It can be seen that we are integrating in the first positive quadrant. Hence the limits of r are 0 and ∞; of θ are 0 and $\pi/2$. Then

$$S^2 = \int_0^{\pi/2}\int_0^\infty e^{-r^2}rdrd\theta = \frac{\pi}{2}\int_0^\infty e^{-r^2}rdr$$

$$= \frac{\pi}{4}\int_0^\infty e^{-z}dz \qquad \text{where} \qquad r^2 = z,$$

$$= \frac{\pi}{4}.$$

Hence

$$S = \int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$

and

$$\int_{-\infty}^\infty e^{-x^2}dx = \sqrt{\pi}.$$

The integration of other powers, e^{-x^3}, e^{-x^4}, etc., can be reduced to that of e^{-x^2}.

Appendix 1–7

To find the law of the distribution of speeds from that of velocities. The probability that a velocity along x lies between u and

$$u + du \qquad \text{is} \qquad \frac{1}{a\sqrt{\pi}} e^{-u^2/a^2} du.$$

Similarly for the other components v and w. Hence the probability that the three components lie between

$$u + du, \qquad v + dv, \qquad w + dv \qquad \text{is} \qquad \frac{1}{a^3\pi^{3/2}} - e^{-\frac{u^2+v^2+w^2}{a^2}} dudvdw.$$

But the speed $C = \sqrt{u^2 + v^2 + w^2}$, and $dudvdw$ is the elementary volume of the velocity space. The elementary volume of the speed space is the spherical shell of radius c and thickness dc, or $4\,\pi c^2 dc$.

Hence the probability that the speed lies between c and $c + dc$ is

$$\frac{4\,\pi}{a^3\pi^{3/2}} c^2 e^{-c^2/a^2} dc,$$

or

$$_cN_{c+dc} = \frac{4\,N}{a^3\pi^{1/2}} c^2 e^{-c^2/a^2} dc$$

when N is the total number of particles. More briefly we write this N_c.

If we wish to find the *average* value of c, we base our work on the following idea: the average weight of x men each of mass m_1, y men each of mass m_2, etc.,

$$\overline{m} \qquad \text{is} \qquad \frac{m_1x + m_2y + \cdots}{x + y + \cdots}.$$

Hence

$$\bar{c} = \frac{\int_0^\infty cN_c}{N} = \int_0^\infty \frac{4}{a^3\pi^{1/2}} c^3 e^{-c^2/a^2} dc = \frac{2}{\sqrt{\pi}}\, a$$

and

$$C^2 = \overline{c^2} = \frac{\int_0^\infty c^2 N_c}{N} = \int_0^\infty \frac{4}{a^3\pi^{1/2}} c^4 e^{-c^2/a^2} dc = \frac{3}{2} a^2.$$

The student should also note that

$$+\,\overline{u} = \frac{\int_0^\infty uN_u}{N/2} = \frac{a}{\sqrt{\pi}}$$

and

$$\overline{u^2} = \frac{\int_0^\infty u^2 N_u}{N/2} = \frac{a^2}{2} \qquad \text{or} \qquad \sqrt{\overline{u^2}} = \frac{a}{\sqrt{2}}.$$

Hence in computing Avogadro's number from the Brownian movement we may find $\bar{x}$, the mean displacement along x (all such displacements being regarded as positive), instead of $\overline{x^2}$, the mean of the squares of such displacements, and write

$$\bar{x}^2 = \frac{2}{\pi}\overline{x^2}.$$

Then

$$N = \frac{4}{\pi}\frac{RT}{Xe}\frac{(v_g + v_c)}{(\bar{x})^2}t \qquad \text{(see Appendix 1–8).}$$

No general proof of the law of *velocity* distribution has been given here. But the expression above can be justified thus: the probability that a particle may have a velocity lying between u and $u + 1$ must be a function of only even powers of u since positive and negative velocities are equally probable; it must have a maximum for $u = 0$ and it must gradually fall away to zero as u approaches ∞. Moreover, if it is an exponential form it must contain u/a where a has the dimensions of velocity. Exponents must be dimensionless. The form e^{-u^2/a^2} satisfies these specifications. The constant is found by making the probability of u falling between $+$ and $-\infty$ equal to 1. Hence the constant $1/a\sqrt{\pi}$.

For a mathematical proof see Loeb, *Kinetic Theory of Gases*, pp. 71–76.

Appendix 1–8

We desire to find Avogadro's number by observing Brownian movements. A particle has been put into motion and is then moving in a viscous medium. The equation of motion is

$$\frac{md^2x}{dt^2} = -K\frac{dx}{dt},$$

where K would be found from Stokes' relation $F = 6\,\pi\mu av$ or from the corrected form. We are going to measure absolute displacements whether positive or negative. Consequently we are interested in x^2. Multiplying by x and remembering that

$$x\frac{d^2x}{dt^2} = \frac{1}{2}\frac{d^2(x^2)}{dt^2} - \left(\frac{dx}{dt}\right)^2$$

we have

$$m\frac{d^2(x^2)}{dt^2} - m\left(\frac{dx}{dt}\right)^2 = \frac{K}{2}\frac{d(x^2)}{dt^2}.$$

Here comes the important assumption by Einstein, that the mean value of the random kinetic energy of this particle is equal to that of a molecule at its temperature. Hence

$$m\left(\frac{dx}{dt}\right)^2 = \frac{RT}{N} \quad \left(\text{because} \quad \frac{1}{2}mC^2 = \frac{3}{2}\frac{RT}{N} \quad \text{and} \quad C^2 = 3\,u^2\right).$$

Then putting $z = d(\overline{x^2})/dt$, we have

$$\frac{dz}{z - 2\dfrac{RT}{Nm}} = -\frac{K}{m}dt, \qquad \text{or} \qquad z = 2\frac{RT}{NK} + Ce^{-Kt/m}.$$

The last term decreases rapidly with t and we have, for any observable time, $z = 2\,RT/NK$. Therefore $\overline{x^2} = 2\,(RT/NK)t$ where t is the time interval of observation.

Millikan and his associates eliminated K by using the oil drop relations, $mg = Kv_g$ and $Xe - mg = Kv_e$, or $Xe = K(v_g + v_e)$. Hence we have

$$\overline{x^2} = 2\frac{RT}{N}\frac{(v_g + v_e)}{Xe}t.$$

Hence Avogadro's number

$$N = \frac{2\,RT}{Xe}\frac{v_g + v_e}{\overline{x^2}}t = \frac{4}{\pi}\frac{RT(v_g + v_e)}{Xe(\overline{x})^2}t.$$

As the mean of the square of a great number of excursions of a particle had been taken, $\overline{x^2}$ was known. All other quantities were experimentally determined, hence N was computed. Fletcher, from 18,837 displacements, found $N = 6.03 \times 10^{23}$. The accepted value is 6.064 ± 0.006.

Again we call attention to this extraordinary performance—using a chaotic motion like the Brownian movement in order to determine one of the important constants of nature.

Appendix 2–1

The corrected form of Stokes' law is

$$F = \frac{6\,\pi\mu av}{1 + \dfrac{b}{pa}}.$$

The experimental value of b is 6.25×10^{-4} when p is measured in cm. of Hg. If v_g is the velocity of fall under gravity, then

$$mg = \frac{4}{3}\pi\rho a^3 g = \frac{6\,\pi\mu a v_g}{1 + \dfrac{b}{pa}},$$

where a is radius of the drop and ρ is the effective density. Solving for a, we have

$$a^2 + \frac{ab}{p} = \frac{9}{2}\frac{\mu v_g}{\rho g} = c,$$

or

$$a = -\frac{b}{2\,p} + \sqrt{c + \frac{b^2}{4\,p^2}}\,.$$

But $b^2/4\,p^2$ is small compared with c. Hence

$$a = \sqrt{\frac{9}{2}\frac{\mu v_g}{\rho g}} - \frac{b}{2\,p}\,\text{cm.}$$

From Stokes' law uncorrected

$$a = \sqrt{\frac{9}{2}\frac{\mu v_g}{\rho g}}\,.$$

The correction, $b/2\,p$, although of the order of 4×10^{-6} cm., amounts to 5 or 10 per cent for ordinary drops. With a known, we substitute in the relation

$$Xe_n - mg = \frac{6\,\pi\mu a v_e}{1 + \dfrac{b}{ap}}$$

for a drop going up with velocity v_e. The numerator has been decreased and the denominator increased by the correction. Then

$$e_n = \frac{6\,\pi\mu a}{1 + \dfrac{b}{ap}}\left(\frac{v_g + v_e}{X}\right)\text{e.m.u.}$$

if X is in e.m.u. of potential. Or $e_n = C(v_g + v_e)$ for a given drop with X constant.

Appendix 3–1

We desire to find the radius of curvature of an ellipse at the extremity of (1) the minor, (2) the major axis. A circle of radius a is "shrunk" along one direction, the y direction, so that all y ordinates are decreased in the ratio of b/a. A small chord, x, of the circle is unchanged but the "sagitta" h is decreased in the ratio of b/a; the ellipse sagitta is therefore $h(b/a)$. Now the radius r of a circle is connected to the sagitta by the relation $2\,hr = x^2$, or $r = x^2/2\,h$. Hence for the radius of curvature of the ellipse at the end of the minor axis we have $r_b = (x^2/2\;bh)a$. For the circle, $r = a = x^2/2\,h$. Therefore $r_b = a^2/b$. Similarly the radius of the circle at A touching the ellipse is b^2/a.

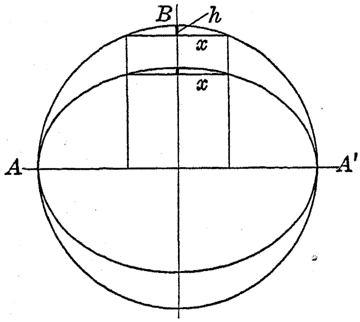


FIG. App. 3–1. When a circle is shrunk to form an ellipse, the curve at B is flattened, the radius of curvature is increased in the ratio of a/b.

Appendix 3–2

The Radius of Curvature of a Parabola at the Vertex.

For a parabola the equation of which is $y^2 = 4\,fx$ we easily find the radius of curvature at the apex thus. Let x be small; then in a circle of radius r, a chord, $2\,y$, cuts off a sagitta x given by $y^2 = 2\,rx$. Hence $r = 2\,f$. Hence a particle shot out horizontally on the earth's surface with a speed of 7 miles/sec. starts out describing a circle of radius equal to the *diameter* of the earth. It may be noted that 7^2 is (nearly) equal to 2×5^2, where 5 miles/sec. (4.95) is the speed required for *circular* motion about the earth.

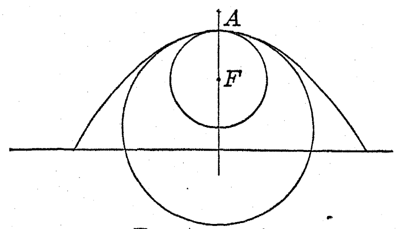


FIG. App. 3–2.

Appendix 3–3

The Law of Equal Areas.

When a force F acts upon a mass m, an acceleration a is generated equal to F/m. When a moment of force L acts about an axis, it

produces an angular acceleration given by $\alpha = L/I$ where I is the moment of inertia about that axis and equal to Σmr^2, the sum of all the mr^2 in the body. When F is zero, the momentum mv is constant. When L is zero, the angular momentum $I\omega$ is constant where ω is the angular velocity. When a mass m describes an orbit (Fig., Appendix 3–3) about a large mass at F_1, the force is along r; there is no component of F perpendicular to r. Hence there is no moment of force and the angular momentum $mr^2\omega$ is constant. Now in a small time t the angle ωt is traced out. The area traced out is $r\omega t \times r/2$ or $\frac{1}{2} r^2\omega t$. Hence the *rate* of description of area is $\frac{1}{2} r^2\omega$. Hence (as m in ordinary cases is constant) if the angular momentum is constant, so also is the rate of description of areas.

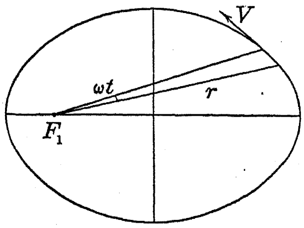


FIG. App. 3–3.

Appendix 3–4

To Find the Period in an Elliptic Orbit.

In an ellipse (Fig. 3–2)

$$FO = a\epsilon = \sqrt{a^2 - b^2} \text{ (since } FB = a); \quad AF = a(1 - \epsilon); b = a\sqrt{1 - \epsilon^2}.$$

If v is the velocity at A of a mass m describing an ellipse, then $v \times AF = 2 \times$ rate of description of areas $= 2R$. If T is the period, then

$$T = \frac{\pi ab}{R} = \frac{2\pi b}{v(1 - \epsilon)}.$$

But since the radius of curvature at A is b^2/a

$$\frac{mv^2}{b^2/a} = \frac{GmM}{a^2(1 - \epsilon)^2}$$

or

$$v = \sqrt{\frac{GMb^2}{a^3(1 - \epsilon)^2}} = \frac{b}{(1 - \epsilon)}\sqrt{\frac{GM}{a^3}}.$$

$$\therefore T = 2\pi\sqrt{\frac{a^3}{GM}}.$$

This is Kepler's Third Law.

Appendix 3–5

To Find the Energy in an Elliptic Orbit.

The total energy in an elliptic orbit is the sum of the kinetic and potential energies. Thus at B it is equal to $\frac{1}{2} mv_B^2 - GMm/a$ and

this is constant around the ellipse. The force on m at B (see Appendix 3–4) is GMm/a^2 along FB. The component of this along BC is $(GMm/a^2) \times (b/a)$. But this must be equal to mv_B^2/r where r = the radius of curvature at B or a^2/b. Hence

$$\frac{mv_B^2}{a^2} \cdot b = \frac{GMm}{a^3} \cdot b \qquad \text{or} \qquad mv_B^2 = \frac{GMm}{a} \cdot$$

Hence the *total energy* at B is

$$\frac{1}{2} mv_B^2 - \frac{GMm}{a} = -\frac{GMm}{2\,a} \cdot$$

This is the total energy in a circle of radius a. Hence the total energy in an elliptic orbit depends only on the major axis and is equal to $-\,GMm/2\,a$.

The velocity at B in the elliptic orbit is the same as that in the circular orbit of diameter $2\,a$.

Since the energy is negative and equals $-\,GMm/2\,a$, it decreases as a decreases. For a parabolic orbit $a = \infty$ and the energy is zero. For a hyperbolic orbit the energy is positive and equals $GMm/2\,a$ or $EE'/2\,a$. Thus from the point of view of energy we might regard the hyperbola as having a *negative* major axis.

We can prove this property of the hyperbola thus.

Appendix 3–6

To Find the Energy in a Hyperbolic Orbit.

Let a particle of charge $+\,E'$ start in from infinity along ID with velocity v towards a center F_2 repelling with a force EE'/r^2. Its kinetic energy will carry it in to a point D such that $\frac{1}{2}\,mv^2 = EE'/d$ where $d = F_2D$. Now let the particle start in along the line J_1O. We can show that it will describe a hyperbola with F_2 as focus where $DF_2 = 2\,a$, the major axis of the hyperbola. For let the particle round the curve at A with velocity v_1. Then, since $F_2O = a\epsilon$, where ϵ is the eccentricity,

$$\frac{1}{2} mv(v^2 - v_1^2) = \frac{EE'}{a(1 + \epsilon)} \text{ (the decrease in kinetic energy} \\ = \text{gain in potential).}$$

But

$$\frac{mv_1^2}{b^2/a} = \frac{EE'}{a^2(1 + \epsilon)^2} \cdot$$

Also, since the area traced out is constant, $vb = v_1a(1 + \epsilon)$. Substi-

tuting, we have

$$\frac{1}{2}m\left(v^2 - \frac{v^2b^2}{a^2(1+\epsilon)^2}\right) = \frac{Ee}{a(1+\epsilon)}.$$

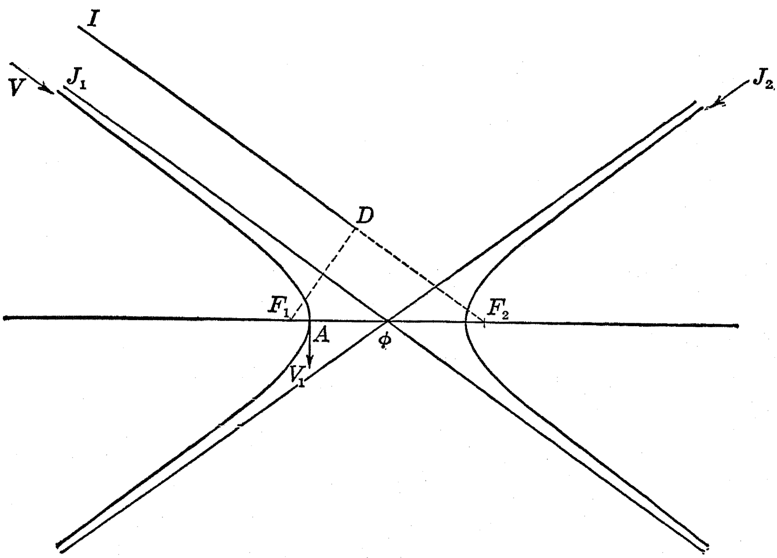


FIG. App. 3–6.

Simplifying, we have $\frac{1}{2}mv^2 = EE'/2a$. Hence $F_2D = 2a$. Thus we see that every hyperbola traced out by a particle, whose speed at infinity is v, repelled from F_2 by a force EE'/r^2, has a major axis such that the total energy in the orbit is $EE'/2a$.

Appendix 3–7

Orbits from the Point of View of Energy.

We summarize some properties of orbits from the point of view of energy.

The energy of a body in an elliptic orbit is negative and equal to $-GMm/2a$ or $-Ee/2a$. It is the same as that for a circle of diameter $2a$. The energy for a parabolic orbit is zero. For a hyperbolic orbit it is $GMm/2a$ or $Ee/2a$.

The hyperbolic orbit may be described by a particle under attraction or repulsion varying as $1/r^2$. For attraction the focus is *inside* the curve, for repulsion outside.

Thus if the particle at infinity has a velocity v along J_1O and is under a force of attraction to F_1 varying as $1/r^2$, it will describe the left branch

of the hyperbola. Then coming in from infinity along J_2O the attracting center must jump to F_2, or else F_1 must be a repelling center. In either case the right branch would be described.

Appendix 3–8

To find the probability that an alpha particle going into a thin film would be scattered at an angle ϕ.

It is seen from the figure (Fig., Appendix 3–6) that an alpha particle repelled from F_2 would be deflected through an angle ϕ and that

$$\tan\left(\frac{180 - \phi}{2}\right) = \frac{b}{a}, \qquad \text{or} \qquad \cot\left(\frac{\phi}{2}\right) = \frac{b}{a}\cdot$$

Now b, the minor axis of the hyperbola, is also the distance between the lines ID and J_1O. Let us suppose that there are n repelling centers per cm.^{3} in a thin foil of thickness t; the number of such centers per cm.^{2} of surface of the foil is nt. Then the probability that the particle at infinity will be directed along a line J_1O within a distance b of the head-on direction ID for one repelling center is proportional to the area πb^2 and for nt centers it is πntb^2. For this is the fraction of one square centimeter covered by all these areas. (We assume that the repelling centers do not shadow one another.)

The deflection of the particles directed along J_1O is ϕ where $\cot(\phi/2) = b/a$. Hence the fraction of the number of particles which will be deflected between angles ϕ_1 and ϕ_2 is

$$\begin{aligned} \pi nt(b_2^2 - b_1^2) &= \pi nta^2\left(\cot^2\frac{\phi_2}{2} - \cot^2\frac{\phi_1}{2}\right) \\ &= \pi nt\frac{d^2}{4}\left(\cot^2\frac{\phi_2}{2} - \cot^2\frac{\phi_1}{2}\right) \end{aligned}$$

where $d = 2a =$ the distance of nearest approach of the alpha particle to the repelling center.

Appendix 4–1

To prove that for adiabatic expansion of black body radiation $pv^{4/3} =$ constant.

Let the energy density be E throughout a volume v enclosed by perfectly reflecting walls. Then the total energy is Ev. The pressure everywhere is $E/3$. Let the volume become $v + dv$. Then the work done by the pressure is $(E/3)dv$.

The total energy in the new volume is

$$Ev + \partial(Ev) = Ev + Edv + vdE.$$

But since work pdv has been done, the energy now is $Ev - pdv$. Hence

$$Ev + \partial(Ev) = Ev - pdv = Ev - \frac{E}{3}dv,$$

or

$$Ev + \frac{4}{3}Edv + vdE = Ev, \qquad \text{or} \qquad \frac{4}{3}Edv + vdE = 0,$$

or

$$\frac{4}{3}\frac{dv}{v} + \frac{dE}{E} = 0, \qquad \text{or} \qquad \frac{4}{3}\log v + \log E = \text{constant.}$$

Then

$$Ev^{4/3} = \text{constant}, \qquad \text{or} \qquad pv^{4/3} = \text{constant.}$$

In a symmetrical body $v \alpha r^3$ where r is a diameter or a side. Then pr^4 is constant. Since $E \propto T^4$ and $p = E/3$, then Tr is constant.

If a spherical volume of black body radiation expands adiabatically, the temperature falls as the radius increases.

In order that we might have adiabatic expansion of radiant energy in such a volume, it would be necessary that the internal surface would be totally reflecting and that there was a very small source of temperature radiation inside. Then let us make the assumption that, as the volume expands, all wave lengths increase in length in the same way as the radius increases. Then, since Tr is constant and λ varies as r, λT is constant. This would give us Wien's law.

But what would happen to photons in this case? If the enclosure were full of *particles*, the average distance between them would increase with r. Would the average distance *between* photons increase with r? Or would the *length* of the photons increase with r? And would they be the same photons? Surely not, since the energy of every photon has decreased. But perhaps we are not justified in basing our arguments upon the *dimensions* of photons since we know nothing about such properties—unless the end justifies the means, unless our picture, our model, satisfies the facts of experiment.

To allow the *length* of a photon to increase with r would lead to the corresponding increase of λ with r and this would satisfy Wien's law.

We summarize certain (superficial) points of resemblance between a perfect gas and black body radiation.

	Gas	B. B. Radiation
For a directed stream of energy density E	$p = 2E$	$p = E$
For motion in all directions of energy density E	$p = \frac{2}{3}E$	$p = E/3$
For adiabatic expansion (where $\gamma = 1.40$)	$pv^{1.40} = \text{constant}$	$pv^{1.33} = \text{constant}$

If now we change E to denote the kinetic energy of a molecule or the energy of a photon, then the number of *molecules* of a perfect gas

having energies between E and $E + dE$ is

$$N = XE^{1/2}e^{-E/kT}dE \qquad \text{(Appendix 9–1).}$$

X is a constant and $E = \frac{1}{2}\,mv^2$. The number of *photons* having such energies is

$$N = YE^2 \frac{1}{e^{E/kT} - 1}\,dE.$$

This is a special form of Planck's law (see Appendix 4–2). If E is large compared with kT, this becomes

$$N = YE^2e^{-E/kT}dE.$$

Y is a constant and $E = hf$.

Appendix 4–2

Just to show how varied were the conclusions arrived at by various theoretical physicists in their attempts to derive a law of distribution for temperature radiation, the following formulae are given:

$$\begin{array}{lll}
\text{Wien's (1896)} & E_\lambda = c_1\lambda^{-5}e^{-c_2/\lambda T} & \\
\text{Thiessen's (1900)} & = c_1\lambda^{-5}(\lambda T)^{1/2}e^{-c_2/\lambda T} & \text{The } c\text{'s are} \\
\text{Rayleigh's (1900)} & = c_1\lambda^{-4}Te^{-c_2/\lambda T} & \text{different constants} \\
\text{Rayleigh-Jean's (1905)} & = 8\,\pi kT\lambda^{-4} & \\
\text{Planck's (1900)} & = 8\,\pi ch\lambda^{-5}/(e^{ch/\lambda kT} - 1) & \text{where } c = 3 \times 10^{10}
\end{array}$$

The experimental curves were not obtained until 1900; they were verified and extended in 1901. Then it was found that the only formula which fitted the curves was Planck's and he had obtained it by "distributing" energy pellets among oscillators, the energy given to an oscillator to be equal to the frequency of that oscillator multiplied by a constant, h. He made his formula fit the curves by this, to him and other physicists, absurd assumption, $e = hf$.

If in Planck's relation we put $\lambda = c/f$, then we have

$$E_f = 8\pi h \frac{f^3}{c^3} \frac{1}{e^{hf/kT} - 1}\,df$$

for the amount of energy lying between frequencies f and $f + df$. But the energy of each photon is hf; hence the *number* of photons having energies between E and $E + dE$ is given by

$$N = \frac{8\pi}{c^3}\frac{E^2}{h^2}\frac{1}{e^{E/kT} - 1}\,dE.$$

Appendix 4–3

To derive Wien's relation $\lambda_m T = 0.2885$ cm. deg. from Planck's formula.

From

$$E_\lambda = c_1\lambda^{-5}(e^{ch/k\lambda T} - 1)^{-1} \qquad \text{and} \qquad \frac{\partial E_\lambda}{d\lambda} = 0$$

we get

$$\left(1 - \frac{ch}{5\,kT\lambda}\right)e^{ch/k\lambda T} = 1$$

or

$$\left(1 - \frac{x}{5}\right)e^{x} = 1 \qquad \text{where} \qquad x = \frac{ch}{k\lambda T},$$

or

$$\left(1 - \frac{x}{5}\right) = e^{-x}.$$

We draw the two curves $y = 1 - x/5$ and $y = e^{-x}$ and we see that they cut at a point for which $x = 5$ nearly. We let $x = 5 - \alpha$ where α is small. Then

$$\left(1 - \frac{5 - \alpha}{5}\right) = e^{-5}e^{\alpha},$$

$$\therefore \quad \alpha = \frac{5}{e^5}\left(1 - \alpha + \frac{\alpha^2}{2}\cdots\right).$$

But $e^5/5 = 29$ nearly. Hence $\alpha = 1/29.3 = 0.0345$. Hence $x = 4.965$,

$$\therefore \quad \frac{ch}{k\lambda_m T} = 4.965 \qquad \text{or} \qquad \lambda_m T = 0.2885 \text{ cm. deg.}$$

Or if we assume $\lambda_m T$ as known, then $h/k = 4.778 \times 10^{-11}$, from which the value of h can be computed, as Planck did in 1900. He found the value of h to be 6.42×10^{-27} erg seconds.

Appendix 4–4

To obtain the Stefan-Boltzmann relation $E = aT^4$ from the Planck formula. Here it is advisable to put Planck's formula in the frequency form. Use $f = c/\lambda$ and it becomes

$$E_f = \frac{8\,\pi h}{c^3}f^3(e^{hf/kT} - 1)^{-1}df.$$

We integrate this from $f = 0$ to $f = \infty$ and note that when e^x is large compared with 1, $(e^x - 1)^{-1} = e^{-x} + e^{-2x} + e^{-3x} + \cdots$. Upon integrating, we obtain a series

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots,$$

and this equals $\pi^4/90$. [To prove this we start with

$$\cos z = \prod_1^\infty \left(1 - \frac{4 z^2}{\overline{2n-1}^2 \pi^2}\right);$$

take logarithms; expand, differentiate; use the relation

$$\tan z = z + \frac{z^3}{3} + \frac{2}{15} z^5, \cdots;$$

equate coefficients and get the above series

$$1 + \frac{1}{2^4} + \cdots = \frac{\pi^4}{90}.\Big]$$

From the operations above we obtain

$$E = aT^4 \qquad \text{where} \qquad a = \frac{48\,\pi}{c^3} \frac{k^4}{h^3} \frac{\pi^4}{90}.$$

From the experimental value of $a = 7.62 \times 10^{-15}$ we can find the value of h; it is 6.54×10^{-27}.

Appendix 6–1

It might make for definiteness if we were to compute the values of various quantities connected with the first Bohr circular orbit and compare with them the corresponding quantities of the other orbits. The general formulae are:

$$a_n = \frac{n^2 h^2}{4\,\pi^2 meE}; \qquad v_n = \frac{2\,\pi eE}{nh}.$$

$$P_n = \frac{2\,\pi a_n}{v_n} = \frac{2\,\pi n^3 a_1}{v_1} = n^3 P_1; \qquad f_n = \frac{1}{P_n} = \frac{1}{n^3} f_1.$$

Let a_1 be the radius, v_1 the velocity in the first orbit, P_1 the period.

$$\begin{array}{lll}
a_1 = 0.528 \times 10^{-8} \text{ cm.} & a_2 = 4\,a_1 & a_3 = \dfrac{9}{a_3} \\
v_1 = 2.19 \times 10^8 \text{ cm./sec.} & v_2 = \dfrac{v_1}{2} & v_3 = \dfrac{v_1}{3} \\
P_1 = 1.52 \times 10^{-16} \text{ sec.} & P_2 = 8\,P_1 & P_3 = 27\,P_1 \\
 & \quad = \left(\dfrac{a_2}{a_1}\right)^{3/2} P_1 & \quad = \left(\dfrac{a_3}{a_1}\right)^{3/2} P_1 \\
f_1 = 6.6 \times 10^{15} \text{ sec.} & f_2 = \dfrac{f_1}{8} & f_3 = \dfrac{f_1}{27} \\
KE_1 = 2.16 \times 10^{-11} \text{ erg} & KE_2 = \dfrac{KE_1}{4} & KE_3 = \dfrac{KE_1}{9} \\
w_1 = -2.16 \times 10^{-11} \text{ erg} & w_2 = -\dfrac{w_1}{4} & w_3 = -\dfrac{w_1}{9}.
\end{array}$$

It is seen that the greatest velocity, that in the first orbit, is small, about 1/137 of that of light; that the periods are in accord with Kepler's law; that the orbital frequencies, starting with 6.6×10^{15} per sec., fall off very rapidly, with the cubes of natural numbers. Now the frequency of yellow light is of the order of 5×10^{14} per sec. So the orbital frequency in the first orbit is large compared with that of visible light—but the frequency of the light emitted does not depend in a simple way upon the orbital frequencies. It is energy differences that control emission frequencies.

Appendix 6–2

If a ring of mass m, radius r, is rotating with angular speed ω radians/sec., the angular momentum is $mr^2\omega$. If the *ring* carries a charge e in e.s.u., the magnetic moment is

$$\frac{e}{c}\frac{1}{2\,\pi r}r\omega \times \pi r^2 = \frac{e}{2\,c}r^2\omega.$$

Hence the ratio of magnetic to mechanical moment $= e/2\,mc$, but if m is uniformly distributed in a disc, mechanical momentum equals $(mr^2/2)\omega$. Then the ratio is e/mc.

Appendix 6–3

Concerning the Fine Structure Constant.

The energy in the n^{th} Bohr orbit is

$$\frac{2\,\pi^2\mu e^4 Z^2}{n^2h^2c^2} = -\frac{RZ^2}{n^2}$$

where $\mu = Mm/(m + M)$ and $Z =$ atomic number. When allowance is made for the relativity increase in mass as the electron goes around the nucleus in the various orbits, the *decrease in energy* is

$$R\frac{\alpha^2 Z^4}{n^4}\left(\frac{n}{k} - \frac{3}{4}\right)$$

where

$$\alpha = \frac{2\,\pi e^2}{hc} = \frac{1}{137} \qquad \left(\text{experimental} = \frac{1}{137.29}\right)$$

and $k = 1, 2, 3, \cdots$ in the old Bohr-Sommerfeld orbits (Fig. 6–3 a).

Thus for the orbit X (Fig. 6–4), $n = 4$, $k = 3$, and the energy in that orbit is

$$-\frac{RZ^2}{4^2}\left[1 + \frac{\alpha^2 Z^2}{4^2}\left(\frac{4}{3} - \frac{3}{4}\right)\right].$$

But for the other sets of orbits shown in Fig. 6–4 b, c, d, slightly different formulae apply. See H. E. White, *Introduction to Atomic Spectra,* pages 76, 77, etc.

[May the author interject a playful remark? In this atomic world the number 137 seems to be a magic one. For the constant α which enters in the fine structure formula is given by $\alpha = 2\,\pi e^2/hc$ and this equals 1/137. After the derivation by Leibniz of the value of $\pi/4$, viz.,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}, \qquad \text{etc.,}$$

some philosophers of the time agreed that the Creator of the universe was a believer in odd numbers. Recently it has seemed that one eminent mathematical physicist believes that the atomic universe is built on the number 137 or its reciprocal.]

Appendix 6–4

The Torque in a Beam of Circularly Polarized Light.

We may prove either by the old wave theory or by the modern quantum theories that a beam of *circularly* polarized light possesses angular momentum about the direction of propagation. If such a beam is passed through a plate of quartz which changes the light from clockwise to counter-clockwise rotation, there should be a torque on the quartz disc. The torque however is very small.

Professor Poynting in 1909 worked out the amount of torque T to be expected on the basis of the old wave theory. His result was that $T = M\lambda/2\,\pi$ where λ is the wave length and M is the energy density and therefore the pressure (we are dealing with a 1 cm.^{2} beam).

The author some years ago worked out the result for photons in the following simple way. The quantum theories give the view that radiation is restricted to the case in which an electron changes in angular momentum by an amount of the order of $h/2\,\pi$. Now the energy of the photon sent out would be hf and the momentum $M = hf/c = h/\lambda$. Hence the torque per photon $= M\lambda/2\,\pi$ where M = momentum. Hence in a 1 cm.^{2} circularly polarized light there is a torque $= M\lambda/2\,\pi$ where M is pressure of the beam. If light were used of wave length 6×10^{-5} cm., the torque in this beam of circularly polarized light would be $M\lambda/2\,\pi = M \times 10^{-5}$ nearly. Or the torque would be 10^{-5} of that due to the pressure in the same light beam at one centimeter from the rotation axis. A quartz disc 2 cm. diameter, of thickness 0.0075 cm. (a so-called half-wave plate), can be supported so as to be horizontal by a long quartz fiber of sufficient strength to support the disc and yet of sufficient fineness to show the torque in an intense beam of circularly polarized light. The period of such a

torsion pendulum would be about ten minutes. A monochromatic source would not give sufficient intensity. An arc lamp or tungsten filament at high temperature could be used, but then the so-called half-wave plate would be a quarter-wave plate for some of the light and a three-quarter plate for some of it. This would involve the computation of the effective phase angle for the entire beam. For this computation we would have to invoke the aid of the wave theory.

The experimental procedure would be as follows. An intense source (a white-hot tungsten ribbon) sends light rendered parallel by a lens up through a Nicol prism, thus becoming plane polarized, then through a quarter-wave plate. (Of course this would be a quarter-wave plate for light of only one wave length.) The emerging light, some circularly or most of it elliptically polarized, passes through the suspended half-wave plate of quartz. The torque on the plate per photon emerging would be $h/2\pi$, but only for those photons for which the suspended plate is a half-wave plate; for the others, a smaller amount depending on the phase between the emerging ordinary and extraordinary beams. We may multiply the effect by a factor of two by changing the phase and reflecting back the light through the quartz plate, but it still remains true that the torque in the beam is only of the order of 10^{-5} or at most 10^{-4} of the torque due to light pressure in the same beam when the torque arm is one centimeter. As the pressure in the standard light beam used by Nichols and Hull was about 10^{-4} dyne/cm.^{2}, the torque in a 1 cm.^{2} beam of circularly polarized light of equal intensity would be between 10^{-8} and 10^{-9} dyne/cm. This can be measured, but there would be difficulties in connecting the observed values with those to be expected from theory.

However, all the vast quantity of data in spectroscopy supports the view that photons have a torque of the order of $h/2\pi$ about the direction of propagation. The torque in a circularly polarized beam is then a necessity.

Appendix 7–1

Mass and Energy.

The Theory of Relativity gives us the idea that mass m may be changed to energy or is the equivalent of energy mc^2, where c is the velocity of light. We can justify this in a general way thus. Energy has the dimensions of mass $\times$ velocity squared. From the point of view of dimensions, the relation is right. But any constant of nature like π or e (the Naperian base) might enter to make the energy of a mass $m = \pi mc^2$ or emc^2. The units would have to be considered also. But in radiation we have this relation, the energy of a photon $= hf$ ergs where h is Planck's constant and f is frequency. Also the momentum

$= hf/c$. Now dimensionally, mass = momentum divided by velocity. Hence we might make out a plausible mass for a photon $= hf/c^2 = m$. Then for a photon, the energy $= mc^2$, and we might extend this rather superficial argument to *mass* and say that any mass, m grams, has an energy mc^2 ergs, or that energy E has a mass $= E/c^2$. However, if we assume this to be true, then the energy of a mass at rest m_0 is m_0c^2 and of a mass in motion is mc^2 where m is the new mass due to motion and therefore is (see page 29) $m_0/\sqrt{1-\beta^2}$ where $\beta = v/c$ and v is the velocity. Hence when a mass m_0 is put into motion, the kinetic energy which is equal to the difference of the energies of the moving mass and the mass at rest is

$$mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right).$$

Now we can easily show that this reduces to $\frac{1}{2}mv^2$ when β or v/c is small. For then

$$\frac{1}{\sqrt{1-\beta^2}} = \left(1 + \frac{\beta^2}{2}\right)$$

[since by the binomial theorem

$$\frac{1}{(1-x)^n} = (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2}x^2 + \cdots;$$

hence when β is small,

$$m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = \frac{m_0\beta^2c^2}{2} = \frac{1}{2}mv^2\Bigg].$$

Thus our plausible assumption that there is energy in a mass m equal to mc^2 is in accord with well-known facts. Later we shall see that it "accounts for" other phenomena—not so well known. It is one of the essential ideas entering into the phenomena connected with the transmutation of the elements.

We might arrive at a "proof" of the relativity relation thus. We accept the experimental fact that when a mass m_0 is given a velocity v, its mass then becomes $m_0/\sqrt{1-\beta^2}$. Now energy is of the form Kmv^2, where v is some velocity. Hence, as before, when a mass m_0 is set in motion the increase in its energy must be

$$\frac{Km_0v^2}{\sqrt{1-\beta^2}} - Km_0v^2.$$

But this must equal $\frac{1}{2}\,mv^2$ when v/c or β is small. Then

$$Km_0v^2\left(1+\frac{v^2}{2\,c^2}-1\right)=\frac{1}{2}\,m_0v^2, \qquad \text{or} \qquad Kv^2=c^2.$$

Hence the energy of m is mc^2.

Appendix 7–2

The Compton Effect.[1]

Let us picture a photon hf striking a mass m_0 at rest, putting it into motion with a velocity v and glancing off as a photon hf'. Let E be the energy of the original photon E' of the bouncing off photon and T the kinetic energy of the mass after impact. The energy and momenta relations hold. See Fig. App. 7–2.

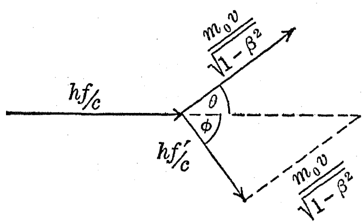


FIG. App. 7–2.

$$(1) \qquad E=E'+T \qquad \text{or} \qquad hf=hf'+m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)$$

$$(2) \qquad \frac{(m_0v)^2}{1-\beta^2}=\frac{(m_0\beta c)^2}{1-\beta^2}=\left(\frac{hf}{c}\right)^2+\left(\frac{hf'}{c}\right)^2-2\,\frac{h^2ff'}{c^2}\cos\phi.$$

$$\text{Let} \quad \frac{hf}{m_0c^2}=x; \quad \frac{hf'}{mc_0{}^2}=x'; \quad b=\frac{1}{\sqrt{1-\beta^2}} \quad \text{or} \quad -1+b^2=\frac{\beta^2}{1-\beta^2}.$$

Then (1) becomes

$$(1.1) \qquad x=x'+b-1,$$

and (2) becomes

$$(2.1) \qquad -1+b^2=x^2+x'^2-2\,xx'\cos\phi.$$

Eliminating b from (1.1) and (2.1) we have

$$(3) \qquad x-x'=xx'(1-\cos\phi)$$

or

$$(4.1) \qquad f-f'=\frac{hff'(1-\cos\phi)}{m_0c^2}$$

[1] Concerning the Compton effect. An experiment has been performed (1936) by Shankland under Compton's direction which throws doubt on the validity of this theory for individual electrons and photons.

or

$$\frac{c}{\lambda} - \frac{c}{\lambda'} = \frac{h}{m_0c^2}\frac{c^2}{\lambda\lambda'}(1 - \cos\phi) \tag{4.2}$$

or

$$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\phi) = 0.0243\ (1 - \cos\phi)\ \text{Å} \tag{5}$$

if m is the mass of an electron. This means that the wave length of the scattered radiation is longer than that of the incident by 0.024 $(1 - \cos\phi)$ Å where ϕ is the angle of scattering. From equation (4.1)

$$E' = \frac{E}{\left[1 + \frac{E}{mc^2}(1 - \cos\phi)\right]}.$$

From (1),

$$T = \frac{E^2(1 - \cos\phi)}{[mc^2 + E(1 - \cos\phi)]}.$$

When $\phi = 180°$ or when the bouncing-off photon is thrown straight back, $\cos\phi = -1$ and $T = 2\ E^2/(mc^2 + 2\ E)$ or

$$\frac{T}{E} = \frac{1}{\left(1 + \frac{hf}{2\ mc^2}\right)}. \tag{6}$$

We can easily show that when the collision is "head on" the photon after impact cannot go forward with the electron (or other mass) but must be thrown back. Hence for that case $\phi = 180°$. If both photon and electron would go forward, we would have

$$hf = hf' + mc^2\left(\frac{1}{\sqrt{1 - \beta^2}} - 1\right) \quad \text{and} \quad \frac{hf}{c} = \frac{hf'}{c} + \frac{m\beta c}{\sqrt{1 - \beta^2}}.$$

Then

$$\frac{\beta}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \beta^2}} - 1 \quad \text{or} \quad 1 - \beta = \sqrt{1 - \beta^2}$$

$$\therefore\quad \beta = 0 \text{ or } 1, \text{ and } f = f'.$$

There would thus be no collision—or in the case of a photon of very great energy the electron would go forward with nearly the speed of light and the photon after collision would have nearly zero energy. To prove the last point let $\beta = 0.99995$, then $f' = f/99$.

Appendix 7–3

Figure Appendix 7–3 represents a crystal of rock salt with sodium ($ACFH$) and chlorine ($BGDE$) atoms equally spaced. Let us regard the lines AB, AD, AF as the x, y, z, axes. Then a plane $BCGF$

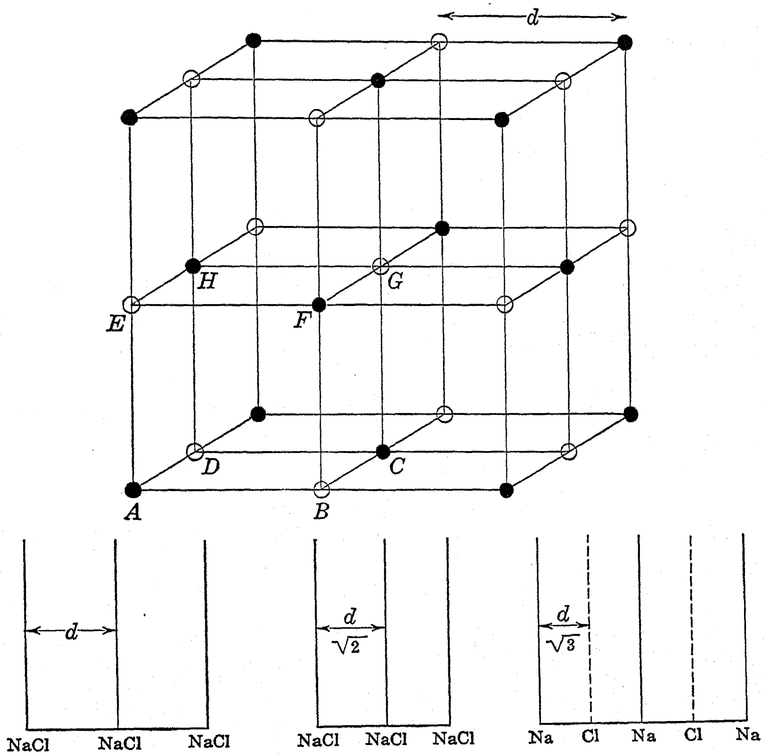


Fig. App. 7–3.

intercepts the x axis at B or one unit distance (d) from A. Its intercepts on the y and z axes are infinity. It is called the (100) plane, these numbers (is zero a number?) being the reciprocals of 1 and infinity. Every parallel plane of atoms is the (100) plane. Similarly the $EFGH$ and the $DCGH$ are (100) planes. These planes contain sodium and chlorine atoms in equal numbers. But we may have planes of atoms parallel to $FBDH$, also containing these atoms in equal numbers. It can be seen at once that the distance between two of these planes is half of AF or is $d/\sqrt{2}$. The $FBDH$ plane is called the (110).

But there are other planes of atoms like BDE which contain all chlorine, and FCH which contain all sodium. We want to find the distance, p, from A to the plane BDE. The volume of the figure $ABDE$ is triangle $ABD \times$ height $AE \div 3 = d^3/6$. This is also equal to the triangle $BDE \times p/3$. Now BDE is an equilateral triangle of side $d\sqrt{2}$. Hence its area is $d^2 \sin 60° = d^2\sqrt{3/2}$. Hence $pd^2\sqrt{3/2} = d^3/6$ or $p = d/\sqrt{3}$. Similarly the distance from G to the plane CFH is $d/\sqrt{3}$. But the total distance from A to G is $d\sqrt{3}$. Then the distance from A to the plane BDE and from G to CFH is one-third of AG. Hence the distance between the planes BDE and CFH is $d/\sqrt{3}$. The plane BDE, since it intercepts unit distances on the axes, is called the (111) plane.

The distances between the various planes are represented (to half a scale) in the figure. It is seen that the distances between similar (111) planes, i.e., between two all-sodium or all-chlorine, is $2\,d/\sqrt{3}$. This distance is identified as one of the d's in the relation $\lambda = 2\,d \sin\theta$.

Appendix 8–1

To derive the relation

$$i = \frac{CV^{3/2}}{x^2}$$

Assume that both cathode and plate are planes facing one another and perpendicular to x. We picture the region between them filled with electrons moving towards the plate. At distance x from the cathode let n = number per cm.^{3}, u = velocity, i = current density, V the potential. Now there is an elementary relation between an electric charge Q and the number of lines of force it sends out; the latter $= 4\,\pi Q$. Hence for all the electrons in 1 cm.^{3} the total number of lines is $4\,\pi ne$, and they are all directed towards P. Hence the number passing through 1 cm.^{2} normal to x at distance x is greater than the number through unit area at distance $x - dx$ by $4\,\pi ne dx$. This then is the increment in X, the intensity, and we have

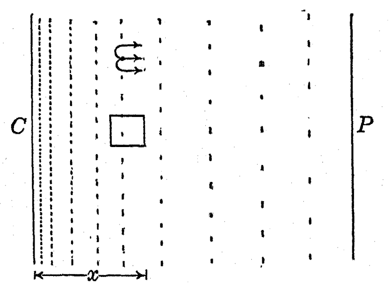


FIG. App. 8–1.

$$4\,\pi ne = \frac{dX}{dx} = \frac{d^2V}{dx^2}\,.$$

This is Poisson's equation with sign changed because we are dealing with electrons. But the current density along x is constant. Hence $i = neu$ and $\frac{1}{2}\, mu^2 = Ve$. From these relations we get

$$i = \sqrt{\frac{2\,Ve}{m}}\,\frac{1}{4\,\pi}\,\frac{d^2V}{dx^2} \qquad \text{or} \qquad 2\,\frac{dV}{dx}\,\frac{d^2V}{dx^2}\,dx = 8\,\pi i\,\sqrt{\frac{m}{2\,e}}\,\frac{dV}{V^{1/2}},$$

$$\therefore\quad \left(\frac{dV}{dx}\right)^2 = 16\,\pi i\,\sqrt{\frac{m}{2\,e}}\,V^{1/2} = AV^{1/2},$$

assuming that $dV/dx = 0$ when $V = 0$. Extracting the square root, separating variables, and again integrating, we get $\frac{4}{3}\,V^{3/4} = A^{1/2}x$ assuming that $V = 0$ when $x = 0$. Substituting the value for A and squaring, we have

$$i = \frac{\sqrt{2}}{9}\,\sqrt{\frac{e}{m}}\,\frac{V^{3/2}}{x^2}.$$

The assumptions which have been made, that $V = dV/dx = 0$ when $x = 0$, show that we are making measurements for a position in space that need not be that of the cathode. For in Fig. 8–5 it is shown that the cathode may be at a higher potential than points outside. Now dV/dx will be zero at a minimum of V and therefore in general at a point some small distance from the cathode, unless the potential of the plate is high, in which case there is no large accumulation of electrons, then we are justified in measuring x from the cathode surface.

In order that we might consider the potential and current as dependent on only one space variable x, it would be necessary for the cathode and plate to be large in area compared with the distance between them. This condition is generally not realized in practice. However, it can be shown that a similar law holds for the case of a cylindrical anode with a fine wire along its axis as cathode, or for a spherical anode with a concentric spherical cathode. Consequently it results that the three-halves power law holds reasonably well in practice.

Appendix 8–2

To find approximately the frequency of the Barkhausen-Kurtz oscillations. We assume that electrons starting from rest at the filament are attracted to the grid on account of its high voltage V; that some of them pass through towards the plate and are then driven back. We regard the plate as at zero potential.

The velocity of the electrons as they pass the grid is given by $\frac{1}{2}mv^2 = Ve$, both V and e being in e.m.u. Then $v = \sqrt{2\,Ve/m}$. If now we make a very rough approximation and take the *average* velocity of the electron to be $v/2$, we would have the period P of the oscillation to be

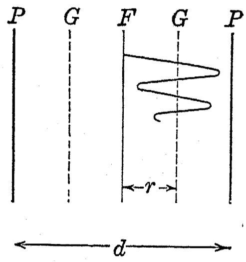


FIG. App. 8–2.

$$P = \frac{4\,r}{v/2} = \frac{8\,r}{v} = \frac{2\,d}{v} = 2\sqrt{\frac{m}{2\,Ve}} \cdot d$$

where d is the diameter of the cylindrical anode.

Now $e/m = 1.77 \times 10^7$. Hence the period is

$$P = \frac{2}{\sqrt{V}} \frac{d}{\sqrt{3.54 \times 10^7}} = \frac{d}{3 \times 10^3 \sqrt{V}} \qquad \text{(nearly)}.$$

But $V = E \times 10^8$ if E is in volts. Then $P = d/(3 \times 10^7 \sqrt{E})$. Hence the wave length $= 3 \times 10^{10} \times P = 1000\,d/\sqrt{E}$ cm. Even if we take the attractive force proportional to the distance from the grid—which is a highly improbable condition—we obtain a result not greatly different from the above. The motion then would be simple harmonic and the period would equal the circumference of the auxiliary circle divided by the maximum velocity. Then

$$P = \frac{2\,\pi r}{v} = \frac{\pi d}{2\,v} = \frac{\pi d}{2\sqrt{V} \times 3.54 \times 10^7} = \frac{d}{6 \times 10^7 \sqrt{E}}.$$

Hence

$$\lambda = 3 \times 10^{10} \times \frac{\pi d}{12 \times 10^7 \sqrt{E}} = 780 \frac{d}{\sqrt{E}} \text{ cm.}$$

The relation $\lambda = 1000\,d/\sqrt{E}$ has been approximately verified. Thus with $d = 1$ cm. and $E = 400$ volts the wave length λ is 50 cm.

Appendix 9–1

Maxwell's law of distribution of speeds is

$$N_c = \frac{4\,N}{\alpha^3\sqrt{\pi}} c^2 e^{-c^2/\alpha^2} dc.$$

We put $\frac{1}{2}mc^2 = E$, the kinetic energy of the particle. But $\frac{1}{2}mC^2$ = average (random translation) energy of a molecule $= 3\,kT/2$ where k is Boltzmann's constant. From chapter 1, $\alpha^2 = \frac{2}{3}C^2$. Hence

$\frac{1}{2}m\alpha^2 = \frac{1}{3}mC^2 = \frac{1}{2}kT$. Hence

$$N_c = \frac{4N}{\alpha^3\sqrt{\pi}}c^2e^{-E/kT}dc \qquad \text{and} \qquad c = \sqrt{\frac{2}{m}E}.$$

But

$$dc = \frac{1}{\sqrt{2mE}}dE.$$

Hence

$$N_E = XE^{1/2}e^{-E/kT}dE$$

where X is a constant. In other words, the number of particles having energies lying between E and $E + 1$ is

$$XE^{1/2}e^{-E/kT}.$$

The Fermi distribution makes it

$$\frac{XE^{1/2}}{e^{(E-E_0)/kT} + 1}$$

where E_0 is obtained as in Chapter 8, viz.,

$$E_0 = \frac{h^2}{2m}\left(\frac{3n\nu}{8\pi}\right)^{3/2}.$$

Appendix 9–2

The Temperature Corresponding to One Volt.

Suppose the ordinary kinetic theory holds for electrons. We desire to find the root-mean-square speed C at a temperature of 20,000 K. We have the relation $\frac{1}{2}mC^2 = \frac{3}{2}kT$ where $k = 1.37 \times 10^{-16}$ and $m = 9 \times 10^{-28}$. Then $C = 9.5 \times 10^7$ cm./sec. = 600 miles/sec. If electrons were speeded up by a potential V, the potential necessary to give the above kinetic energy is given by

$$Ve = \frac{3}{2}kT \qquad \text{or} \qquad V = 2.6 \text{ volts.}$$

The excitation potential of mercury vapor is 4.9 volts. This represents a speed of 1.3×10^8 cm./sec. or 800 miles/sec. and a temperature of 38,000 K. A criticism that may be made of this kind of computation is that temperature motions are random; motion due to an electrical field is uni-directional, it is drift or convection motion. But we have seen that an electron of 4.9 volts may excite a mercury atom and the latter may give its energy to an electron in any direction. Hence we see that in a small electrical field electrons may acquire a random energy corresponding to a very high temperature. Consequently the

law of equipartition of energy (Chapter 1) does not apply to a mixture of electrons and atoms in a region of even a small electric field.

To justify the relation $I = ne\bar{c}/4$ (page 190). Let us find the number of particles passing through 1 cm.^{2} per second. Consider a square centimeter normal to x. The number of particles per cm.^{3} having velocities along x between u and $u + du$ is

$$\frac{N}{\alpha\sqrt{\pi}} e^{-u^2/\alpha^2} du$$

if N is the total number per cm.^{3}. The number of such particles that would pass through 1 cm.^{2} normal to x per second is the above multiplied by u. Hence the total number is the integral of

$$\frac{N}{\alpha\sqrt{\pi}} ue^{-u^2/\alpha^2} du$$

from 0 to ∞. This equals $N\alpha/2\sqrt{\pi}$. Using the relations of Chapter 1,

$$\alpha = \bar{c}\frac{\sqrt{\pi}}{2} = C\sqrt{\frac{2}{3}}$$

$$\frac{N\alpha}{2\sqrt{\pi}} = \frac{N\bar{c}}{4} = \frac{NC}{\sqrt{6\pi}}.$$

But $\frac{1}{2}mC^2 = (3/2)kT$. Hence the number $= N\sqrt{kT/2\pi m}$.

If each particle carries a charge e, then the charge per second crossing 1 square centimeter, that is, the current, is $Ne\bar{c}/4$. This method may be criticized, since we have considered motion along one direction, whereas what is contemplated in the text is the case of particles driving in all directions through unit area. But suppose the unit area is wrapped around a probe; the number driving into it would be the same as it would be were the area cut and spread open. Then instead of dealing with the particles going in all directions we can resolve all motions along x. We then come to the above computation.

If in the above we limit the number of particles which can cross unit area by the condition that their energies must be greater than $\phi\epsilon$, as was done by Richardson (page 137), we integrate

$$\frac{N}{\alpha\sqrt{\pi}} ue^{-u^2/\alpha^2} du$$

from u_0 to ∞ where u_0 is such that $\frac{1}{2}mu_0^2 = \phi e$. When we do that, we obtain at once Richardson's law,

$$i = \frac{N\alpha}{2\sqrt{\pi}} e^{-u^2/\alpha^2} = N\sqrt{\frac{k}{2\pi m}}\, T^{1/2} e^{(\frac{1}{2}mu_0^2/\frac{1}{3}mC^2)} = N\sqrt{\frac{k}{2\pi m}}\, T^{1/2} e^{-\epsilon\phi/kT}.$$

In the above relations k is Boltzmann's constant, equal to 1.37×10^{-16}. Since $\epsilon\phi$ must be of the same dimension as kT, it follows that one volt may be expressed in terms of temperature. Thus the temperature corresponding to one volt $= e/k = (1.59 \times 10^{-12})/(1.37 \times 10^{-16}) = 11{,}606$ degrees.

Appendix 9–3

Vapor Pressure of Helium in mm. of Hg	Absolute Temperature
500	3.79° K.
450	3.69
400	3.59
350	3.48
300	3.36
250	3.22
200	3.06
260	2.87
100	2.64
75	2.48
50	2.30
25	2.01

To show how greatly in error at very low temperatures data would be if based on gas laws or known relations between vapor pressure and temperatures, we present the table below giving the vapor pressure of helium for low temperatures, the data being *computed* from a formula derived by Keesom of Leiden. The formula has received some experimental confirmation.

p (in mm.)	K.
0.15	1.0
3.2×10^{-3}	0.7
2.5×10^{-5}	0.5
7×10^{-10}	0.3
3×10^{-31}	0.1
4×10^{-62}	0.05
6×10^{-103}	0.03

It is estimated that the vapor pressure of iron at room temperature is about 10^{-62} mm. Thus helium at 0.05° K. would evaporate as rapidly as iron at room temperature!

Appendix 10–1

The Zeeman Effect.

We may derive the change in energy of an electron in an orbit in the following elementary way.

If we have a current i flowing in a circle of radius r, the magnetic moment is $i\pi r^2$. Placed in and parallel to a magnetic field H, it would

have a potential energy $\pi r^2 iH$. Now an electron revolving in a circular orbit, radius r, with velocity v, is equivalent to an electric current $(e/2\,\pi r)v$, and its magnetic moment is this multiplied by πr^2, or $erv/2$. Then the potential energy becomes $Herv/2$. But the mechanical or orbital moment is mvr, which in the simplest case equals $h/2\,\pi$. Substituting for rv, the potential energy becomes $(eh/4\,\pi m)H$. If we regard this as the necessary energy change due to the magnetic field and put it equal to hf', we have $f' = (e/4\,\pi m)H$. This is the frequency change which we obtained when using the classical method (page 206). But in the Bohr theory or in the quantum theory based on the Bohr model, the frequency belonging to a quantum is due to the transition from one energy state to another. If all energy states had their energies altered every one by the above amount due to the imposing of a magnetic field on them, there would be no Zeeman effect, as judged by this theory. But (see page 96) though an electron may change from a large energy orbit to a smaller one, its angular momentum change is limited to one Bohr unit $\pm\, h/2\,\pi$.

Appendix 10–2

The relation $\delta\alpha = (2\,a^2/\mu)v\delta v$ (see page 214) was derived from the same fundamental relation

$$v^2 = \frac{Ee}{m}\left(\frac{2}{r} - \frac{1}{a}\right)$$

as holds for the Bohr orbits. For the special case of the circular orbit, $v^2 = Ee/ma$. From this $2\,v\delta v = (\mu/a^2)\delta a$, where $\mu = Ee/m$. The constants used by the author in the computation, viz., $f = 10^{15}$, $a = 10^{-8}$, $v = 10^8$, are of the same order as those in the first Bohr orbit, $f = 6.6 \times 10^{15}$, $a = 0.53 \times 10^{-8}$, $v = 2.2 \times 10^8$ (see Appendix 6–1).

Appendix 10–3

Stark Effect.

An electrical doublet

$$\underset{+e \qquad -e}{\overset{\longleftarrow r \longrightarrow}{\bullet\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!\bullet}}$$

has an electrical moment of er and in an electrical field X has a potential energy $= Xer\cos\theta$ where θ is the angle between the field and r. Now when an electron is describing an ellipse round a nucleus, r and θ both change. A computation shows that the *average* energy is $(3/2)\epsilon eaX\cos\phi$ where ϵ is the eccentricity of the orbit, a the semi-major axis, and ϕ the angle between a and X. The manner in which a changes as we go from one *main* energy state to another is shown in Chapter 6. But the

eccentricity changes on account of the field and the computation now becomes complex.

Appendix 10–4

Concerning the Rotational Energy of a Molecule.

Here as in the original Bohr theory we assume that the angular momentum of the molecule can take only definite values, multiples of $h/2\pi$. Hence if the moment of inertia of the molecule is I and the angular speed is ω radians per second, we have $I\omega = nh/2\pi$ for the n^{th} state. The kinetic energy is $\frac{1}{2} I\omega^2$, which equals $n^2h^2/8\pi^2I$. But it is found necessary to change this to $(r + \frac{1}{2})^2h^2/8\pi^2I$ where $r = 0, 1, 2,$ etc. Thus the energies of the successive states are proportional to $R/4, \cdots, 9R/4 \cdots 25R/4$, etc., where $R = h^2/8\pi^2I$. The frequency difference in the vibration-rotation band is $h/4\pi^2I$. Hence measuring the frequency difference of successive maxima, we are able to compute I, the moment of inertia of a molecule. For example, for the HCl molecule this frequency difference is 6×10^{11} per sec. As this equals $h/4\pi^2I$, we find $I = 2.7 \times 10^{-40}$ gm. cm.^{2}. In the case of a symmetrical molecule like oxygen, a rotation through 180 degrees brings the molecule into a condition or alignment similar to its initial position. The probability of that position is doubled as compared with an asymetrical molecule. Hence we take $\pi I\omega = nh$. Therefore $\frac{1}{2} I\omega^2 = n^2h^2/2\pi^2I$ in place of $n^2h^2/8\pi^2I$. The distance in wave numbers between successive lines in the Raman rotation spectrum is h/π^2Ic.

Can we establish any relation between the rotational frequency of the molecule and the frequency of the radiation emitted when the molecule changes from one rotational state to another? We have the relation $2\pi I\omega = (r + \frac{1}{2})h$. The orbital frequency is $\omega/2\pi$ or $(r + \frac{1}{2})h/4\pi^2I$. Hence the *average* orbital frequency as the molecule changes from the $r + \frac{1}{2}$ to the $r - \frac{1}{2}$ state is $rh/4\pi^2I$. Now the energy change is

$$\frac{(r + \frac{1}{2})^2h^2}{8\pi^2I} - \frac{(r - \frac{1}{2})^2h^2}{8\pi^2I} \quad \text{or} \quad \frac{rh^2}{4\pi^2I}.$$

Since the frequency radiated is the energy change divided by h we have f the frequency of the photon $= rh/4\pi^2I$ which is the *average* orbital frequency.

Appendix 10–5

Vibrations of a Molecule.

To interpret the vibration-rotation bands of HCl, we picture a HCl molecule as below. Two masses m_1m_2 are held by a spring and are set

vibrating along their connecting band. The center of mass remains fixed at C. We measure distances x_1, x_2, out from C. Then we have

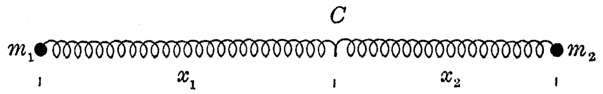


FIG. App. 10-5.

the relations

$$m_1 \frac{d^2x_1}{dt^2} = -g_1x_1, \qquad m_2 \frac{d^2x_2}{dt^2} = -g_2x_2,$$

also $m_1x_1 = m_2x_2$ and, since the force in m_1 [or $m_1(d^2x_1/dt^2)$] is equal to the force in m_2, $g_1x_1 = g_2x_2 = gl$ where $x_1 + x_2 = l$. Then

$$\frac{d^2(x_1 + x_2)}{dt^2} = -\frac{g_1x_1}{m_1} - \frac{g_2x_2}{m_2} = -gl\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

or

$$\frac{d^2l}{dt^2} = -\frac{gl}{M} \qquad \text{where} \qquad \frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2}.$$

The frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{M}} = \frac{k}{\sqrt{M}}.$$

For the case where the chlorine atom is of atomic weight 35, we have

$$\frac{1}{M_1} = \frac{1}{M_{\mathrm{H}}} + \frac{1}{M_{35}};$$

for the other molecule

$$\frac{1}{M} = \frac{1}{M_{\mathrm{H}}} + \frac{1}{M_{37}}.$$

The difference in the frequencies for the two molecules is given by

$$\frac{\partial f}{f} = \frac{M}{2}\,\partial\left(\frac{(1)}{(M)}\right) = \left(\frac{36}{35}\right)\frac{1}{2}\left(\frac{1}{35} - \frac{1}{37}\right) = \frac{1}{1259}.$$

Since

$$\frac{\partial\lambda}{\lambda} = -\frac{\partial f}{f}$$

and the mean value of λ for a certain band is 0.000176 cm. = 1.76 $\times$ 10^4 Å, the value of

$$\partial\lambda = \frac{1.76 \times 10^4}{1260} = 13.9 \text{ Å}.$$

In other words, the vibration frequency for the HCl 35 molecule ought to differ from that of the HCl 37 molecule so as to give a wave length difference of 13.9 angstroms. This should be the difference between the small and large humps in the vibration-rotation band of HCl.

Appendix 10–6

The g Factor.

In the famous Landé g factor the value of g is given by

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

where s is the spin angular momentum
l is the orbital angular momentum
and j is dependent upon these and the space quantization.

For example, for the sodium $S_{1/2}$ level $j = \frac{1}{2}$, $s = \frac{1}{2}$, $l = 0$; then $g = 2$. For the $P_{1/2}$ level, $j = \frac{1}{2}$, $s = \frac{1}{2}$, $l = 1$; then $g = \frac{2}{3}$. For the $P_{3/2}$ level, $j = \frac{3}{2}$, $s = \frac{1}{2}$, $l = 1$; then $g = \frac{4}{3}$.

A very clear statement concerning this rule is given in White's *Introduction to Atomic Spectra*, Chapter X.

Appendix 11–1

The Law of Radioactive Decay.

At a time which we call $t = 0$, there are N_0 atoms which have not been transformed. The number dN which changes over in time dt is proportional to N; therefore equal to λN. This is a decrease in N. Hence $dN/dt = -\lambda N$ or $N = N_0 e^{-\lambda t}$.

The time required to reduce the number of unmodified atoms one-half is given by

$$e^{-\lambda t} = \frac{1}{2}, \qquad \text{or} \qquad t = \frac{1}{\lambda}\log_e 2, \qquad \text{or} \qquad t = \frac{0.692}{\lambda}.$$

This is called the period or half-life of the radioactive element.

Though an atom may be transformed in the next second or may remain unchanged for a million years, it has a definite *average* life. For at a time t hence, the number of atoms unchanged will be N or $N_0 e^{-\lambda t}$. The number that will be transformed in time dt is $N_0 e^{-\lambda t} dt$. These have had a life t seconds since we started observations; their total time is $tN_0 e^{-\lambda t} dt$ and the average life of all the atoms is

$$\frac{\int_0^\infty tN_0 e^{-\lambda t}\,dt}{N_0} = \frac{1}{\lambda}.$$

Appendix 12–1

Comparison of the Lyman Series of the Hydrogens, H^1 and H^2.

The first four lines of the ultraviolet series, obtained with a vacuum spectrograph by Ballard and White, are shown in the figure. The measured and calculated differences in wave lengths of six lines are given in the table below. It is seen that the difference is very nearly 1/3685 or 1/3700 of the wave length of either line. This is the difference between the two R's or between the two μ's where $\mu = Mm/(M + m)$.

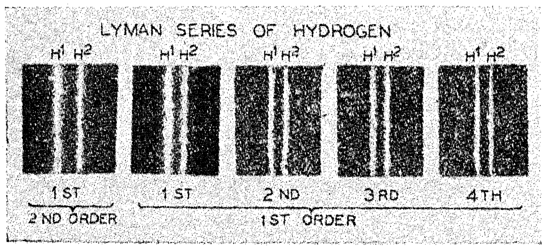


FIG. App. 12–1. Photographs of the first four members of the two hydrogen atoms H^1 and H^2. The separation is accounted for on the Bohr theory as due to the difference of the masses. (Ballard and White.)

Wave lengths and frequencies of lines in the Lyman series of H^1 and H^2

$\lambda(H^1)$ VAC.	$\nu(H^1)$	$\lambda(H^2)$ VAC.	$\Delta\lambda$ CALCULATED	$\Delta\lambda$ OBSERVED
1215.664A	82,259.57 cm.$^{-1}$	1215.334A	0.330A	0.330A
1025.718	97,492.71	1025.439	0.279	0.276
972.533	102,824.31	972.269	0.264	0.266
949.739	105,292.07	949.481	0.258	0.262
937.800	106,632.59	937.545	0.255	0.274 *
930.744	107,440.88	930.491	0.253	0.258

* The large discrepancy here may be due to the fact that these lines were faint and difficult to measure with accuracy.

Appendix 12–2

Comparison of a Head-on Collision of (a) a Photon, (b) an Electron, (c) a Proton, with a Particle.

(a). The kinetic energy given to a particle of mass m struck head-on by a photon of energy E_0 is

$$E = hf\left(\frac{1}{1 + \dfrac{mc^2}{2\,hf}}\right) = E_0\left(\frac{1}{1 + \dfrac{mc^2}{2\,E_0}}\right) \qquad \text{(see Appendix 7–2).}$$

Now mc^2 for an electron is about 5×10^5 volts; for a proton about 9.2×10^8 volts. Hence if E_0 is small compared with 5×10^5 volts, E is small compared with E_0. In other words, if E_0 is that of a soft X-ray of 10,000 volts, E for an electron would be 200 volts. But for $E_0 = 10^7$ volts, E for an electron $= 9.75 \times 10^6$ volts. A cosmic ray photon (secondary) of more than 10^7 volts would give up nearly all its energy to an electron but only about one per cent to a proton.

(b), (c). The energy given to a particle of mass m_2 at rest when struck by one of mass m_1 is given by

$$\frac{E}{E_0} = \frac{4\,x}{(1 + x)^2} = y$$

where $x = m_1/m_2$ or m_2/m_1.

When $x = 1$, $y = 1$. That is, an electron striking an electron at rest would come to rest and give all its energy to the struck electron, but striking a proton would give to the latter only 1/460 of its energy. By considerations as above Chadwick discovered the neutron.

Appendix 13–1

Curvature of Path of a High-Energy Electron in a Magnetic Field.

When a mass m, of charge e and velocity v, describes a circle due to a magnetic field H at right angles to v, the relation $mv^2/r = Hev$ holds. But $m = m_0(1 - \beta^2)^{-1/2}$ where m_0 is the rest mass and $\beta = v/c$. The kinetic energy $E = m_0c^2\{(1 - \beta^2)^{-1/2} - 1\}$. From the first two relations we obtain

$$v = \frac{Herc}{(m_0^2c^2 + H^2e^2r^2)^{1/2}} \qquad \text{and} \qquad 1 - \beta^2 = \frac{m_0^2c^2}{m_0^2c^2 + H^2e^2r^2}.$$

Suppose that the track is beaded, thus indicating that the particle is an electron. Let $H = 10{,}000$ gauss and $r = 5$ cm. Then $Her/m_0c = 30$.

Since H is measured in magnetic units, e also must be measured in those units. Then $Her/m_0c = 30$ nearly and $1 - \beta^2 = 1/900$. Hence $\beta = 1 - 1/1800$; or the velocity of the electron is 1799/1800 of the velocity of light.

For such velocities we may simplify the expressions for v and E. We have

$$c - v = \frac{1}{2}\left(\frac{m_0c}{Her}\right)^2$$

and $E = Herc$ ergs $= 300\,Hr$ electron volts.

For $H = 10{,}000$ and $r = 5$ cm., $E = 24 \times 10^{-6}$ ergs. Since 1 *ev* $= 1.6 \times 10^{-12}$ ergs, $E = 15 \times 10^6$ volts.

If the electron departs from a straight line by 1 mm. in 5 cm., then $r = 125$ cm. and $E = 300\,Hr = 3.75 \times 10^8$ volts.

Appendix 13–2

Circuits for Cosmic Ray Counters.

SUGGESTED VALUES FOR THE CIRCUIT

Condensers	Capacity in Farads		
C_1	10^{-12}	C_5	2×10^{-6}
C_2	5×10^{-11}	C_6	16×10^{-6}
C_3	2×10^{-6}	C_7	2×10^{-6}
C_4	16×10^{-6}	C_8	2×10^{-6}
Resistors	**Resistance in Ohms**		
R_1	10^{10}	R_7	2×10^4
R_2	10^7	R_8	2×10^2
R_3	2×10^6	R_9	3.5×10^3
R_4	2×10^4	R_{10}	7600
R_5	5×10^4	R_{11}	1100
R_6	5×10^4	R_{12}	7600
Inductances	**Henries**		
L_1	10	L_2	30
Batteries	**Potential in Volts**		
B_1	70	B_4	8
B_2	45	B_5	45
B_3	21	B_6	22.5

G C Geiger Counter, w wire, Cy cylinder
M Monitor relay, Western Electric relay No. 178 BY
M R Message Recorder Western Electric type 5 T
C R Chronograph using a telegraph sounder

It is best however to eliminate all batteries by using rectified AC voltage stabilized by ballast tubes, voltage regulating tubes, or grid glow tubes (Chapter 9).

Appendix 14–1

Atomic weights of the lighter nuclei as computed by H. Bethe from transmutation energies, referred to $O^{16} = 16$ (*Phys. Rev.*, Vol. 43, p. 634, 1935). It will be noted that the old value for He as determined by Aston was 4.00216; his new (May, 1935) value is 4.0041.

	$Be^8 = 8.0082$
$n^1 = 1.0085$	$Be^9 = 9.0135$
$H^1 = 1.00807$	$B^{10} = 10.0146$
$H^2 = 2.01423$	$B^{11} = 11.0111$
$H^3 = 3.0161$	$C^{12} = 12.0037$
$He^3 = 3.01699$	$C^{13} = 13.1069$
$He^4 = 4.00336$	$N^{14} = 14.0076$
$Li^6 = 6.01614$	$N^{15} = 15.0053$
$Li^7 = 7.01694$	$O^{17} = 17.004$

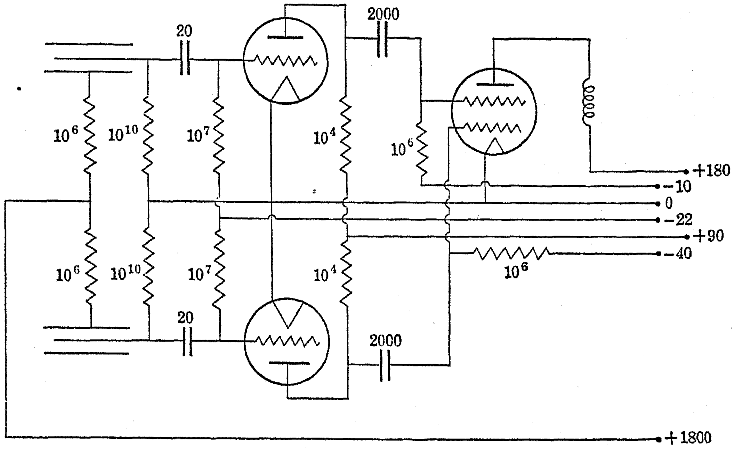


FIG. App. 13–2 a. A simple coincidence counter. The voltages depend on the tubes used. Capacities are in mmf.

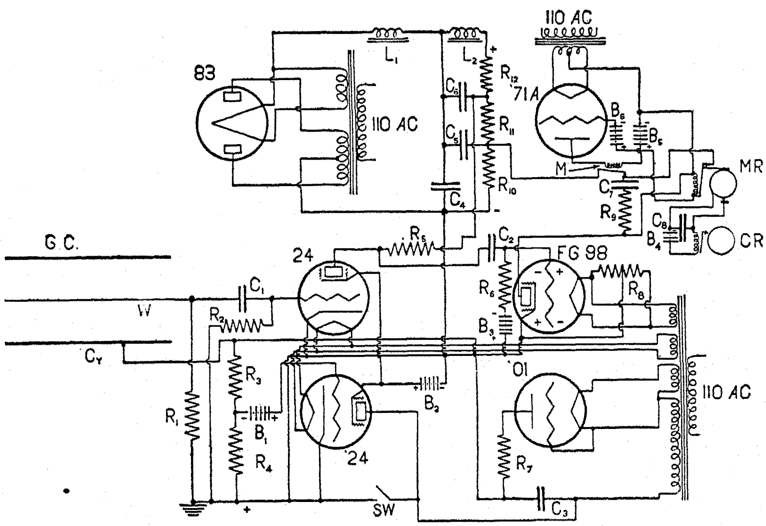


FIG. App. 13–2 b. A circuit for counting and continuously recording cosmic rays. Suggested values for the circuit are given on page 442. Notice that the high potential (up to 2000 volts) necessary for the Geiger Counter is obtained by rectifying a high voltage AC by a simple tube ('01). The lower '24 tube reduces fluctuations in voltage to very small values.

Appendix 15–1

Schroedinger's Equation.

Let us consider the motion along a stretched string, the tension being T, and the mass per unit length ρ. At a certain point the displacement y is given by the relation

$$\frac{d^2y}{dt^2} = \frac{T}{\rho}\frac{d^2y}{dx^2} = v^2\frac{d^2y}{dx^2} \qquad \text{where} \qquad v = \sqrt{\frac{T}{\rho}}\,.$$

The solution is $y = f_1(x - vt) + f_2(x + vt)$. The first member represents a disturbed form $y = f_1(x)$ moving along the x axis with velocity v; the other member represents the form $f_2(x)$ moving in the opposite direction. If we simplify the form and impose the boundary conditions that $y = 0$ for $x = 0$ and $x = L$ where $L =$ the length of the string, then the solution takes the form

$$y = A \sin\frac{\pi nvt}{L}\sin\frac{n\pi x}{L} \qquad \text{where} \qquad n = 1, 2, \cdots.$$

This illustrates the point that the solution of an equation may be restricted by boundary conditions and that a series of similar forms may be obtained characterized by integers—or by definitely related numbers.

If in the preceding discussion we confine our attention to one particle of the string, it will have a periodic motion (let us assume that it is simple) given by the simple harmonic motion equation $d^2y/dt^2 = -k^2y$, where the period $P = 2\pi/k$ or $k = 2\pi f = 2\pi(v/\lambda)$, f being the frequency and λ the wave length. We can then change the first equation to the form

$$\frac{d^2y}{dx^2} + \frac{k^2}{v^2}y = 0 \qquad \text{or} \qquad \frac{d^2y}{dx^2} + \frac{4\pi^2}{\lambda^2}y = 0.$$

Let us now impose the de Broglie relation, $\lambda = h/mv$. Since $\frac{1}{2}mv^2 = E - V$, E being the total and V the potential energy, we have

$$\lambda = \frac{h}{\sqrt{2m(E - V)}} \qquad \text{and} \qquad \frac{d^2y}{dx^2} + \frac{8\pi^2m(E - V)}{h^2}y = 0.$$

This is Schroedinger's equation.

It is true that in the original relation we regarded λ as the wave length of a gross vibrating body, a string, and in the de Broglie relation we contemplate application to a minute particle, but we are justified in carrying ideas over from the gross to the microscopic realm, at least provisionally.

The Schroedinger equation has the favorable quality that *time* is not directly involved. It seeks to determine the relations between the allowed energy states of an electron or atom in terms of space. The general form of the Schroedinger equation is

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\,\pi^2 m(E - V)}{h^2}\psi = 0$$

or

$$\nabla^2\psi + \frac{8\,\pi^2 m(E - V)}{h^2}\psi = 0.$$

We desire to transform this to spherical coordinates. Now $\nabla^2\psi$ represents the flow or integral of $d\psi/dx$, $d\psi/dy$, $d\psi/dz$, into and out of the volume $dxdydz$ through the faces $dydz$, $dxdz$, $dxdy$. We find the same sum for the elementary volume bounded by the rectangular sides dr, $rd\theta$, $r \sin \theta d\phi$. The elementary volume is then $r^2 \sin \theta dr d\theta d\phi$. The areas at right angles to these faces are $r^2 \sin \theta d\phi d\theta$, $r \sin \theta dr d\phi$, $r dr d\theta$. The rates of flow, corresponding to $d\psi/dx$, $d\psi/dy$, $d\psi/dz$, are

$$\frac{d\psi}{dr}, \qquad \frac{1}{r}\frac{d\psi}{d\theta}, \qquad \frac{1}{r \sin \theta}\frac{d\psi}{d\phi}.$$

Hence the flow into the volume is

$$r^2 \sin \theta \frac{d\psi}{dr} d\theta d\phi, \qquad r \sin \theta \frac{1}{r}\frac{d\psi}{d\theta} dr d\phi, \qquad \frac{1}{r \sin \theta}\frac{r d\psi}{d\phi} dr d\theta.$$

Then the excess flow out is the sum of the variations of these with regard to the necessary variable. This excess is

$$\left[\sin \theta \frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) + \frac{d}{d\theta}\left(\sin \theta \frac{d\psi}{d\theta}\right) + \frac{1}{\sin \theta}\frac{d^2\psi}{d\phi^2}\right] dr d\theta d\phi.$$

We must divide by the volume. Then we have

$$\nabla^2\psi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) + \frac{1}{r^2}\sin \theta \frac{d}{d\theta}\left(\sin \theta \frac{d\psi}{d\theta}\right) + \frac{1}{r^2 \sin^2 \theta}\frac{d^2\psi}{d\phi^2}.$$

Since the picture of the hydrogen atom is still that of a nucleus Ze and electron e at distance r, then the potential energy is $- Ze^2/r$ and the Schroedinger equation becomes

$$\nabla^2\psi + \frac{8\,\pi^2 m}{h^2}\left(E + \frac{Ze^2}{r}\right)\psi = 0.$$

The three variables r, θ, ϕ, are independent. We therefore assume that the solution takes the form of $\psi = R.\,\Theta.\,\Phi$, where R is a function

of r only, etc. The equation then takes the form

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Theta}\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)$$
$$+\frac{1}{\Phi}\sin^2\phi\frac{d^2\Phi}{d\phi^2}+\frac{8\,\pi^2 m}{h^2}\left(E+\frac{Ze^2}{r}\right)r^2=0.$$

We separate this into r, θ, ϕ, members. The simplest is

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=C_1 \qquad \left(\text{since } \sin^2\theta \text{ is constant for } \phi\right).$$

Now ϕ may take any value from 0 to $2\,\pi$ and the solution must be periodic in ϕ. Hence C_1 must have a value of $-\,m^2$ and we have the solution in the form $\Phi = A \sin m\phi$, m corresponding to the magnetic quantum member. The second equation now becomes

$$\frac{1}{\Theta}\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)=C_2.$$

The solution now is more involved. Here we can merely state that solutions are possible only when $C_2 = -\,l(l+1)$, l corresponding to the azimuthal quantum number. Again we replace the constants in the original equation and find

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\left(A+2\frac{B}{r}+\frac{C}{r^2}\right)R=0$$

where

$$A=\frac{8\,\pi^2 m}{h^2}E, \qquad B=\frac{4\,\pi^2 mZe^2}{h^2}, \qquad C=-\,l(l+1).$$

For the case $E < 0$. Now we find that solutions are possible when

$$E=-\,\frac{2\,\pi^2 me^4Z^2}{n^2h^2} \qquad \text{where} \qquad n=1,2,3,\cdots.$$

These are the energies of the various orbits as obtained by Bohr. But we look upon them as the energies of the steady states. They are determined as the values of a constant occurring in a differential equation, values of a constant which allow possible solutions. No orbits are involved—though they may exist. No picture is necessary, though there are implied probable positions of the electron. These probable positions are pictured in Fig. 15–6.

The above is just an outline of the method of dealing with the Schroedinger equation. For optical details see *Introduction to Atomic Spectra* by H. E. White.

Appendix 15–2

The extraordinary contrast in points of view as between the electromagnetic nature of light and the quantum nature is set forth admirably in the two following songs. An electron struggling to get free from an atom causes the ether to tremble, ordinary light results; or rushing against a plate it causes the ether's pulse to agitate—an X-ray is produced. But then we might sum it all up in "$h\nu$."

THE REVOLUTION OF THE CORPUSCLE

By A. A. R.

A corpuscle once did oscillate so quickly to and fro,
He always raised disturbances wherever he did go.
He struggled hard for freedom against a powerful foe—
An atom—who would not let him go.
The aether trembled at his agitations
In a manner so familiar that I only need to say,
In accordance with Clerk Maxwell's six equations
It tickled people's optics far away.
    You can feel the way it's done,
    You may trace them as they run—
$d\gamma$ by dy less $d\beta$ by dz is equal $K \cdot dX/dt$.
    . . . .
    . . . .
While the curl of (X, Y, Z) is the minus d/dt of the vector (a, b, c).

Some professional agitators only holler till they're hoarse,
But this plucky little corpuscle pursued another course,
And finally resorted to electromotive force,
Resorted to electromotive force.
The medium quaked in dread anticipation,
It feared that its equations might be somewhat too abstruse,
And not admit of finite integration
In case the little corpuscle got loose.
    For there was a lot of gas
    Through which he had to pass,
    And in case he was too rash,
    There was sure to be a smash,
    Resulting in a flash.
Then $d\gamma$ by dy less $d\beta$ by dz would equal $K \cdot dX/dt$.
    . . . .
    . . . .
While the curl of (X, Y, Z) would be minus d/dt of the vector (a, b, c).

The corpuscle radiated until he had conceived
A plan by which his freedom might be easily achieved,
I'll not go into details, for I might not be believed,
Indeed, I'm sure I should not be believed.
However, there was one decisive action.
The atom and the corpuscle each made a single charge,
But the atom could not hold him in subjection
Though something like a thousand times as large.

The corpuscle won the day
And in freedom went away
And became a cathode ray.
But his life was rather gay,
And he went at such a rate,
That he ran against a plate;
When the aether saw his fate
Its pulse did palpitate.
And $d\gamma$ by dy less $d\beta$ by dz was equal $K \cdot dX/dt$.

. . . .

. . . .

While the curl of (X, Y, Z) was the minus d/dt of the vector (a, b, c).

"$h\nu$"

(Pronounce "h new". In the symbols used in this text $h\nu = hf$.)

All black body radiations,
All the spectrum variations,
All atomic oscillations
Vary as "$h\nu$."

CHORUS

Here's the right relation,
Governs radiation,
Here's the new
And only true
Electrodynamical equation;
Never mind your d/dt^2,
Ve or half mv^2
(If you watch the factor "c^2")
's equal to "$h\nu$."

Ultraviolet vibrations,
X and gamma ray pulsations,
Ordinary light sensations
All obey "$h\nu$."

Even in matters calorific,
Such things as the heat specific
Yield to treatment scientific
If you use "$h\nu$."

In all questions energetic,
Whether static or kinetic,
Or electric, or magnetic,
You must use "$h\nu$."

There would be a mighty clearance,
We should all be Planck's adherents,
Were it not that interference
Still defies "$h\nu$."

G. S.

Appendix 15-3

To show that mathematicians are also guilty, we give you

"π TO THIRTY PLACES"

(The Number of Letters in the Words Gives the Successive Integers)

Que j'aime à faire apprendre un nombre utile aux sages!
Immortel Archimède, artiste ingénieur,
Qui de ton jugement peut priser la valeur?
Pour moi, ton problème eut de pareils advantages.

Appendix 16-1

An electron is driven towards an anode with a potential of 2000 volts, accurate to $\Delta V = 1$ volt. What is the greatest accuracy with which its position, Δx, may be measured at that potential?

The energy, $\frac{1}{2}\,mv^2 = 3.2 \times 10^{-9}$ ergs (since 1 e.v. $= 1.6 \times 10^{-12}$). Then $v = \sqrt{(6.4 \times 10^{-9}/9 \times 10^{-28})} = 2.7 \times 10^9$ cm./sec. The fractional accuracy in voltage, therefore in energy, is 1/2000; the fractional accuracy in v is then 1/4000. Hence $\Delta v = 6.8 \times 10^5$ cm./sec. Then $\Delta x = 6.5 \times 10^{-27}/9 \times 10^{-28} \times 6.8 \times 10^5 = 1 \times 10^{-5}$ cm. Problems similar to this may be stated in regard to a proton of certain voltage or of a rifle bullet at a certain point in its flight. It will be seen that as the mass increases, the uncertainty of position or velocity decreases.

Appendix—General

NOBEL PRIZES

The Swedish scientist, Alfred B. Nobel, inventor of dynamite, died on December 10, 1896, bequeathing \$9,000,000, the interest of which should be distributed yearly to those who during the preceding year had made the most important contributions in the fields of physics, chemistry, medicine, literature, and peace. The prize amounts to about \$41,000. Below are given the physics and chemistry awards to physicists. The nationality of the recipient is denoted by the letter after the name.

The reader should be able to name and expound the contribution made by each recipient.

1901	Roentgen (G)
1902	Lorentz (H)
	Zeeman (H)
1903	Becquerel (F)
	P. and M. Curie (F)
1904	Lord Rayleigh (E)
	Sir William Ramsay (chem.) (E)
1905	Lenard (G)
1906	J. J. Thomson (E)
1907	Michelson (A)
1908	Lippmann (F)
	Rutherford (chem.) (E)

1909	Marconi (It.)
	Braun (G)
1910	Van der Waals (H)
1911	Wien (G)
	M. Curie (chem.) (F)
1912	Dalén (Swe)
1913	Onnes (H)
1914	Laue (G)
1915	W. H. Bragg (E)
	W. L. Bragg (E)
1917	Barkla (E)
1918	Planck (G)
1919	Stark (G)
1920	Guillaume (Swi)
1921	Einstein (G)
	Soddy (chem.) (E)
1922	Bohr (D)
	Aston (chem.) (E)
1923	Millikan (A)
1924	Siegbahn (Swe)
1925	Frank (G)
	Hertz (G)
1926	Perrin (F)
1927	A. H. Compton (A)
	C. T. R. Wilson (E)
1928	Richardson (E)
1929	L. V. de Broglie (F)
1930	Raman (Ind)
1932	Heisenberg (G)
	Langmuir (chem.) (A)
1933	Dirac (E)
	Schroedinger (G) and (E)
1934	Urey (chem.) (A)
1935	Chadwick (E)
	M. Joliot, Mme. Curie-Joliot (chem.) (F)

PHYSICAL CONSTANTS

(See also Tables in Chapter 1)

Velocity of light	$c = (2.99796 \pm 0.00004) \times 10^{10}$ cm. sec.$^{-1}$
Gravitation constant	$G = (6.664 \pm 0.002) \times 10^{-8}$ dyne cm.^{2} g.$^{-2}$
Volume of 1 mole	$v = 22.414 \pm 0.007$ liters mole^{-1}
Faraday constant	$F = 96{,}490$ coulombs
Electronic charge	$e = (4.770 \pm 0.005) \times 10^{-10}$ e.s.u.
	$= 1.591 \times 10^{-20}$ e.m.u.
Mass of electron (average)	$m_0 = 9.02 \times 10^{-28}$ gm.
Ratio of charge to mass of electron	$e/m = 1.765 \times 10^{7}$ e.m.u. g.$^{-1}$
Ampere second deposits	$(1.1180) \times 10^{-3}$ gm. of silver
Planck's constant	$h = (6.547 \pm 0.008) \times 10^{-27}$ erg sec.
Rydberg constant for hydrogen	$R_{\mathrm{H}} = 109{,}677.76$
Rydberg constant for helium	$R_{\mathrm{He}} = 109{,}722.40$

PHYSICAL CONSTANTS (*Continued*)

Rydberg constant for infinite mass	$R_\infty = 109{,}737.42$
Grating space of calcite (20°)	$d = 3.0279 \times 10^{-8}$ cm.
Avogadro's number	$N = 6.064 \times 10^{23}$
Number of molecules of a perfect gas in 1 cm.³ at 0° C. and 1 atmosphere	$= (6.064 \times 10^{23})/22{,}413.5$
	$= 2.705 \times 10^{19}$
Ratio of mass H atom/electron	
(spectroscopic)	$= 1840$
(deflection)	$= 1848$
1 electron volt (e.v.)	$= 1.591 \times 10^{-12}$ erg
	$= 11{,}606$ degrees C. or K.
1 M.E.V. (10^6 e.v.)	$= 0.00107$ statom
For photons	$\lambda = \dfrac{12{,}336}{V}$ Å (V in volts)
For electrons	$\lambda = \sqrt{\dfrac{150}{V}}$ Å (V in volts)

INDEX

(Starred names are Nobel prize men. See pp. 449–450.)

Allis, 190
Alphas, complex, 252
Aluminum, radioactive, 308
Alvarez and Compton, 295
Amplification factor, 153, 155
Anderson, C. D., 297
Anderson, Capt., 402
Antineutrino, 332
*Aston, 33
Atom building, 327
Audiometer, 383
Avogadro, 21
Avogadro's number, 13, 20, 411

Bainbridge, 31
Ballard, 440
Balmer series, 86
Band spectra, 219, 231
Barkhausen-Kurtz, 156
  oscillations, 431
*Barkla, 111
Becker, J. A., 144
*Becquerel, 236
Beta ray spectra, 250, 254
Beth, 57
Birge, 20
Black body radiation and perfect gas, 419
Blackett, 257
Blackwell, O. B., 381
*Bohr, Chapter 6
Bohr-Sommerfeld orbits, 95
Bonner, 314
Bothe and Becker, 262
Bowen, 303
*Bragg, W. H. and W. L.
Bragg, 109, 120
Bragg's spectrometer, 131
*Braun
Brickwedde, 264
Bridgman, 203
*de Broglie, Louis, 338, 348
Brownian movement, 10
  Einstein equation, 412
Brubaker, 314
Burton, 201, 277

Cadmium, absorber of neutrons, 319, 321
Cathode ray oscillograph, 165
*Chadwick, 236, 262
Chlorine isotopes from band spectra, 438
Clark, G. W., 340
Clay, 281, 288, 289, 295
Closest packing, 110
Cloud tracks, *Frontispiece*, 183
Cockcroft, 267, 314
Collisions, elastic, 256
*Compton, A. H., 280
Compton effect, 120, 427, 451
Compton, K. T., 185
Compton-Stephenson, 303
Condenser discharge, 408
Contact difference of potential, 81
Cooke, 277
Coolidge, 106
Cosmic ray particle, Chapter 13, 441
Cosmic ray telescope, 285
Crane, 314, 335
Crooks, Sir William, 22
Cuprous oxide cell, 85
*Curie, P. and Mme., 236
Curie unit, 237
Currents without batteries, 147
Cyclotron, 268

Dahl, 273
*Dalen
Dalton, 21
Davisson, 141, 399
Debye, 132
Decay, law of, 244
Dee, 313, 314, 315
Delsasso, 314, 315
Dempster, 34
Deuteron as projectile, 313, 328
Diffraction of electrons, 360

Diffraction of light, 359
*Dirac, 140
Dunning, 319, 321

Edison effect, 138
*Einstein, 62
  equation, 70
  theory of Brownian movement, 11, App. 1–8
Eisenhut and Kaupp, 341
Elastic collisions, 174
Electric waves discovered, 54
Electromagnetic units, 67
Electron discovered, 23
Electron optics, 396
Electron spin, 100
Electrostatic units, 67
Element 93, 323
Ellis, 236
Energy, conservation of, 1
  loss of kinetic, 4
Eötvös, 389
Equal areas, law of, 414
Errors, distribution of, 6
Ether string theory of light, 108
Evans, R. D., 329
Eve and Keys, 390
Excitation potentials, 175

Faraday effect, 204
Fermat, 338
Fermi, 322
Fermi-Dirac law, 140
Fine structure constant, 423
Fink, 319, 321
Fitzgerald-Lorentz shortening, 369
Fowler, 314, 315
*Frank, H., 174

Gamma rays as projectiles, 328
Gas-filled tubes, 159
Geiger, 47
Geiger-Müller counter, 240, 442
Geophysics, 388
Gerlach, 209
Germer, 141, 339
Glockel, 402
Glow lamps, 187
Gravity apparatus, 387
Grayson-Smith, 198
Grondahl, 85
Group velocity, 49, 347
*Guillaume

Hafstad, 273
Half-life, 245
*Heisenberg
Helium compared with hydrogen, 35
Helium, liquid, 201
  vapor pressure of, 435
*Hertz, G., 174
Hertz, Heinrich, 54, 65
Hess, 402
Howell, 281
Hull, A. W., 132, 340
Hull, G. F. (author), 54, 214, 295
Hull, G. F., Jr., 218
Hydrogen, atom pictures, 356
  heavy, discovery, 264
  heavy, spectrum, 440

Ignitrons, 161
Illinium, discovered, 112
Infra red absorption, 220
Intensity meter, 384
Ionization, by collisions, 174
  by X-rays, 173
  due to α rays, 243
  potentials, 176
Isotope of neon, discovery, 32

Jeans, 61
Johnson, 295
*Joliot, M. and Mme., 261, 307, 333, 335

Keesom, 202
Kempton, 314
Kent, R. H., 386
Keys, 390
Killian, 191
Kinsey, 314
Knipping, 340
Kolhörster, 277, 286, 402
Konel metal, 143
Korff, 295
Kurie, 320

Lamar, 190
*Langmuir, 185, 189
Larmor, 205
*Laue, 109
Lauritsen, 314, 315, 335
Lawrence, 268, 317
Least action, 338
Least time, 338
Lebedew, 54

Lees, 257
Lehmann, 340
Lemaitre and Vallarta, 291
*Lenard, 66
Limitations imposed by nature, 358
*Lippmann
Livingston, 268, 329
*Lorentz, H. A., 128, 205
Lorentz equations, 370

*Marconi
Marsden, 47
Mass, and energy, 425
  increases with speed, 29
  rest, 30
Materialization of energy, 333
Maxwell's theory of light, 54
McLennan and Burton, 277
Mean Free Path, 15, 19
Meitner and Philipp, 308
Mercury arc rectifiers, 160
Merritt, 66
*Michelson, 50, 51, 87
Michelson-Morley experiment, 365
Miller, D. C., 366
*Millikan, 25, 27, 71, 278, 402
Mitchell, 319, 321
Models of molecules, 230
Modified line in X rays, 121, 123
Molecule, diameter of, 15, 19
Momentum, conservation of, 3
Morley, 365
Morse, Allis and Lamar, 190
Moseley, 29, 111
Mott-Smith and Howell, 281
Murphy, 264
Mutations by X rays, 395

Neher, 284
Neutrino, 332
Neutron, discovery, 261
  properties, 263
Neutrons as projectiles, 319
Newton, *Optiks*, 374
  quantity of motion, 3
Nichols, E. F., 54
Nobel prize men, 449–450
Nuclear atom discovered, 46
Nucleus, barriers, 324
  density of gold, 328
  energy levels, 253
  protons and neutrons, 326
  resonance levels, 325

Oil drop experiment, 26
Oliphant, 314
*Onnes, 202
Orbit, circular about earth, 38
  elliptic, 39
  hyperbolic, 44
  parabolic, 43
Orbit, hyperbolic, energy, 416
Orbit models, 97
Oscillators, 153, 156, 159

Packing fraction, 35, 333
Parabola method, 32
Particles as waves, 339
Pegram, 319, 321
*Perrin, 12, 28
Phosphorus, radioactive, 308
Photoelectric cell amplifier, 167
Photoelectric sensitivity, 82, 83
Photoelectrons, maximum speed, 68
  number of, 74
Photo-rectifier, 85
Photo-resistant cell, 84
Photo-voltaic cell, 84
Photon, dimensions?, 78
  model, 80
  picture, 74, 75
  polarized, 79
  properties, 130
  split?, 77
Photons vs. electrons, 345
Piccard, 402
Pickering series, 92
Piezo-electric effect, 385
"Pilfering" collisions, 180
*Planck, 61
Planck's law, 420
Positron, discovery, 297
Potential barrier, 137
Powdered crystal method, 132
Pressure of light, 54
Prism, constant deviation, 69
Pulse, theory of X-rays, 107, 116
"Pumpkin" atom, 28, 45
Push pull oscillators, 158

Quantization of space, 209
Quantum theory, beginning of, 61

Radiation due to temperature, 58
Radioactivity, artificial, 307
  tables, 247

Radon, 237
*Raman, Chapter 10
  spectrum, 223
*Ramsay
Randall, 340
*Rayleigh
Rayleigh-Jeans law, 61
Recoil electrons, 124
Recombination of ions, 172
Rectifiers, 149
Regener, 279, 301, 402
Relativity, theory of, 371, 373
  theory, new, 377
Resistance, electrical, 196, 197
Resonance collisions, 174
*Richardson, O. W., 136
*Roentgen, 105
Rossi, 295
Rotational energies, 220
Rowland, 87
Royds, 239
*Rutherford, 29, 46, 314
  and Cooke, 277
  and Royds, 239
  discovery of transmutation, 259
Rydberg, 86
  constants, 93

Saturation currents, 149
Scherrer, 132
*Schroedinger, 350
  Equation, 350, 444
Screen grid, 154
Seismic methods, 391
Selenium cell, 84
Shankland, 451
Sheathed cables, 380
Showers, cosmic ray, 299
*Siegbahn, 113
Smoke eradicator, 171
*Soddy, 249
Sodium, radioactive, 316
Soldner, 374
Space charge, 148
  equation, 430
Specific heat of a gas, 233
Spectra, ionized atoms, 101, 102
Spectral line shift, 372
Spectroscopic atom, 103
Speed, root-mean-square, 7
Speeds, distribution of, 6
*Stark
Stark effect, 194, 436

Statom, unit of atom mass, 312
Stefan-Boltzmann, 63
Stephenson, 303
Stern, 209
Stevens-Anderson, 402
Stevenson, 295, 300
Stewart, 66
Stokes, 26, 27
Stokes' law, correction, 412
Stoney, Johnston, 22
Stopping potential, 73
Street, 300
Superconductivity, 197
Surface film, influence of, 144
Sweep circuit, 166

Tarr, 198
Telescope for cosmic rays, 285
Television, 382
Temperature, significance of, 9
Thermions, 136
Thibaud, 401
Thomson, G. P., 339, 342
*Thomson, Sir J. J., 23, 32, 192
Threshhold frequencies, 72
Thyratrons, 162
Torque in a beam of light, 424
Townsend, 25
Trillat and Hirsch, 341
Tuve, Hafstad, and Dahl, 273

Uncertainty principle, 362
Uranium, slow and fast, 322
*Urey, 264

Vallarta, 291
Van de Graaff, 268
*Van der Waals
Velocities, distribution of, 6
Velocity selector, 31
Vibration-rotation, 221, 227
Viscosity, 17
Voltage regulators, 188

Walton, 267, 314, 315
White, 355, 440
*Wien, 64
Williams, Dr. H. B., 239
*Wilson, C. T. R., 25
Wilson, H. A., 25, 137
Wood, R. W., 88
Woodward, 300

X-rays, 105
  absorption, 116
  application, 395
  fluorescent, 119
  general, 114
  polarized, 126
  refracted, 127
  scattered, 115

Young, Dr. Thomas, 52

Zartman, 9
*Zeeman, 204, 205